# Force & Translational Dynamics

AP Physics C: Mechanics ▪ Unit 2

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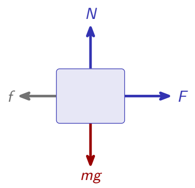
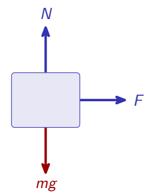





## What we will cover






# 1 Center of mass



## Systems and center of mass

A **system** 系统 moves as if all its mass sat at its **center of mass** 质心:

**Discrete and continuous**

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1}{M}\int x\,dm$$

Only the **net external force** 净外力 accelerates the center of mass – internal forces cannot.

**Worked example – a uniform rod**

With $\lambda = M/L$, $dm = \lambda\,dx$:

$$x_{cm} = \frac{1}{M}\int_0^L x\,\lambda\,dx = \frac{L}{2}$$

The integral confirms it balances at the middle.

# 2 Newton's laws



## Free-body diagrams and Newton's laws

Draw every force on a **free-body diagram** 受力图, resolve into components, add to the **net force** 合力.

**The three laws**

1st: inertia $\vec{F}_{net} = m\vec{a}$

3rd: equal-opposite pairs, different bodies

*Rocket force*

## Worked example – two bodies, one string

**A $6.0$ kg cart on a frictionless table is pulled by a light string over a pulley to a hanging $2.0$ kg mass. Find $a$ and the tension.**

- treat them as one system: $a = \dfrac{m_2 g}{m_1 + m_2} = \dfrac{(2.0)(9.8)}{8.0} = 2.45\ \text{m/s}^2$
- then one body alone for $T$: cart, $T = m_1 a = 14.7$ N
- check with the hanging mass: $m_2 g - T = 19.6 - 14.7 = 4.9 = m_2 a$ ✓

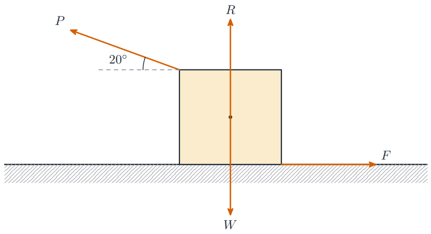
The string constraint – same $|a|$, same $|T|$ – is what lets you do this. Solve for $a$ with the whole system, then use one body to get an internal force.

## Newton's laws, one axis at a time

| | |
|---|---|
| third law 牛顿第三定律 | A pushes B, B pushes back equally and oppositely. The pair acts on different objects, so it never cancels inside one free-body diagram |
| second law | $\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$ for constant mass – apply it one axis at a time |
| when $F$ depends on $v$ | $m\,dv/dt = F(v)$ is a differential equation; separate and integrate |
| translational equilibrium 平动平衡 | $\sum \vec{F} = 0$, so $\vec{a} = 0$ – at rest *or* at constant velocity |
| normal force 法向力 | perpendicular to the surface; it is whatever it needs to be, and is not always $mg$ |

An action–reaction pair on one diagram is the commonest error in the unit. If both arrows are on the same body, one of them does not belong there.

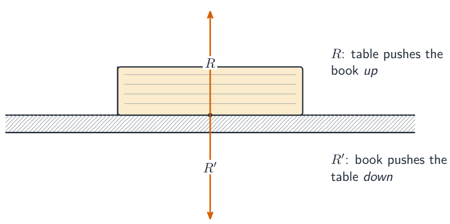
## Forces and Free-Body Diagrams




A free-body diagram shows every force acting on one object



## A Newton's third-law pair




A Newton's third-law pair: equal and opposite forces on two different objects

# 3 Gravity



## Gravitation

**Universal gravitation**

$$F_g = \frac{G m_1 m_2}{r^2}, \qquad g = \frac{GM}{r^2}$$

- **mass** 质量 is intrinsic; **weight** 重量 $= mg$
- a sphere pulls as if all mass sat at its centre
- $g$ falls off as $1/r^2$ with **altitude** 海拔

**Worked example**

At one Earth radius up, $r$ doubles:

$$g = \frac{9.8}{2^2} = 2.45\,\text{m/s}^2$$

The gentler pull is what lets satellites orbit high up.

# 4 Friction & springs



## Friction and spring forces

**Friction**

$$f_s \le \mu_s N, \qquad f_k = \mu_k N$$

Depends on $N$ and $\mu$ – not on the contact area.

**Hooke's law**

$$F_s = -kx$$

Combining springs: parallel adds ($k_{eq} = k_1 + k_2$, stiffer); series is softer ($\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$).

Two $100\,\mathrm{N/m}$ springs: parallel $200$, series $50\,\mathrm{N/m}$.



## Worked example – a block sliding down a rough incline

**A block slides down a $30^\circ$ incline with $\mu_k = 0.20$. Find its acceleration.**

- perpendicular: $N = mg\cos 30^\circ$
- along: $ma = mg\sin 30^\circ - \mu_k mg\cos 30^\circ$
- $a = g(\sin 30^\circ - 0.20\cos 30^\circ) = 3.2\ \mathrm{m/s^2}$

The mass cancels – the answer is the same for a marble and a boulder. Note $N \neq mg$ on an incline; using $mg$ there is the standard lost mark.

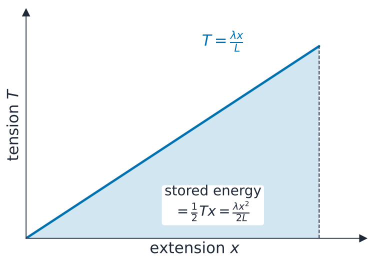


## Static and kinetic friction are not the same law

| | |
|---|---|
| **kinetic 动摩擦** | $f_k = \mu_k N$ – a fixed value while sliding, opposing the slide |
| **static 静摩擦** | $f_s \le \mu_s N$ – an inequality: it takes whatever value stops the slide, up to that maximum |
| **usually** | $\mu_s > \mu_k$, which is why a block is harder to start than to keep going |
| **on an incline 斜面** | resolve along and across the ramp: $N = mg\cos\theta$, and the driving component is $mg\sin\theta$ |

Writing $f_s = \mu_s N$ when nothing is on the point of slipping is wrong: static friction is only *at* its maximum at the instant of breaking away.

## Hooke's law




Hooke's law: extension is proportional to load up to the limit of proportionality



## Combining springs, and the sign that trips everyone

| | |
|---|---|
| **in parallel** | side by side, sharing the load: $k_{eq} = k_1 + k_2$ – stiffer than either |
| **in series** | end to end, sharing the stretch: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$ – softer than either |
| **the spring constant 弹簧常数** | $F = -kx$; the minus sign says the force is a restoring one, always back toward natural length |
| **the memory hook** | exactly the *opposite* of resistors – springs in parallel add, resistors in series add |

Two identical springs side by side hold twice the load at the same stretch; end to end they stretch twice as far under the same load.

# 5 Resistive forces



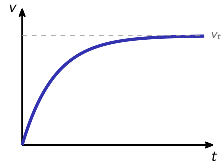
## Resistive forces and terminal velocity

Drag grows with speed ($F = -bv$), so Newton's law becomes a **differential equation** 微分方程:

**Falling with drag**

$$m\frac{dv}{dt} = mg - bv$$

At **terminal velocity** 终极速度, $\frac{dv}{dt} = 0$, so $v_t = \frac{mg}{b}$.




$v$ rises smoothly toward $v_t$ and levels off.



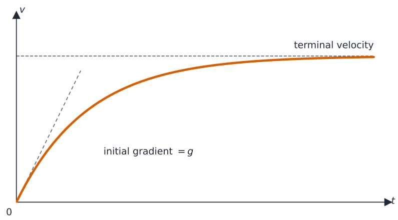
## Separating the variables, and why you must

| | |
|---|---|
| **the problem** | with drag, $m\frac{dv}{dt} = mg - bv$ – the acceleration depends on $v$, so no SUVAT equation applies |
| **separate the variables 分离变量** | $\int_0^v \frac{dv'}{mg - bv'} = \int_0^t \frac{dt'}{m}$ |
| **the result** | $v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$ |
| **terminal velocity** | set $dv/dt = 0$: $v_T = \frac{mg}{b}$ – reachable without solving the equation at all |

If a question asks only for the terminal value, set the acceleration to zero and stop. Integrate only when it asks for $v(t)$.



## An object falling through a fluid speeds




An object falling through a fluid speeds up to a terminal velocity

# 6 Circular motion



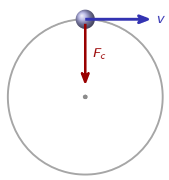
## Circular motion

**Uniform circular motion** 匀速圆周运动: steady speed, changing direction – so it accelerates toward the centre:

**Centripetal quantities**

$$a_c = \frac{v^2}{r}, \qquad F_c = \frac{mv^2}{r}$$

Non-uniform motion also has a **tangential** 切向 force that changes the speed.




A car needs $F_c$ from tyre friction – else it skids.

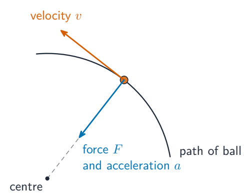
## Circular motion needs a net inward force

| | |
|---|---|
| **centripetal acceleration 向心加速度** | $a_c = \frac{v^2}{r} = \omega^2 r$, always pointing to the centre |
| **the force** | $F_c = \frac{mv^2}{r}$ – not a new force. It is the name for whatever the net inward force already is |
| **identify it** | tension, gravity, friction, the normal force – say *which*, every time |
| **Newton's law of gravitation 万有引力定律** | $F = \frac{Gm_1 m_2}{r^2}$ – the inward force for an orbit |

> There is no outward force. Writing 'centrifugal force' on a free-body diagram loses the mark; the sensation of being flung out is the body's inertia, not a force.



## The velocity points along the tangent; the

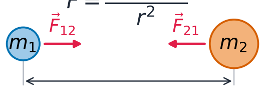



> The velocity points along the tangent; the centripetal force points to the centre



## Gravitational Force

$$F = \frac{G\,m_1 m_2}{r^2}$$




$\vec{F}_{12} = -\vec{F}_{21}$ (third-law pair; equal magnitude, opposite direction)

$F = Gm_1m_2/r^2 \;\cdot\; g = GM/r^2$ outside sphere

> Two masses attract each other with equal, opposite, inverse-square forces along the line joining them



## Resistive Forces

> Skydivers in freefall: drag grows with speed until it balances weight at terminal velocity



## Exam tips

- Locate the **center of mass** with $x_{cm} = \frac{1}{M}\int x\,dm$ (or $\frac{\sum m_i x_i}{\sum m_i}$ for point masses) and use $\lambda, \sigma, \rho$ for the mass element.
- The net external force moves the center of mass as if all mass sat there: $\vec{F}_{net} = M\vec{a}_{cm}$.
- Define your **system** clearly – internal forces cancel, so only external forces change its momentum.
- Exploit symmetry to shortcut a center-of-mass integral.
- Distinguish center of mass from center of gravity (identical in a uniform field).

# 7 Recap

## Unit 2 – key takeaways

**1 Center of mass**

$x_{cm} = \frac{1}{M}\int x\,dm$; moved only by external force

**2 Newton**

$\vec{F}_{net} = m\vec{a}$; third-law pairs act on different bodies

**3 Drag**

$m\,dv/dt = mg - bv$; terminal velocity $v_t = mg/b$

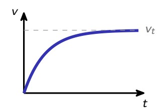
**4 Circular**

$a_c = v^2/r$ toward the centre; a role, not a new force



## Check yourself

1. What integral gives the center of mass of a continuous object?
2. At terminal velocity, what is the acceleration?
3. Two springs in parallel – stiffer or softer than one?
4. Rise altitude by one Earth radius – what happens to $g$?




**Answers** **1.** $\frac{1}{M}\int x\,dm$ **2.** zero **3.** stiffer **4.** it drops to one quarter