Question 2: Translation Between Representations (TBR) — 12 points

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

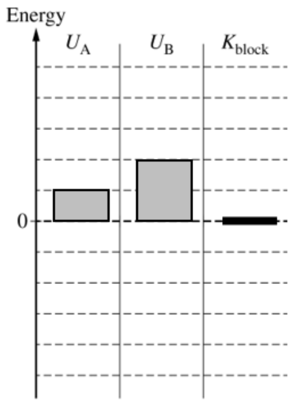


Figure 3

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$$E_{\text{tot},0} = U_A(x_1) + U_B(x_1) + K_{\text{block}}(x_1)$$
$$E_{\text{tot},f} = U_A\left(\frac{1}{2}x_1\right) + U_B\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$$

$$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$$
$$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$E_{\text{tot},0} = E_{\text{tot},f}$$
$$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$
$$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\frac{1}{2}x_1}$$

$$K_{\text{block},\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$$
$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

OR

$$x = x_{\max}\cos(\omega t + \phi)$$

$$x(t) = x_1\cos\omega t$$
$$v(t) = -x_1\omega\sin\omega t$$

$$k_{\text{eff}} = k + 2k = 3k$$
$$\omega = \sqrt{\frac{3k}{m}}$$

$$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$$
$$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$$
$$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$$

$$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$$
$$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$$

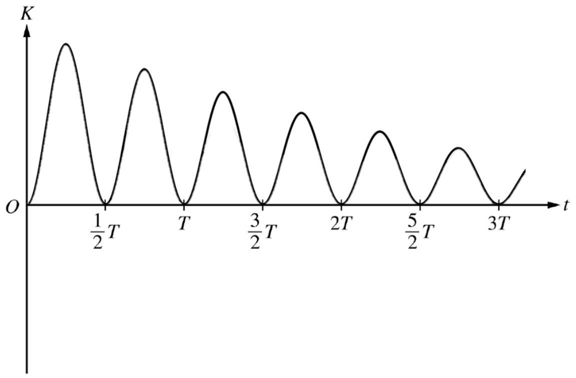
$$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

C For sketching a curve that starts at zero and is always positive or zero **Point C1**

For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$ **Point C2**

For sketching a periodic curve with a decreasing amplitude **Point C3**

Example Response



Scenario 2

Figure 5

D For indicating **one** of the following: **Point D1**

- The period increases.
- The rate at which the graph approaches zero increases.
- The maximum value of K over each period is less than the graph drawn.

For a correct justification relevant to the feature indicated **Point D2**

Example Responses

The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.

OR

The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.

Question 3: Experimental Design and Analysis (LAB) — 10 points

A For describing a procedure that includes measuring h and the speed of the block as the block moves across the surface **Point A1**

For a procedure that indicates a reasonable method of reducing experimental uncertainty **Point A2**

Examples of acceptable responses may include the following:

- Repeating the experiment multiple times for the same value of h
- Repeating the experiment for multiple values of h

Example Response

Measure the height h at which the block-box system is released. Measure the speed of the block as the block slides across the horizontal surface. Repeat the measurement of the speed of the block for varying release heights.

B For describing a graph that has linear trend that can be used to find g **Point B1**

Examples of acceptable responses include the following:

- v^2 vs. $2h$
- $\frac{1}{2}v^2$ vs. h
- v^2 vs. h
- v vs. $\sqrt{h}$

Scoring Notes:

- Reponses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.
- This point may be earned independently of the response in part A.

For correctly relating the slope of the best-fit line to g **Point B2**

Examples of acceptable responses include the following:

Graph	Analysis
v^2 vs. $2h$	slope $= g$
$\frac{1}{2}v^2$ vs. h	slope $= g$
v^2 vs. h	slope $= 2g$
v vs. $\sqrt{h}$	slope $= \sqrt{2g}$

Example Response

Plot v^2 on the vertical axis and $2h$ on the horizontal axis. The slope of the best-fit line is equal to g.

C **(i)** For indicating appropriate quantities that could be plotted to produce a linear graph that can be used to determine μ, such as h **vs.** x_{max} — **Point C1**

Scoring Note: Reponses that include the reciprocal of the preceding example, in addition to other equivalent graphs, also earn this point.

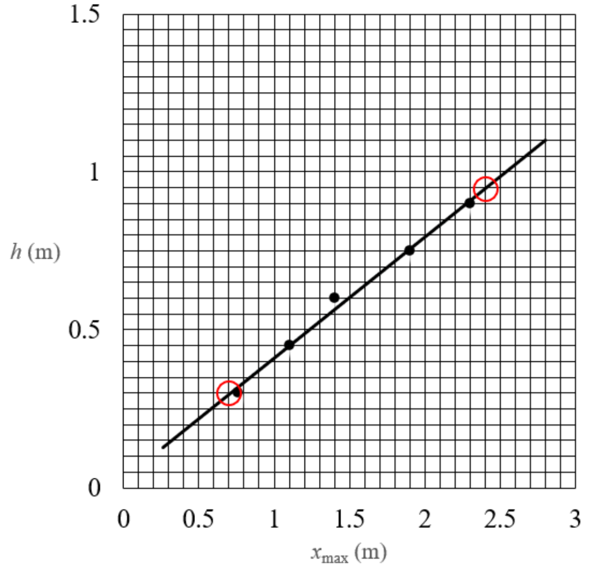
(ii) For labeling the axes (including units) with a linear scale — **Point C2**

For plotting data points consistent with **one** of the following: — **Point C3**

- The quantities indicated in part C (i)
- The quantities provided in Table 2
- The axes indicated on the grid

(iii) For drawing a line or curve that approximates the trend of the plotted data — **Point C4**

Example Response



D For correctly relating the slope of the best-fit line to the value of μ — **Point D1**

Examples of acceptable responses includes the following:

Graph	Analysis
h vs. x_{max}	slope $= \mu$

For a value of μ within a range of 0.30 to 0.50 — **Point D2**

Example Response

$$E_0 + W = E_f$$
$$mgh - F_f x_{max} = 0$$
$$mgh = \mu mg x_{max}$$
$$h = \mu x_{max}$$

$$y = mx + b$$
$$h = \mu x_{max}$$
$$\text{slope} = \mu$$
$$\text{slope} = \frac{0.95\ \text{m} - 0.30\ \text{m}}{2.4\ \text{m} - 0.70\ \text{m}}$$
$$\text{slope} = 0.38$$
$$\mu = \text{slope} = 0.38$$

Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A

For indicating $f_D < f_R$ — **Point A1**

For a justification that compares **one** of the following: — **Point A2**

- The motions of the disk and the ring using translational kinematics
- The motions of the disk and the ring using rotational kinematics
- The rotational inertias of the disk and the ring

For a justification that includes **one** of the following: — **Point A3**

- Reasoning that attempts Newton's second law in translational form
- Reasoning that attempts Newton's second law in rotational form
- Reasoning that attempts conservation of energy

Example Response

The ring has a greater rotational inertia because it has more mass distributed towards the edge. So the ring travels farther and has less acceleration down the ramp. Because the ring and the disk have the same mass, the gravitational forces exerted on the shapes are the same. From Newton's second law, the ring must have less net force down the ramp and more friction up the ramp.

B

For a multistep derivation that includes Newton's second law in both translational and rotational forms — **Point B1**

For indicating opposite signs for the gravitational force and frictional force in an expression for Newton's second law — **Point B2**

For using the relationship $a = r\alpha$ in an attempt to solve a system of equations — **Point B3**

Scoring Note: Responses that include conservation of energy may earn full credit.

Example Response

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}}$$

$$\left(\frac{a}{R}\right) = \frac{fR}{I}$$

$$a = \frac{fR^2}{I}$$

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}}$$

$$f - Mg\sin\theta = -Ma$$

$$f - Mg\sin\theta = -M\frac{fR^2}{I}$$

$$f + M\frac{fR^2}{I} = Mg\sin\theta$$

$$f\left(1 + \frac{MR^2}{I}\right) = Mg\sin\theta$$

$$f = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}}$$

C

For indicating "Equal to" — **Point C1**

For a justification that includes that kinetic friction is only dependent on the surfaces (or equivalently, the coefficient of kinetic friction between those surfaces) and normal force — **Point C2**

Example Response

The forces of kinetic friction are equal. If slipping, the friction is kinetic, and the force of kinetic friction on the hoop and the disk are the same because the coefficient of kinetic friction is only dependent on the surfaces, and the normal forces on both are the same.

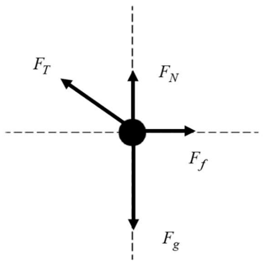
Question 1: Free-Response Question 15 points

(a) For correctly drawing and labeling the force of gravity and the normal force on the block of mass m — **1 point**

For correctly drawing and labeling the force of friction on the block of mass m — **1 point**

For correctly drawing and labeling the force of tension on the block of mass m — **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth" $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, $F_{Tension}$, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

(b) For a trigonometric expression relating the angle to the horizontal distance of the block from the left corner of the table, x. — **1 point**

For any correct trigonometric expression for θ in terms of the given quantities — **1 point**

Example Responses

First Point	Second Point
$\sin\theta = \dfrac{H}{\sqrt{H^2+x^2}}$	$\theta = \sin^{-1}\left(\dfrac{H}{\sqrt{H^2+x^2}}\right)$
$\cos\theta = \dfrac{x}{\sqrt{H^2+x^2}}$	$\theta = \cos^{-1}\left(\dfrac{x}{\sqrt{H^2+x^2}}\right)$
$\tan\theta = \dfrac{H}{x}$	$\theta = \tan^{-1}\left(\dfrac{H}{x}\right)$

Total for part (b) 2 points

(c)(i) For using Newton's second law to sum the forces in the vertical direction and write an equation that is consistent with part (a) — **1 point**

$$\Sigma F_y = ma$$
$$F_{net,y} = F_N + F_{T,y} - F_g = 0$$
$$F_N = F_g - F_{T,y}$$

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) — **1 point**

$$F_N = mg - F_T\sin\theta$$
$$F_N = mg - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)$$

(c)(ii) For using Newton's second law to sum the forces in the horizontal direction and write an equation that is consistent with part (a) — **1 point**

$$F_{net,x} = F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables consistent with part (b) — **1 point**

$$F_{net,x} = F_T\cos\theta - F_f$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction — **1 point**

$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - \mu_k F_N$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - \mu_k\left(mg - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)\right)$$

Total for part (c) 5 points

(d) For any indication that the work done on the block by the string is due only to the horizontal component of the tension in the string — **1 point**

$$W = \int F_{T,x}dx$$

For using the horizontal component of the force of tension consistent with part (c) — **1 point**

For indicating that work is the integral of the force with respect to x, including limits or a constant of integration — **1 point**

$$W = \int_{x=L}^{x=0} -F_T\left(\frac{x}{\sqrt{H^2+x^2}}\right)dx$$

Total for part (d) 3 points

(e) For selecting "More work …" with an attempt at a relevant justification **1 point**

For a correct justification relating the smaller angle to a larger component of the force of tension, thus resulting in greater work. **1 point**

Example Response

F_T stays the same for both halves, the displacement is the same in both halves, but from $x = L$ to $x = L/2$ the angle is smaller, resulting in a larger component of the tension force that aligns with the displacement.

Total for part (e) 2 points

Total for question 1 15 points

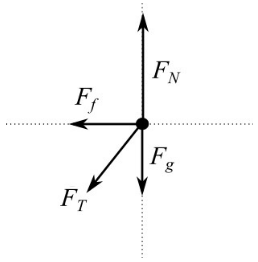
Question 1: Free-Response Question — 15 points

(a) For correctly drawing and labeling the force of gravity and the normal force on the sled **1 point**

For correctly drawing and labeling the force of friction on the on the sled **1 point**

For correctly drawing and labeling the force of tension on the on the sled **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{Earth\ on\ block}$, $F_{E,Block}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, $F_{Tension}$, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) 3 points

(b) For any correct trigonometric expression for θ in terms of the given quantities **1 point**

$$\sin\theta = \frac{x}{\sqrt{y^2 + x^2}} \qquad \therefore\ \theta = \sin^{-1}\left(\frac{x}{\sqrt{y^2 + x^2}}\right)$$

OR

$$\cos\theta = \frac{y}{\sqrt{y^2 + x^2}} \qquad \therefore\ \theta = \cos^{-1}\frac{y}{\sqrt{y^2 + x^2}}$$

OR

$$\tan\theta = \frac{x}{y} \qquad \therefore\ \theta = \tan^{-1}\frac{x}{y}$$

Total for part (b) 1 point

(c)(i) For beginning the derivation with Newton's second law to write an equation that is consistent with part (a). **1 point**

$$\Sigma F = ma$$
$$F_N - F_{T,y} - F_g = 0$$
$$F_N = F_{T,y} + F_g$$

Scoring Note: Derivation must start with a statement of Newton's second law to earn this point.

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) **1 point**

$$F_N = mg + F_T \cos\theta$$
$$F_N = mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)$$

(c)(ii) For summing the forces in the horizontal direction, consistent with parts (a) and (b) **1 point**

$$F_{net} = -F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables **1 point**

$$F_{net} = -F_T \sin\theta - F_f$$
$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction **1 point**

$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k F_N$$
$$F_{net} = -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right) - \mu_k\left(mg + F_T\left(\frac{y}{\sqrt{y^2 + x^2}}\right)\right)$$

Total for part (c) 5 points

AP Physics C: **Mechanics: 2022 Set 2**

(d) For using the integral definition of work to derive an expression for the work done by the force of tension **1 point**

$$W = \int F \cdot dx$$

For any indication that the work done on the sled by the string is due only to the horizontal component of the tension in the string **1 point**

$$W = \int F_{T,x} dx$$

For using the horizontal component of the force of tension consistent with part (b) or (c) **1 point**

$$W = \int_{x=0}^{x=L} -F_T\left(\frac{x}{\sqrt{y^2 + x^2}}\right)dx$$

For correct limits of integration, x= 0 to x= L, and indicating that the work done is negative **1 point**

let $u = y^2 + x^2$, then $du = 2x\ dx$

$$W = -F_T \int_{x=0}^{x=L} \frac{1}{2}\left(\frac{1}{\sqrt{u}}\right)du$$
$$W = -F_T\left(\sqrt{y^2 + x^2}\right)\Big|_{x=0}^{x=L}$$
$$W = -F_T\left(\sqrt{y^2 + L^2} - \sqrt{y^2}\right)$$

Total for part (d) 4 points

(e) For selecting "$E_1 > E_2$" with an attempt at a relevant justification **1 point**

For a justification that correctly relates the force of friction to the normal force, and the normal force to the position or the angle **1 point**

Example Response

The vertical component of the string force is the largest when the string is more vertical ($0 < x < L$). A larger vertical component of the string tension leads to a larger normal force and, hence, a larger friction force.

Total for part (e) 2 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a) For correctly evaluating Newton's second law equations for the system at rest — **1 point**

$m_2 g - f = (m_1 + m_2)a = 0 \therefore m_2 g = f$

For correctly substituting for static friction into above equation: — **1 point**

$m_2 g = f = \mu_s F_N = \mu_s m_1 g$

$\mu_s = \dfrac{m_2}{m_1} = \dfrac{(0.20 \text{ kg})}{(0.44 \text{ kg})} = 0.45$

Total for part (a) — 2 points

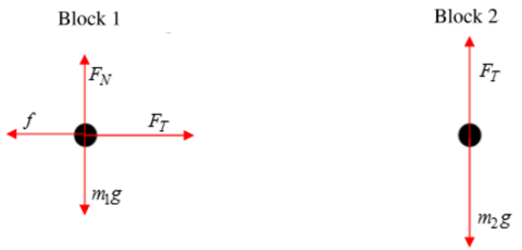
(b) For correctly drawing and labeling the horizontal forces of friction and tension on block of mass m_1 — **1 point**

For correctly drawing and labeling the vertical forces of weight and normal force on block of mass m_1 — **1 point**

For correctly drawing and labeling forces of weight and tension on block of mass m_2 — **1 point**

For indicating that the gravitational forces on each block are different — **1 point**

Example responses for part (b)



Scoring note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force.

Total for part (b) — 4 points

AP Physics C: **Mechanics: 2021 Set 2**

(c) For correctly evaluating Newton's second law equation for block 1: — **1 point**

$T - f = m_1 a$

For correctly evaluating Newton's second law equation for block 2: — **1 point**

$m_2 g - T = m_2 a$

Combining the two equations

$m_2 g - f = (m_1 + m_2)a \therefore f = m_2 g - (m_1 + m_2)a$

Scoring note: Both points are earned for a single correct Newton's second law equation for the two-block system.

For correctly substituting for kinetic friction into above equation — **1 point**

$f = \mu_k F_N = \mu_k m_1 g = m_2 g - (m_1 + m_2)a \therefore \mu_k = \dfrac{m_2 g - (m_1 + m_2)a}{m_1 g}$

$\mu_k = \dfrac{(0.20 \text{ kg})(9.8 \text{ m/s}^2) - (0.44 \text{ kg} + 0.20 \text{ kg})(2.3 \text{ m/s}^2)}{(0.44 \text{ kg})(9.8 \text{ m/s}^2)} = 0.11$

Total for part (c) — 3 points

(d) For selecting "Yes" and attempting a relevant justification — **1 point**

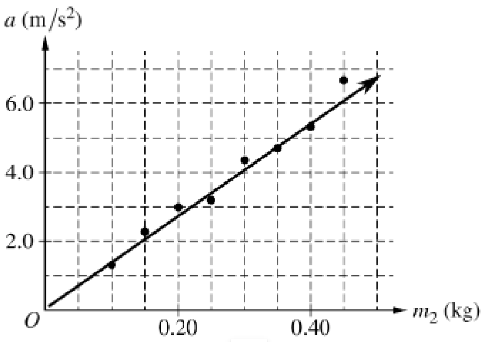
For a correct justification — **1 point**

Example response for part (d)

If the track is not level, the angle of the track must be incorporated into the equation for acceleration, and this could account for the larger coefficient of kinetic friction.

Total for part (d) — 2 points

(e)	**i.**	For drawing an appropriate best-fit line	**1 point**



ii. For calculating slope using two points from the best-fit line — **1 point**

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(6-2)\left(\text{m/s}^2\right)}{(0.45-0.15)(\text{kg})} = 13.3\ \text{m/kg}\bullet\text{s}^2$$

For correctly using an expression that relates the slope to the acceleration due to gravity — **1 point**

From $y = mx + b$

$a =$ (slope)m_2 + (y-intercept)

$$a = \frac{m_2 g}{(m_1 + m_2)} \therefore \text{slope} = \frac{g}{(m_1 + m_2)}$$

$$g = \text{slope} \times (m_1 + m_2) = \left(13.3\ \text{m/kg}\bullet\text{s}^2\right)(0.44\ \text{kg} + 0.20\ \text{kg}) = 8.5\ \text{m/s}^2$$

Total for part (e) 3 points

(f) For a correct justification — **1 point**

Example response for part (f)

The acceleration would be greater because there would be a component of the gravitational force on block 1 along the surface, which would be in the same direction as the tension force.

Total for question 1 15 points

2019

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AP® Physics C: Mechanics

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES

Question 1

15 points

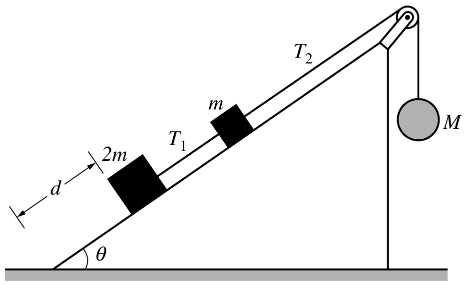
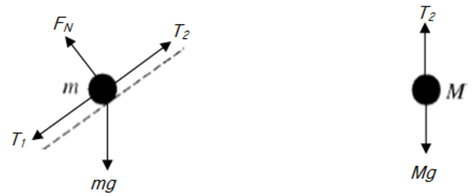


Figure 1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

(a) LO INT-1.A, SP 3.D
3 points

On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



For correctly drawing and labeling vectors representing the normal force and the gravitational force on the block of mass m		1 point
For correctly drawing and labeling vectors representing the forces of tension on the block of mass m		1 point
For correctly drawing and labeling vectors representing the tension force and the gravitational force on the sphere of mass M		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors.		

Question 1 (continued)

(b)

Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. LO INT-1.C.e, SP 1.D, 5.E
2 points

The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

For using an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{net} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g\sin\theta = 0$		
For a correct answer with supporting work		1 point
$T_2 = 3mg\sin\theta$		

ii. LO INT-1.D, SP 5.E
1 point

The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

For using a correct statement of Newton's second law for the whole system to derive an expression for M		1 point
$F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$		
$M = 3m\sin\theta$		
Alternate Solution		*Alternate Points*
For a correct statement of Newton's second law for the sphere and an answer consistent with part (b)(i)		1 point
$T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g$		
$M = 3m\sin\theta$		

Question 1 (continued)

(c)

Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. LO INT-1.D, SP 7.A
1 point

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

For correctly stating that the magnitude of T_1 drops to zero		1 point

ii. LO INT-1.D, SP 7.A
1 point

Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Up the ramp"		1 point

iii. LO INT-1.D, SP 7.A
1 point

Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Down the ramp"		1 point

(d) LO INT-1.D, SP 5.A, 5.E
3 points

Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the system		1 point
$F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$		
For attempting to substitute in for the force of friction		1 point
$3mg\sin\theta - \mu_s F_{N2} - \mu_s F_{N1} = Mg$		
$3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$		
For a correct answer with supporting work		1 point
$M = 3m(\sin\theta - \mu_s\cos\theta)$		

Question 1 (continued)

(e) LO INT-1.D, CHA-1.A.b, SP 5.A, 5.E
3 points

The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{\text{net}} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$		
Solve for the acceleration		
$3mg\sin\theta - \mu_k(2mg\cos\theta) - \mu_k mg\cos\theta = 3ma$		
$a = g(\sin\theta - \mu_k\cos\theta)$		
For using a correct kinematics equation to solve for the final velocity		1 point
$v_2^2 = v_1^2 + 2ad$		
$v_2^2 = 0^2 + 2g(\sin\theta - \mu_k\cos\theta)d$		
For a correct answer with supporting work		1 point
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		
Alternate Solution		*Alternate Points*
For an attempt at a correct statement of conservation of energy for the two blocks		*1 point*
$U_1 + K_1 = U_2 + K_2 + E_{\text{lost}}$		
$2mgh + mgh + 0 = 0 + \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 + fd$		
For correct substitutions of height and friction		*1 point*
$3mgd\sin\theta = \frac{1}{2}(3m)v^2 + \mu_k(2mg\cos\theta)d + \mu_k(mg\cos\theta)d$		
$3gd\sin\theta - 3\mu_k gd\cos\theta = \frac{3}{2}v^2$		
For a correct answer with supporting work		*1 point*
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		

Question 1 (continued)

Learning Objectives

CHA-1.A.b: Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.
INT-1.A: Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).
INT-1.C.e: Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).
INT-1.D: Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood Machines, falling with air resistance, pulley systems, mass in elevator, etc.)

Science Practices

1.D: Select relevant features of a representation to answer a question or solve a problem.
3.C: Sketch a graph that shows a functional relationship between two quantities.
3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration *a* for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and *t* is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.		1 point
$J = \int F(t)\,dt$		
$J = \int ma(t)\,dt = (0.50)\int \left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$J = (0.50)\int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6}$		
$J = (0.50)\left[\left((9.0)(6) - \left(\frac{1}{12}\right)(6)^3\right)\right] - 0 = 18\ \text{N•s}$		
Alternate Solution (using an alternate solution from part (b))		*Alternate Points*
For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (b) into equation above		1 point
$J = (0.50\ \text{kg})(36 - 0)\ \therefore\ J = 18N•s$		

Question 2 (continued)

(b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (a) into equation above		1 point
$(18\ \text{N•s}) = (0.50\ \text{kg})(v_2 - 0)\ \therefore\ v_2 = 36\ \text{m/s}$		
Alternate Solution		*Alternate Points*
Integrate expression for a(t) (This may have already been done in solving part (a).)		1 point
$v = \int a\,dt = \int \left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$v = \int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = \left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6} = 36\ \text{m/s}$		

(c)

i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy		1 point
For substituting the answer from part (b) into the equation for kinetic energy		1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50\ \text{kg})(36\ \text{m/s})^2 = 324\ \text{J}$		

ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position		1 point
For integrating with correct limits or including a constant of integration		1 point
$v(t) = \int a(t)\,dt = \int\left(9.0 - 0.25t^2\right)dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$		
$\Delta y = \int v(t)\,dt = \int_{t=0}^{t=6}\left(9.0t - \frac{1}{12}t^3\right)dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_{t=0}^{t=6}$		
$\Delta y = \left[\left(\frac{9}{2}\right)(6)^2 - \left(\frac{1}{48}\right)(6)^4\right] - 0 = 135\ \text{m}$		
For substituting into equation for potential energy		1 point
$\Delta U_g = mg\Delta y = (0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(135\ \text{m}) = 660\ \text{J}$		

Question 2 (continued)

(d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

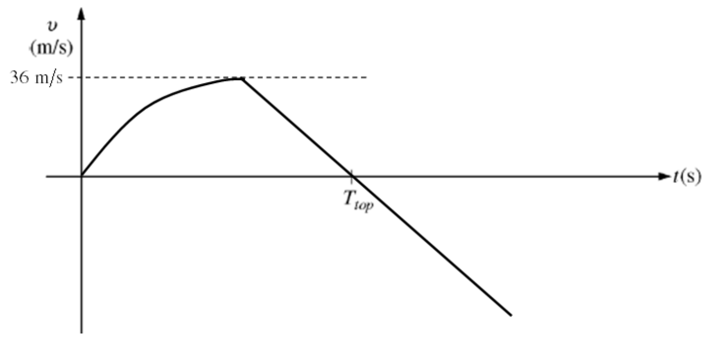
Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height		1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$		
$y_2 - y_1 = -\frac{v_1^2}{2a} \therefore y_2 = -\frac{v_1^2}{2a} + y_1$		
For substituting the speed from part (b)		1 point
$y_2 = -\frac{(36\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} + y_1$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 1		*Alternate Points*
For using energy conservation to find maximum height, consistent with the speed found in part (b).		1 point
$mg\Delta y = \frac{1}{2}mv^2 \therefore \Delta y = \frac{v^2}{2g}$		
For substituting the speed from part (b)		1 point
$\Delta y = \frac{(36\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66\ \text{m}$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 2		*Alternate Points*
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)		1 point
$K_1 + U_1 = 0 + U_{top}$		
For substituting the kinetic energy from part (c)(i) into the equation above		1 point
For substituting the potential energy from part (c)(ii) into the equation above		1 point
$324 + 660 = (0.5)(9.8)\Delta y$		
$\Delta y = 201\ \text{m}$ (199.8 m if g = 10m/s^2)		

Question 2 (continued)

(e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.		1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.		1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}		1 point

Question 2 (continued)

Learning Objectives

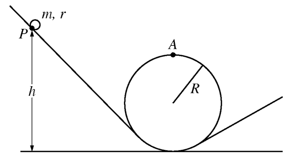
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.
CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.
INT-4.C.c: Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.
INT-5.A.a: Calculate the total momentum of an object or system of objects.
INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) LO INT-2.D, SP 5.A, 5.E
2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force		1 point
$F_C = mg + F_N = \dfrac{mv^2}{R}$		
$mg + 0 = \dfrac{mv^2}{R}$		
For a correct substitution into a correct expression.		1 point
$v = \sqrt{Rg}$		

Question 3 (continued)

(b) LO INT-7.E, SP 5.A, 5.E
3 points

Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object		1 point
$K_1 + U_1 = K_2 + U_2$		
For correct substitutions, including rotational kinetic energy		1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$		
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(bmr^2\right)\left(\frac{v}{r}\right)^2 + mgh_2$		
For correctly substituting for the final height and the answer from part (a) for the velocity		1 point
$gh = \frac{1}{2}\left(\sqrt{Rg}\right)^2 + \frac{1}{2}(b)\left(\sqrt{Rg}\right)^2 + g(2R)$		
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$		

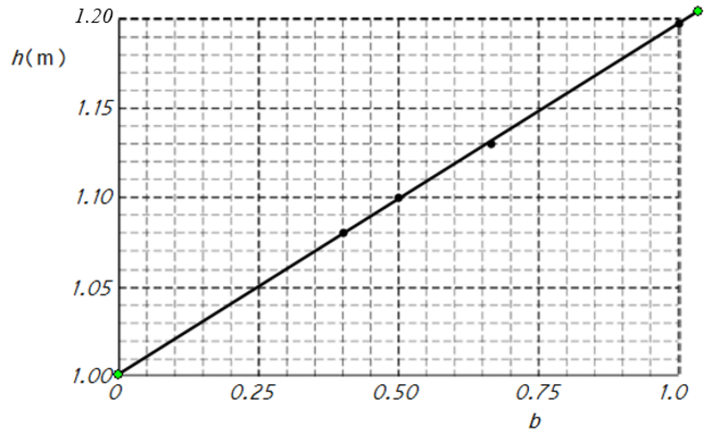
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h, no units for b		1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a best-fit straight line that represents the data		1 point

Question 3 (continued)

(d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation		1 point
slope $= \dfrac{1.20 - 1.00}{1.0 - 0.0} = 0.20$ m		
$y = mx + b \therefore h = 0.20b + 1.00$		
$1.16 = 0.20b + 1.00$		
For a correct answer for b		1 point
$b = 0.80$		
Note: Full credit can be earned for the correct answer if there is an indication that this was read from the graph.		

(e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate R, the radius of the loop.

For substituting into the expression from part (b)		1 point
$h = \dfrac{1}{2}(b+5)R \therefore R = \dfrac{2(1.16\text{ m})}{(0.80+5)}$		
For an answer consistent with previous parts		1 point
$R = 0.40$ m		
Alternate Solution		*Alternate Points*
For using an expression for the y-intercept (set b = 0)		1 point
$h = \dfrac{1}{2}(b+5)R = \dfrac{1}{2}(0+5)R = \dfrac{5}{2}R$		
For determining the value of the intercept (1.0 m) from the graph		1 point
$1.0\text{ m} = \dfrac{5}{2}R \therefore R = \dfrac{2}{5}\text{ m} = 0.40$ m		

Question 3 (continued)

(f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

For selecting "Less" and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.
INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

2018

AP® CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

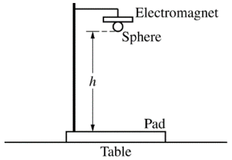
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

AP Physics C: **Mechanics: 2018**

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

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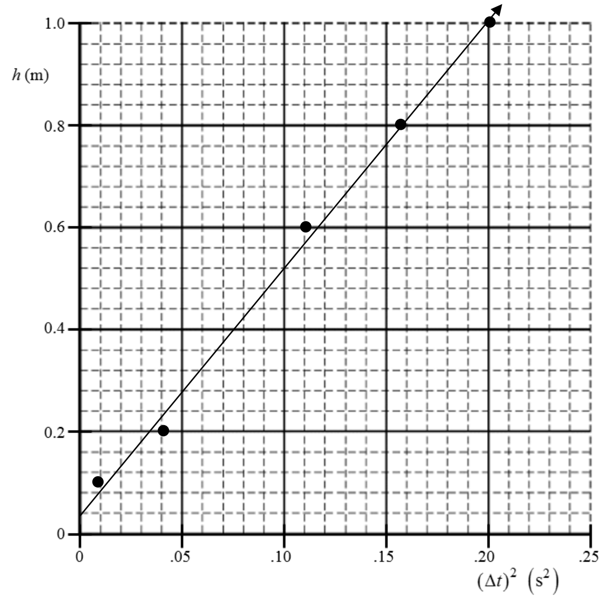
Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

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Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
$\text{slope} = \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\ \text{m}}{(0.160 - 0.039)\ \text{s}^2} = 4.96\ \text{m/s}^2$ (Linear regression $= 4.83\ \text{m/s}^2$)		
For correctly relating the slope to the acceleration due to gravity		1 point
$\text{slope} = \dfrac{1}{2}g \therefore g = 2 \times \text{slope} = 2 \times \left(4.96\ \text{m/s}^2\right)$		
$g = 9.92\ \text{m/s}^2$ (Linear regression $= 9.66\ \text{m/s}^2$)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\ \text{m/s}^2\right)t - 0.524\ \text{m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		

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Question 1 (continued)

Distribution of points

iii. 1 point

Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \dfrac{\lvert \text{acc} - \text{exp} \rvert}{\text{acc}} \times 100\% = \dfrac{\left\lvert \left(11.5\ \text{m/s}^2\right) - \left(9.81\ \text{m/s}^2\right) \right\rvert}{\left(9.81\ \text{m/s}^2\right)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C\ \therefore v_1 = B = -0.524\ \text{m/s}$		
$a = 2A = 2 \times \left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524\ \text{m/s})^2 + (2)\left(11.5\ \text{m/s}^2\right)(1.40\ \text{m} - 0.080\ \text{m})}$		
$v = 5.54\ \text{m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = \left(5.75\ \text{m/s}^2\right)t^2 + (-0.524\ \text{m/s})t + (0.080\ \text{m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53\ \text{s}$ or $-0.44\ \text{s}\ \therefore t = 0.53\ \text{s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = \left(11.5\ \text{m/s}^2\right)(0.53\ \text{s}) - 0.524\ \text{m/s}$		
$v = 5.54\ \text{m/s}$		

AP® PHYSICS C: MECHANICS
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(e)

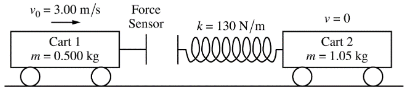
iv. (continued)

Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C\ \therefore v_1 = B = -0.524\ \text{m/s}$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54\ \text{m/s}$		

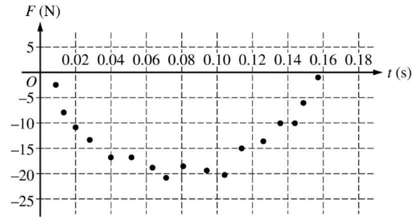
2018 SCORING GUIDELINES

Question 2

15 points total | **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - \left(500\ \text{N/s}\right)t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int \left(3200t^2 - 500t\right) dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\ \text{s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\bullet\text{s}$		

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1\left(v_{1f} - v_{1i}\right) \therefore v_{1f} = \frac{J}{m_1} + v_{1i} = \frac{(-2.03\ \text{N}\bullet\text{s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06\ \text{m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1\left(v_{1f} - v_{1i}\right) = m_2 v_{2f} \therefore v_{2f} = \frac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \frac{(2.03\ \text{N}\bullet\text{s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93\ \text{m/s}$		

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Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \therefore v_{2f} = \dfrac{m_1(v_{1i} - v_{1f})}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500\text{ kg})((3.00\text{ m/s}) - (-1.06\text{ m/s}))}{(1.05\text{ kg})} = 1.93\text{ m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 = \left(\frac{1}{2}\right)(0.500\text{ kg})(-1.06\text{ m/s})^2 + \left(\frac{1}{2}\right)(1.05\text{ kg})(1.93\text{ m/s})^2$		
$2.25\text{ J} \approx 2.24\text{ J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \therefore v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500\text{ kg})(3.00\text{ m/s})}{(0.500\text{ kg} + 1.05\text{ kg})} = 0.97\text{ m/s}$		

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2) v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\dfrac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX} \rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\dfrac{F_{MAX}}{k} = -\dfrac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2} kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

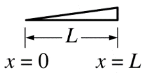
Units point: 1 point for correct units on all calculated answers

2018 SCORING GUIDELINES

Question 3

15 points total — **Distribution of points**

$$\lambda = \left(\frac{2M}{L^2}\right)x$$



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int (2M/L^2) x dx \therefore dm = (2M/L^2) x dx$		
$I = \int (2M/L^2) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} (2M/L^2) x^3 dx = \left[\frac{(2M/L^2)x^4}{4}\right]_{x=0}^{x=L} = \frac{2M}{L^2}\left(\frac{L^4}{4} - 0\right) = ML^2/2$		

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Question 3 (continued)

Distribution of points

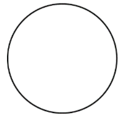
Figure 1

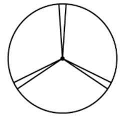
Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3(ML^2/2) + ML^2$		
For a correct answer with work		1 point
$I = \frac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) 3 points

Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \frac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \frac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \frac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \frac{2F\Delta t}{5ML}$		

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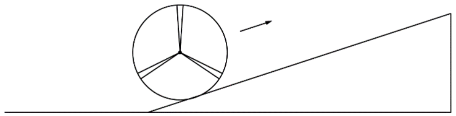
Question 3 (continued)

Distribution of points

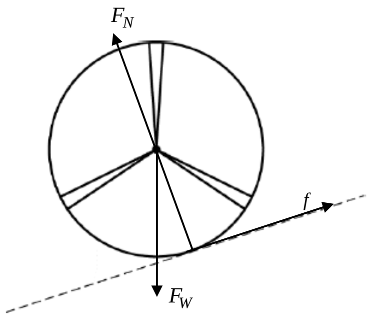


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		

2017

AP® CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

AP® PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

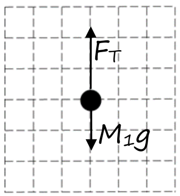
SCORING GUIDELINES

Question 1

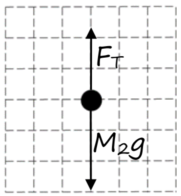
15 points total — **Distribution of points**

(a) 3 points

Block of Mass M_1



Block of Mass M_2



For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_1 with the tension larger than the weight	1 point
For correctly drawing and labeling the vectors for the weight of the block and the tension for block M_2 with the weight larger than the tension	1 point
For correctly drawing tension on the two blocks as equal in magnitude	1 point

Note: If any extraneous vectors are drawn, only a maximum of two points may be earned.

(b) 3 points

For correctly applying Newton's second law to block 1 — 1 point

$F_T - M_1 g = M_1 a$

For correctly applying Newton's second law to block 2 — 1 point

$M_2 g - F_T = M_2 a$

For combining the two equations above in such a way that will lead to the correct equation — 1 point

$M_2 g - M_1 g = (M_1 + M_2) a$

$$a = \frac{(M_2 - M_1)}{(M_1 + M_2)} g$$

(c) 1 point

For indicating variables that will create a straight line whose slope can be used to determine g — 1 point

Example: Vertical axis: a

Horizontal axis: $\dfrac{(M_2 - M_1)}{(M_1 + M_2)}$

Note: Full credit is earned if axes are reversed, or if the student uses another acceptable combination.

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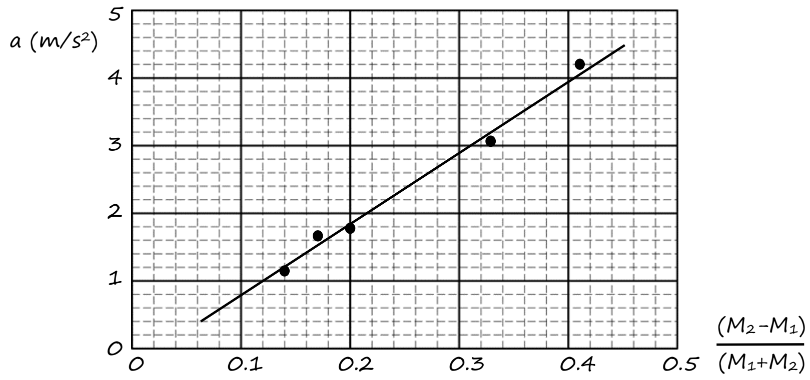
Question 1 (continued)

Distribution of points

(d) 3 points



For using a correct scale that uses more than half the grid and for correctly labeling the axes, including units as appropriate	1 point
For correctly plotting the data	1 point
For drawing a straight best-fit line consistent with the plotted data	1 point

(e) 2 points

For correctly calculating the slope using the best-fit line and not the data points, unless the points fall on the best-fit line — 1 point

(Note: Credit may be given for the linear regression only if the student states linear regression is used.)

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(3.0 - 1.0)\ \text{m/s}^2}{(0.31 - 0.12)} = 10.5\ \text{m/s}^2$$

For correctly relating the slope to g — 1 point

$g = \text{slope} = 10.5\ \text{m/s}^2$ (using linear regression yields $10.1\ \text{m/s}^2$)

2017 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(f) 2 points

Correct answer: “Lower”

For including a correct statement about the acceleration of the blocks — 1 point
For including a correct statement about the forces on the blocks — 1 point

Example: Because the block M_1 is on the table, the net force on the system increases, and therefore the acceleration increases. Because the acceleration of M_2 increases, the net force on M_2 must increase. Therefore, there must be a greater difference in the magnitude of the two forces on M_2. Because the weight of block M_2 stays the same, the retarding force — the tension in the string — must decrease.

(g) 1 point

For a correct justification — 1 point

Example: Block M_1 will experience friction with the table. The acceleration of the system will decrease and this will decrease the slope of the line; therefore, the value of g is determined by the experiment.

SCORING GUIDELINES

Question

15 points total

Distribution of points

(a)

i. 2 points

For correctly using conservation of energy for the block moving down the incline — 1 point

$U_g = K$

$mgh = \frac{1}{2}mv^2$

For a correct answer — 1 point

$v = \sqrt{2gh}$

ii. 1 point

Correct answer: “Greater than”
For a correct justification — 1 point

Example: The speed is proportional to the square root of the change in height. So if the height is reduced by a factor of 2, the speed is reduced by a factor of $\sqrt{2} \approx 1.41$. Therefore, the speed halfway down the ramp is more than half the speed at the bottom of the ramp.
Note: If the incorrect selection is made, the justification cannot earn credit.

(b) 1 point

For correctly using conservation of energy, consistent with part (a), for the block compressing the spring — 1 point

$U_g = U_S$

$mgh = \frac{1}{2}kx_{max}^2$

$x_{max} = \sqrt{\frac{2mgh}{k}}$

(c) 2 points

For indicating a simple harmonic motion approach — 1 point
For a correct answer — 1 point

$t = \frac{1}{4}T = \left(\frac{1}{4}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

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Question continued)

Distribution of points

(d)

i. 3 points

For correctly applying Newton's second law for the horizontally sliding block — 1 point

For correctly indicating that the direction of F_{net} is opposite the direction of motion — 1 point

$F_{net} = ma$

$-\beta v^2 = ma$

For expressing the equation as a differential equation — 1 point

$-\beta v^2 = m\dfrac{dv}{dt}$

ii. 3 points

For correctly separating variables — 1 point

$-\dfrac{\beta}{m}dt = \dfrac{1}{v^2}dv$

For correctly integrating the equation above — 1 point

$\int -\dfrac{\beta}{m}dt = \int \dfrac{1}{v^2}dv$

$-\dfrac{\beta}{m}[t] = \left[-\dfrac{1}{v}\right]$

For using the correct limits or constant of integration — 1 point

$-\dfrac{\beta}{m}[t]_0^t = \left[-\dfrac{1}{v}\right]_{v_0}^{v(t)}$

$-\dfrac{\beta t}{m} = -\dfrac{1}{v(t)} - \left(-\dfrac{1}{v_0}\right)$

$\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$

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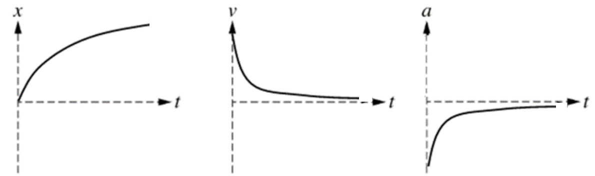
Question continued)

Distribution of points

(e) 3 points



For a displacement curve that is concave down and approaching a horizontal asymptote — 1 point

For a velocity curve that is concave up and has the horizontal axis as an asymptote — 1 point

For an acceleration curve that is concave down and has the horizontal axis as an asymptote — 1 point

Note: If an incorrect nonlinear velocity graph is generated, 1 point is earned if the position and acceleration graphs are consistent with the velocity graph.

Note: Full credit is earned if all three graphs are flipped vertically.

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Question 3

15 points total — **Distribution of points**

(a) 2 points

For correctly applying conservation of energy to the cylinder rolling down the incline — 1 point

$U_{g\_top} = K_{table}$

$K_{table} = mgh = (0.50\text{ kg})\left(9.8\text{ m/s}^2\right)(1.0\text{ m})(\sin 30)$

For a correct answer with units — 1 point

$K_{table} = 2.45\text{ J}$ (or 2.5 J using $g = 10\text{ m/s}^2$)

(b) 3 points

For correctly setting the kinetic energy of the cylinder equal to the sum of both the linear and rotational kinetic energy — 1 point

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For correctly substituting into the above equation for the linear velocity and moment of inertia of the cylinder — 1 point

$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$

$\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{(4)(2.45\text{ J})}{(3)(0.50\text{ kg})(0.10\text{ m})^2}}$

For correct substitution into the equation above — 1 point

$\omega = 25.6\text{ rad/s}$ (or 26.0 rad/s using $g = 10\text{ m/s}^2$)

(c) 2 points

For using a correct expression for the ratio of the rotational kinetic energy to the total kinetic energy of the cylinder — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh} = \frac{\frac{1}{4}M(R\omega)^2}{\frac{3}{4}M(R\omega)^2 + Mgh_{\text{table}}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{\text{table}}}$$

For substituting into the above equation — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{[(0.10\text{ m})(25.6\text{ rad/s})]^2}{(3)(0.10\text{ m})^2(25.6\text{ rad/s})^2 + (4)\left(9.81\text{ m/s}^2\right)(0.75\text{ m})}$$

$\frac{K_{rot}}{K_{tot}} = 0.133$ (or 0.135 using $g = 10\text{ m/s}^2$)

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Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate Points*

For using a correct expression for the ratio of the rotational kinetic energy to the total potential energy of the cylinder — *1 point*

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{U_{total}} = \frac{\frac{1}{4}M(R\omega)^2}{Mg(h+y)}$$

For substituting into the above equation — *1 point*

$$\frac{K_{rot}}{U_{total}} = \frac{\frac{1}{4}(R\omega)^2}{g(h+y)} = \frac{\frac{1}{4}[(0.10\text{ m})(25.6\text{ rad/s})]^2}{\left(9.81\text{ m/s}^2\right)\left((1.0\text{ m})\sin\left(30^\circ\right) + (0.75\text{ m})\right)}$$

$\frac{K_{rot}}{U_{total}} = 0.13$ *(or* 0.135 *using* $g = 10\text{ m/s}^2$ *)*

(d) 2 points

For correctly using motion in the vertical direction to calculate the time for the cylinder to reach the floor — 1 point

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y - y_0 = 0 - \frac{1}{2}gt^2$

Determine the time for the cylinder to reach the floor

$y - y_0 = -\frac{1}{2}gt^2$

$0 - y = -\frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(0.75\text{ m})}{\left(9.81\text{ m/s}^2\right)}} = 0.39\text{ s}$

For correctly using the equation for constant speed in the horizontal direction — 1 point

$x - x_0 = v_{0x}t$

$x - x_0 = R\omega t$

$x - x_0 = R\omega\sqrt{\frac{2y}{g}}$

$D = R\omega t = (0.10\text{ m})(25.6\text{ rad/s})(0.39\text{ s})$

$D = 1.0\text{ m}$

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Question 3 (continued)

	Distribution of points
(e)	
i. 2 points	
For selecting “Equal to” and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere falls the same height as the cylinder and because they have the same mass, the sphere-Earth system has the same initial potential energy and, therefore, the same total kinetic energy when it reaches the floor.	
ii. 2 points	
For selecting “Less than” and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the rotational inertia of the sphere is less than the rotational inertia of the cylinder, the sphere will rotate faster and, because $v = r\omega$, will move with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere will have a greater linear kinetic energy. Because the total kinetic energies of the sphere and cylinder are the same, the sphere must have less rotational kinetic energy.	
iii. 2 points	
For selecting “Greater than” and attempting a relevant justification	1 point
For a correct justification	1 point
Example: Because the sphere has a greater linear speed as it leaves the table, it will travel a greater horizontal distance before it reaches the floor.	