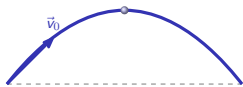


Kinematics

AP Physics C: Mechanics • Unit 1

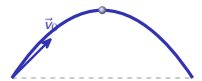
AP Physics C: Mechanics



Kinematics

What we will cover

- 1 Vectors, components, products
- 2 Velocity and acceleration as derivatives
- 3 Motion graphs
- 4 Reference frames and relative motion
- 5 Motion in two dimensions



1 Vectors

Kinematics

↳ Vectors

Vectors and components

Resolve every **vector** 矢量 onto the axes with **unit vectors** 单位矢量 $\hat{i}, \hat{j}, \hat{k}$:

Components

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Add vectors axis by axis to get the **resultant** 合矢量.



Speedometer

Vectors are added component by component

components 分量 resolve every vector onto the axes first: $v_x = v \cos \theta$, $v_y = v \sin \theta$

adding add the x's, add the y's – never add magnitudes

magnitude $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

direction $\theta = \tan^{-1}(v_y/v_x)$ – and check the quadrant against the signs of the components

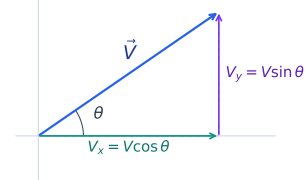
A **scalar** 标量 carries only size (mass, time, energy); a **vector** 矢量 carries size and direction. Half the sign errors in this course come from treating one as the other.

Kinematics

↳ Vectors

Resolving a vector into its x and y components

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2} \cdot V_x = V \cos \theta, V_y = V \sin \theta$$



Resolving a vector into its x and y components

Two ways to multiply vectors

Dot product → a scalar

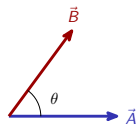
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \theta$$

Work is defined this way.

Cross product → a vector

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta$$

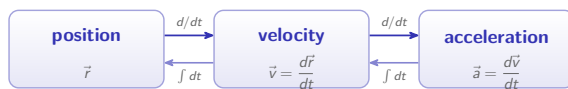
⊥ to both (right-hand rule). Torque and angular momentum use this.



2 Derivatives

Velocity and acceleration are derivatives

Differentiate to move down the chain; integrate to move back up:



So $\vec{a} = \frac{d^2 \vec{r}}{dt^2}$, and running it backwards: $\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a} dt'$ – valid even when a is not constant.

In this course, motion is calculus

the derivatives 导数 $v = \frac{dx}{dt}$, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

reversed $v = \int a dt$ and $x = \int v dt$, each with a constant fixed by the initial condition

average 平均 velocity $\Delta x / \Delta t$ over an interval – a chord on the graph

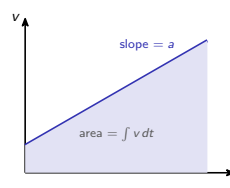
instantaneous 瞬时 velocity the derivative at one instant – the tangent

the SUVAT equations $v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2} at^2$, $v^2 = v_0^2 + 2a\Delta x$ – valid only for constant a

The most important habit in this unit: check whether a is constant before reaching for a SUVAT equation. If it is not, integrate.

3 Motion graphs

Motion graphs: slope and area



$x-t$: slope = $v = \frac{dx}{dt}$

$v-t$: slope = a ; area = $\int v dt = \Delta x$

Constant a (a special case)

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2} at^2$$

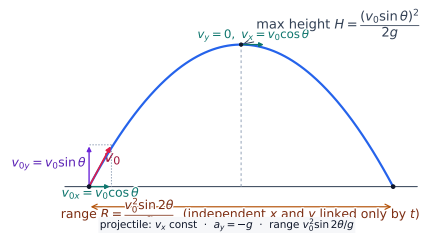
Worked example – differentiate, then integrate

(a) $x(t) = 2t^3 - 3t^2$ m: find v and a . (b) From rest at the origin with $a(t) = 6t$: find v and x at $t = 2$ s.

- (a) $v = \frac{dx}{dt} = 6t^2 - 6t$, $a = \frac{dv}{dt} = 12t - 6$
- (b) $v = \int 6t dt = 3t^2$,
 $x = \int 3t^2 dt = t^3$
- at $t = 2$: $v = 12$ m/s, $x = 8$ m

Slopes are derivatives; areas are integrals – the two are inverse operations. Both constants of integration are zero here only because the particle starts from rest at the origin.

Motion in Two or Three Dimensions – one



A projectile launched at an angle: velocity components, maximum height, and range

A projectile launched at an angle: velocity components, maximum height, and range

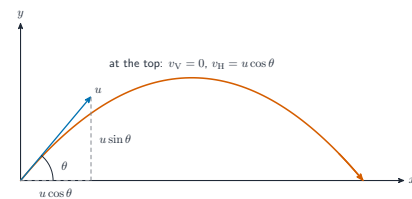
Worked example – motion in two dimensions

$\vec{r}(t) = (2t^2)\hat{i} + (4t - t^3)\hat{j}$ metres. Find $\vec{v}$ and $\vec{a}$.

- differentiate each component separately:
- $\vec{v} = 4t\hat{i} + (4 - 3t^2)\hat{j}$
- $\vec{a} = 4\hat{i} - 6t\hat{j}$

Components are independent: the $\hat{i}$ motion never enters the $\hat{j}$ derivative. That independence is the whole reason projectile motion splits into two one-dimensional problems.

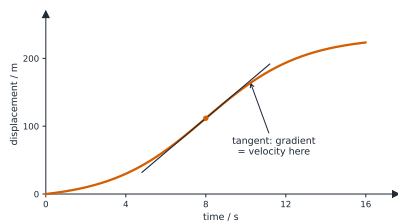
Motion in Two or Three Dimensions – two



A projectile launched at an angle: the horizontal and vertical motions are independent

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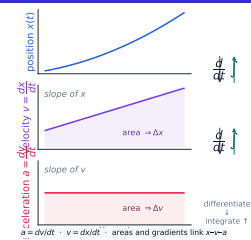
Representing Motion – one



A displacement-time graph: the slope at any instant is the velocity

A displacement-time graph: the slope at any instant is the velocity

Representing Motion – two



Position, velocity, and acceleration are linked by differentiation

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4 Relative motion

Kinematics
Relative motion

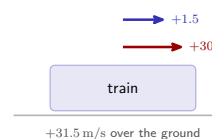
Reference frames and relative motion

Velocities add as vectors between frames:

Frame conversion

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

Frames moving at **constant** relative velocity share the same acceleration – they are **inertial** 惯性系, and Newton's laws hold identically.



Kinematics
Relative motion

Relative motion is vector addition

the rule $\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$ – the inner subscripts cancel

reversing $\vec{v}_{B/A} = -\vec{v}_{A/B}$

a boat crossing add the boat's velocity through the water to the water's velocity over the ground

reference frame 参考系 all motion is measured relative to one; there is no absolute rest

Write the subscripts before the numbers. If the inner pair does not cancel, you have chained the frames the wrong way round.

Kinematics
Relative motion

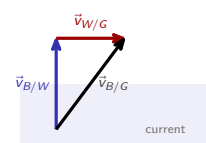
A boat crossing a river

The boat's ground velocity is the **vector sum** of its velocity through the water and the current:

$$\vec{v}_{B/G} = \vec{v}_{B/W} + \vec{v}_{W/G}$$

Across 3.0, downstream 2.0

$$|\vec{v}_{B/G}| = \sqrt{3.0^2 + 2.0^2} \approx 3.6 \text{ m/s}$$



5 Projectiles

Kinematics
Projectiles

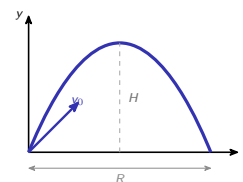
Projectile motion

The axes are **independent**, joined only by time t :

- horizontal: $a_x = 0$, so v_x constant
- vertical: $a_y = -g$ (free fall)

Eliminating t makes y a quadratic in x – the **trajectory** 轨迹 is a **parabola** 抛物线.

A dropped and a launched ball land **together**.



Projectile formulas

Fired from the ground at speed v_0 , angle θ :

Three key results

$$t_{\text{flight}} = \frac{2v_0 \sin \theta}{g}$$

$$H = \frac{v_0^2 \sin^2 \theta}{2g}, \quad R = \frac{v_0^2 \sin 2\theta}{g}$$

Range 射程 is greatest at $\theta = 45^\circ$; angles like 30° and 60° land at the **same** distance.

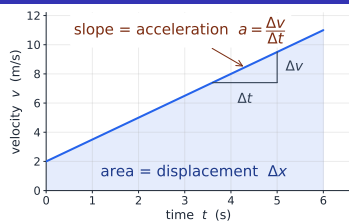
Uniform circular motion 匀速圆周运动:
 $a_c = \frac{v^2}{r}$ toward the centre – a preview of dynamics.

Displacement, Velocity, and Acceleration



A high-speed train: kinematics describes displacement, velocity and acceleration over time

On a velocity-time graph the gradient is



On a velocity-time graph the gradient is the acceleration and the area is the displacement

Exam tips

- Resolve every vector into components before adding – never add magnitudes at an angle directly.
- On Physics C you are expected to use **calculus**: velocity is $\vec{v} = \frac{d\vec{r}}{dt}$ and acceleration $\vec{a} = \frac{d\vec{v}}{dt}$; reverse with integration.
- Carry and check **units** and treat direction with signs (choose a positive axis and stick to it).
- Use the dot product for work-type quantities and the cross product for torque and angular momentum.
- Sketch the vectors – a diagram catches sign and direction errors the algebra hides.

6

Recap

Unit 1 – key takeaways

1 Vectors

components $\hat{i}, \hat{j}, \hat{k}$; dot $\rightarrow$ scalar,
cross $\rightarrow$ vector

2 Calculus

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$; integrate to
reverse

3 Relative

$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$; inertial frames

4 Projectiles

independent axes; range peaks at
 45°

Check yourself

- 1 What operation takes velocity back to position?
- 2 Dot product or cross product – which gives a scalar?
- 3 Two frames move at constant relative velocity – do they measure the same acceleration?
- 4 Which launch angle gives the greatest range?



Answers 1. integration 2. the dot product 3. yes 4. 45°