Kinematics

AP Physics C: Mechanics ▪ Unit 1

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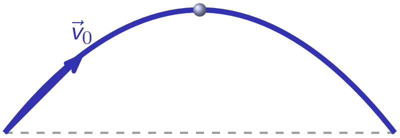
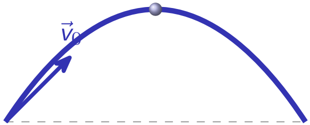


What we will cover

1. Vectors, components, products
2. Velocity and acceleration as derivatives
3. Motion graphs
4. Reference frames and relative motion
5. Motion in two dimensions



1 Vectors

Vectors and components

Resolve every **vector** 矢量 onto the axes with **unit vectors** 单位矢量 $\hat{i}, \hat{j}, \hat{k}$:

Components

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Add vectors axis by axis to get the **resultant** 合矢量.

Speedometer

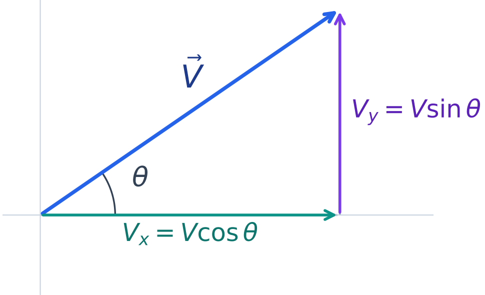
Vectors are added component by component

components 分量	resolve every vector onto the axes first: $v_x = v\cos\theta,\ v_y = v\sin\theta$
adding	add the x's, add the y's – never add magnitudes
magnitude	$\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2 + v_z^2}$
direction	$\theta = \tan^{-1}(v_y/v_x)$ – and check the quadrant against the signs of the components

> A **scalar** 标量 carries only size (mass, time, energy); a **vector** 矢量 carries size and direction. Half the sign errors in this course come from treating one as the other.

Resolving a vector into its x and y components

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2} \quad \cdot \quad V_x = V\cos\theta,\ V_y = V\sin\theta$$



Resolving a vector into its x and y components

Two ways to multiply vectors

Dot product → a *scalar*

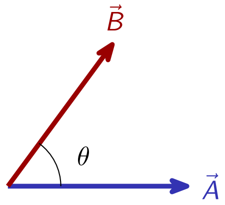
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Work is defined this way.

Cross product → a *vector*

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

$\perp$ to both (right-hand rule). Torque and angular momentum use this.



2

Derivatives

Velocity and acceleration are derivatives

Differentiate to move down the chain; integrate to move back up:

$$\begin{tikzcd}
\begin{array}{c}\textbf{position}\\ \vec{r}\end{array} \arrow[r, shift left, "d/dt"] & \begin{array}{c}\textbf{velocity}\\ \vec{v} = \dfrac{d\vec{r}}{dt}\end{array} \arrow[l, shift left, "\int dt"] \arrow[r, shift left, "d/dt"] & \begin{array}{c}\textbf{acceleration}\\ \vec{a} = \dfrac{d\vec{v}}{dt}\end{array} \arrow[l, shift left, "\int dt"]
\end{tikzcd}$$

So $\vec{a} = \dfrac{d^2\vec{r}}{dt^2}$, and running it backwards: $\vec{v}(t) = \vec{v}_0 + \displaystyle\int_0^t \vec{a}\, dt'$ – valid even when a is not constant.

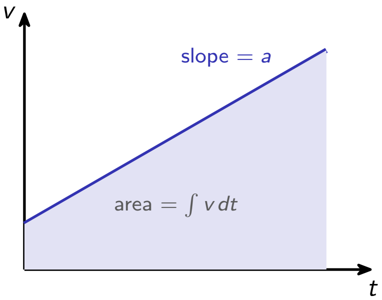
In this course, motion is calculus

the derivatives 导数	$v = \frac{dx}{dt}$, $\quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
reversed	$v = \int a\,dt$ and $x = \int v\,dt$, each with a constant fixed by the initial condition
average 平均 velocity	$\Delta x/\Delta t$ over an interval – a chord on the graph
instantaneous 瞬时 velocity	the derivative at one instant – the tangent
the SUVAT equations	$v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\Delta x$ – valid only for constant a

The most important habit in this unit: check whether a is constant before reaching for a SUVAT equation. If it is not, integrate.

3 Motion graphs

Motion graphs: slope and area



x–t: slope $= v = \dfrac{dx}{dt}$.

v–t: slope $= a$; area $= \displaystyle\int v\,dt = \Delta x$.

Constant a (a special case)

$$v = v_0 + at, \;\; x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Worked example – differentiate, then integrate

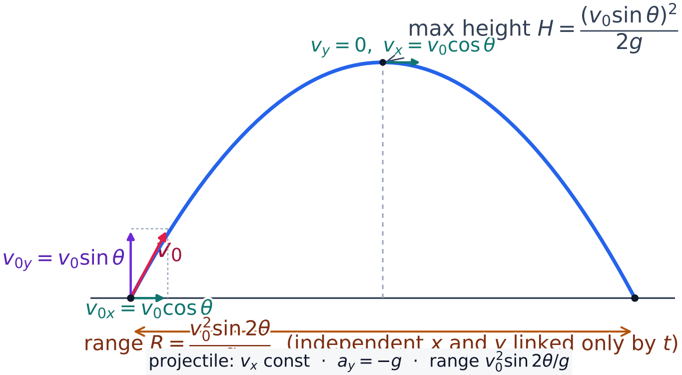
(a) $x(t) = 2t^3 - 3t^2$ m: find v and a. (b) From rest at the origin with $a(t) = 6t$: find v and x at $t = 2$ s.

- (a) $v = \dfrac{dx}{dt} = 6t^2 - 6t, \quad a = \dfrac{dv}{dt} = 12t - 6$
- (b) $v = \displaystyle\int 6t\,dt = 3t^2,$

  $x = \displaystyle\int 3t^2\,dt = t^3$
- at $t = 2$: $v = 12$ m/s, $x = 8$ m

Slopes are derivatives; areas are integrals – the two are inverse operations. Both constants of integration are zero here only because the particle starts from rest at the origin.

Motion in Two or Three Dimensions – one



A projectile launched at an angle: velocity components, maximum height, and range

A projectile launched at an angle: velocity components, maximum height, and range

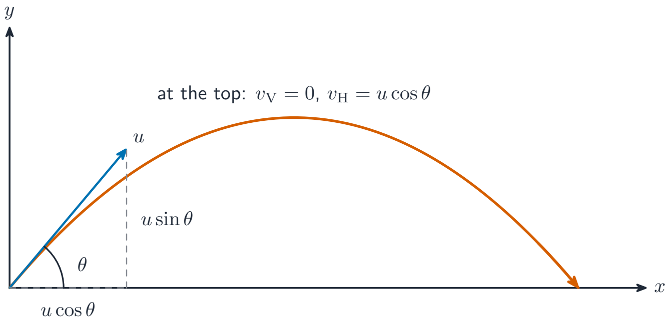
Worked example – motion in two dimensions

$\vec{r}(t) = (2t^2)\hat{i} + (4t - t^3)\hat{j}$ metres. Find $\vec{v}$ and $\vec{a}$.

- differentiate each component separately:
- $\vec{v} = 4t\,\hat{i} + (4 - 3t^2)\,\hat{j}$
- $\vec{a} = 4\,\hat{i} - 6t\,\hat{j}$

Components are independent: the $\hat{i}$ motion never enters the $\hat{j}$ derivative. That independence is the whole reason projectile motion splits into two one-dimensional problems.

Motion in Two or Three Dimensions – two



A projectile launched at an angle: the horizontal and vertical motions are independent

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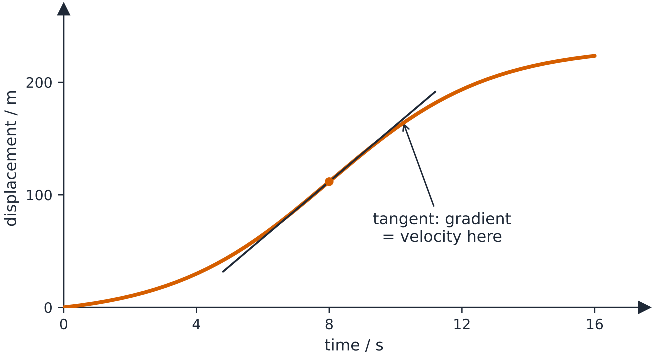
Representing Motion – one



A displacement-time graph: the slope at any instant is the velocity

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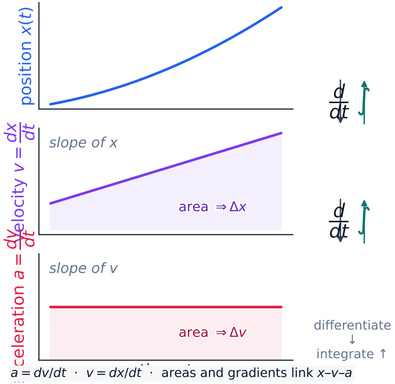
Representing Motion – two



Position, velocity, and acceleration are linked by differentiation

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4 Relative motion

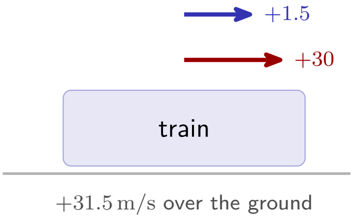
Reference frames and relative motion

Velocities add as vectors between frames:

Frame conversion

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

Frames moving at constant relative velocity share the same acceleration – they are **inertial** 惯性系, and Newton's laws hold identically.



Relative motion is vector addition

the rule	$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$ – the inner subscripts cancel
reversing	$\vec{v}_{B/A} = -\vec{v}_{A/B}$
a boat crossing	add the boat's velocity through the water to the water's velocity over the ground
reference frame 参考系	all motion is measured relative to one; there is no absolute rest

> Write the subscripts before the numbers. If the inner pair does not cancel, you have chained the frames the wrong way round.

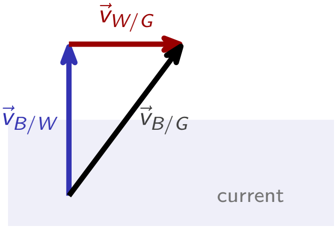
A boat crossing a river

The boat's ground velocity is the vector sum of its velocity through the water and the current:

$$\vec{v}_{B/G} = \vec{v}_{B/W} + \vec{v}_{W/G}$$

Across 3.0, downstream 2.0

$$|\vec{v}_{B/G}| = \sqrt{3.0^2 + 2.0^2} \approx 3.6\,\mathrm{m/s}$$



5 Projectiles

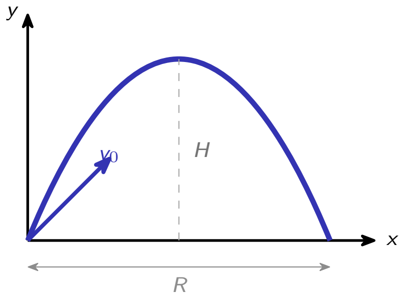
Projectile motion

The axes are independent, joined only by time t:

- horizontal: $a_x = 0$, so v_x constant
- vertical: $a_y = -g$ (free fall)

Eliminating t makes y a quadratic in x – the **trajectory** 轨迹 is a **parabola** 抛物线.

> A dropped and a launched ball land together.



Projectile formulas

Fired from the ground at speed v_0, angle θ:

Three key results

$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g}$$

$$H = \frac{v_0^2 \sin^2\theta}{2g}, \qquad R = \frac{v_0^2 \sin 2\theta}{g}$$

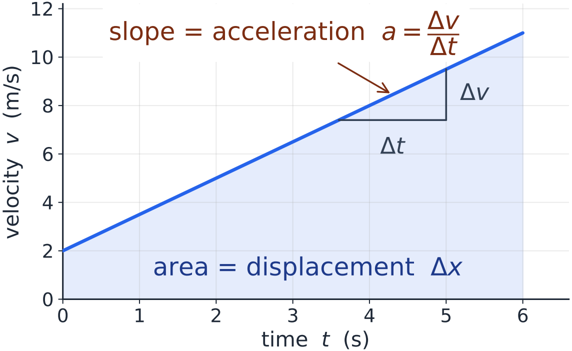
Range 射程 is greatest at $\theta = 45^\circ$; angles like 30° and 60° land at the same distance.

Uniform circular motion 匀速圆周运动: $a_c = \dfrac{v^2}{r}$ toward the centre – a preview of dynamics.

Displacement, Velocity, and Acceleration

A high-speed train: kinematics describes displacement, velocity and acceleration over time

On a velocity-time graph the gradient is



On a velocity-time graph the gradient is the acceleration and the area is the displacement

Exam tips

- Resolve every vector into components before adding – never add magnitudes at an angle directly.
- On Physics C you are expected to use **calculus**: velocity is $\vec{v} = \frac{d\vec{r}}{dt}$ and acceleration $\vec{a} = \frac{d\vec{v}}{dt}$; reverse with integration.
- Carry and check **units** and treat direction with signs (choose a positive axis and stick to it).
- Use the dot product for work-type quantities and the cross product for torque and angular momentum.
- Sketch the vectors – a diagram catches sign and direction errors the algebra hides.

6 Recap

Unit 1 – key takeaways

1 **Vectors**

components $\hat{i}, \hat{j}, \hat{k}$; dot $\rightarrow$ scalar, cross $\rightarrow$ vector

2 **Calculus**

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$; integrate to reverse

3 **Relative**

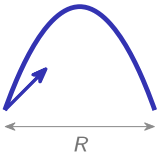
$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$; inertial frames

4 **Projectiles**

independent axes; range peaks at $45°$

Check yourself

1. What operation takes velocity back to position?
2. Dot product or cross product – which gives a scalar?
3. Two frames move at constant relative velocity – do they measure the same acceleration?
4. Which launch angle gives the greatest range?



Answers **1.** integration **2.** the dot product **3.** yes **4.** $45°$