Linear Momentum

AP Physics C: Mechanics

Linear Momentum

Linear momentum 动量 is mass times velocity:

$$\vec{p} = m\vec{v}.$$

It is a **vector** 矢量 pointing along the velocity, and it is *the* quantity for analysing **collisions** 碰撞 and **explosions** 爆炸. A collection of objects can be treated as one system moving with the velocity of its **center of mass** 质心:

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{\sum m_i},$$

so the system's total momentum is its total mass times $\vec{v}_{\text{cm}}$ – one object's worth of bookkeeping for any number of parts.

Change in Momentum and Impulse

Newton's second law is really a statement about momentum:

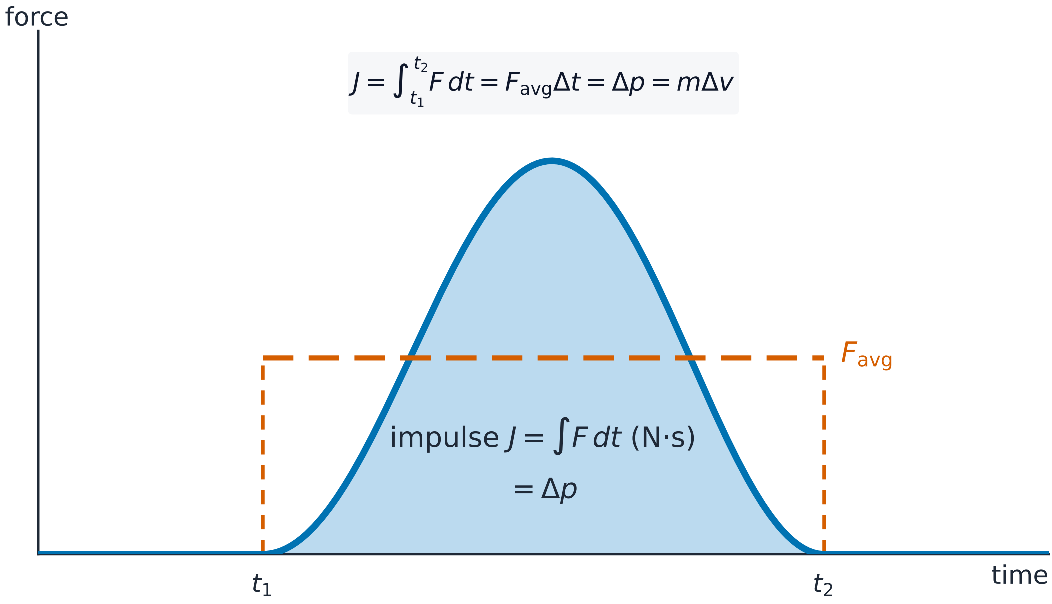
$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt},$$

which reduces to $m\vec{a}$ when the mass is constant – and handles a *changing* mass when it is not (the special case $\vec{F} = \frac{dm}{dt}\vec{v}$ holds only when the velocity is constant, e.g. a chain piling onto a scale or sand landing on a belt moving at fixed speed; a rocket does not fit, since it accelerates). The **impulse** 冲量 delivered by a force is its time integral, and the **impulse–momentum theorem** 冲量-动量定理 says it equals the change in momentum:

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}\, dt = \Delta\vec{p}.$$

Impulse is a vector along the net force. Read it off graphs both ways:

- On a force–time graph, impulse is the **area** under the curve.
- On a momentum–time graph, the net force is the **slope** 斜率 at each instant.



Impulse is the area under the force-time curve, equal to the average force times the contact time

Spreading the same Δp over a longer time lowers the force – that is why airbags, crumple zones, and soft landings work, and why you bend your knees when you land.

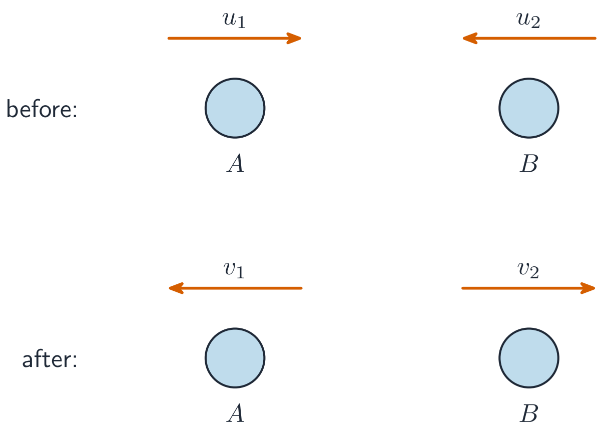
Worked example. A time-varying force $F(t) = 10t$ N acts on a 2.0 kg object for 2.0 s. The impulse is $J = \int_0^2 10t\,dt = \left[5t^2\right]_0^2 = 20\ \mathrm{N \cdot s}$, so the speed changes by $\Delta v = J/m = 10$ m/s.

Conservation of Linear Momentum

Internal forces come in Newton's-third-law pairs, so they cancel inside any system: they can shuffle momentum *between* parts but never change the total. Momentum only enters or leaves a system through a net *external* force ($\vec{J} = \Delta\vec{p}$). So, with **zero net external force**, total momentum is **conserved** 守恒:

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}.$$

Momentum is conserved in *every* collision, however violent – choose the system large enough that the collision forces are internal. Apply conservation separately along each axis; AP asks for quantitative work in one or two dimensions.



A head-on collision: total momentum before equals total momentum after

Worked example (explosion). A 6.0 kg shell at rest splits into a 2.0 kg piece moving at 9.0 m/s east and a 4.0 kg piece. Total momentum stays zero, so the heavy piece moves west at $v = \dfrac{2.0(9.0)}{4.0} = 4.5$ m/s. The kinetic energy came from stored (chemical or spring) energy – momentum conservation does not require kinetic-energy conservation.

Elastic and Inelastic Collisions

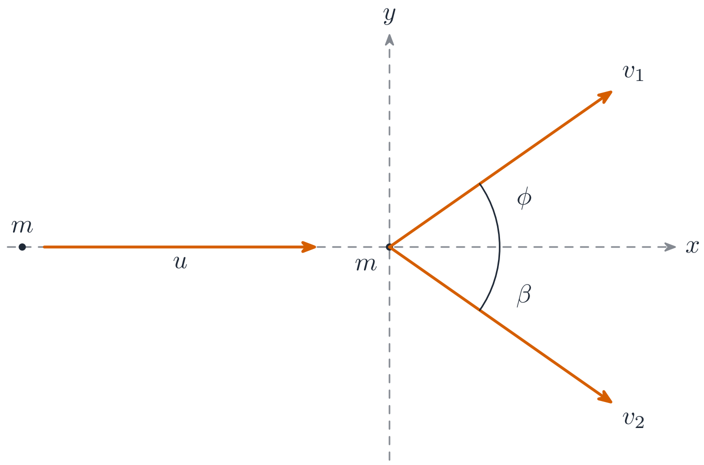
All collisions conserve momentum; they differ in what happens to the **kinetic energy** 动能:

type	momentum	kinetic energy
elastic collision 弹性碰撞	conserved	total conserved (individual shares may change)
inelastic collision 非弹性碰撞	conserved	decreases – some becomes heat, sound, **deformation** 形变
perfectly inelastic collision 完全非弹性碰撞	conserved	largest possible loss – the objects stick and share one velocity

Strategy: *always* write momentum conservation first; add the kinetic-energy equation only when the problem says "elastic". Two useful elastic facts: equal masses in a 1D elastic collision simply exchange velocities, and in the center-of-mass frame each object just reverses its velocity.

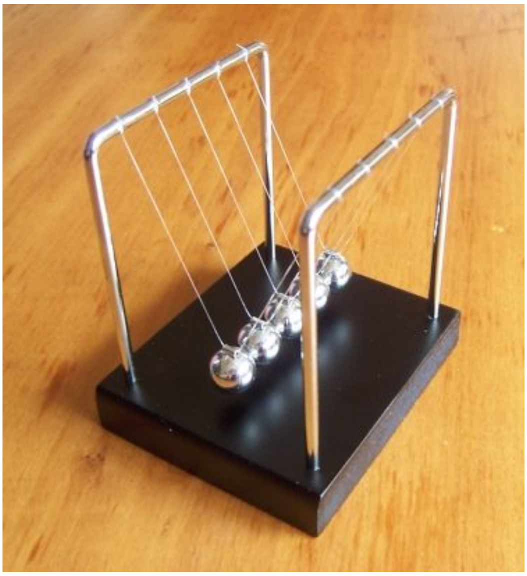
Worked example. A 1000 kg car at 20 m/s strikes a stationary 1500 kg car and they lock together – perfectly inelastic. Momentum: $v = \dfrac{1000(20)}{2500} = 8.0$ m/s. Kinetic energy falls from 2.0×10^5 J to $\frac{1}{2}(2500)(8.0)^2 = 8.0 \times 10^4$ J: about 60% is lost, even though momentum is exactly conserved.

Worked example (2D). A puck moving east at 4.0 m/s strikes an identical puck at rest; after the glancing hit, one moves at 2.0 m/s at 60° north of east. Conserve each axis: east–west, $m(4.0) = m(2.0)\cos 60° + mv_x$, so $v_x = 3.0$ m/s; north–south, $0 = m(2.0)\sin 60° - mv_y$, so $v_y = 1.7$ m/s. The second puck moves at $\sqrt{3.0^2 + 1.7^2} = 3.5$ m/s, about 30° south of east.



A glancing collision, resolved along two perpendicular axes

Exam skill. On FRQs, justify with the *condition*, not the slogan: “the net external force on the two-puck system is zero during the collision, so its total momentum is constant.” If asked whether the collision is elastic, *compute* the kinetic energy before and after and compare – never assume.



A Newton’s cradle shows momentum and kinetic energy passing through a near-elastic collision

Exam tips

- Impulse equals the momentum change: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$, and it is the area under a force–time graph.
- **Momentum is conserved** whenever the net external force is zero — always the go-to for collisions.
- Distinguish **elastic** (kinetic energy conserved) from **inelastic** (kinetic energy decreases) collisions; in a **perfectly inelastic** collision the objects stick and move with one velocity.
- Apply conservation to each component (x and y) separately in 2-D.
- Connect to center of mass: total momentum = $M\vec{v}_{cm}$.