Week 1

AP Physics C: Mechanics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Physics C: Mechanics Name: ______________ Score: ______

1. Which of these is a **vector**?

   ☐ temperature

   ☐ velocity

   ☐ mass

   ☐ time

2. You jog once around a 400 m track and stop where you began. What is your displacement (in m)?

   ______________ m

3. On a position-versus-time graph, the slope represents the...

   ☐ acceleration

   ☐ velocity

   ☐ displacement

   ☐ distance

4. You walk forward at 1.5 $\frac{\text{m}}{\text{s}}$ in a train moving at 25 $\frac{\text{m}}{\text{s}}$. Your speed relative to the ground (in m/s)?

   ______________ m/s

5. A ball is thrown horizontally from a 5 m table ($g = 10$). How long until it lands (in s)?

   ______________ s

6. Select **all** scalars.

   ☐ speed

   ☐ energy

   ☐ force

   ☐ displacement

   Tick every correct option.

7. If position is $x(t)$, how do you get the acceleration?

   ☐ differentiate x once

   ☐ differentiate x twice

   ☐ integrate x once

☐ multiply x by t

8. A velocity-time graph is a flat line at 5 $\frac{\mathrm{m}}{\mathrm{s}}$ for 6 s. What displacement does it show (in m)?

_________________ m

9. Now you walk **backward** at 1.5 $\frac{\mathrm{m}}{\mathrm{s}}$ in the same 25 $\frac{\mathrm{m}}{\mathrm{s}}$ train. Your speed relative to the ground (in m/s)?

_________________ m/s

10. A ball dropped from a height and one thrown horizontally from the same height hit the ground at the same time.

☐ True ☐ False

AP Physics C: Mechanics

1. **velocity**

   Velocity has a size and a direction, so it is a vector. Temperature, mass, and time are scalars.

2. **0 m**

   Displacement is start-to-finish. You end where you began, so it is 0 – though the distance was 400 m.

3. **velocity**

   Slope of position vs time is dx/dt – the velocity.

4. **26.5 m/s**

   Same direction, so add them, $1.5 + 25 = 26.5\ \frac{\mathrm{m}}{\mathrm{s}}$.

5. **1 s**

   Looking at the vertical direction, $5 = \frac{1}{2}(10)t^2$, so $t^2 = 1$ and $t = 1$ s. The horizontal speed does not matter for the time.

6. **speed; energy**

   Speed and energy are scalars (size only). Force and displacement are vectors.

7. **differentiate x twice**

   $v = dx/dt$ and $a = dv/dt$, so a is the *second* derivative of position.

8. **30 m**

   Displacement is the area under the graph, $5 \times 6 = 30$ m.

9. **23.5 m/s**

   Opposite directions subtract, $25 - 1.5 = 23.5\ \frac{\mathrm{m}}{\mathrm{s}}$.

10. **True**

    Horizontal and vertical motion are independent – both have the same vertical drop, so the same fall time.

1.1 Scalars and Vectors

Name: ______________ Class: __________ Date: ____________ **Total: 16 marks**

KEY WORDS vector 矢量 • magnitude 大小 • components 分量 • scalar 标量 • direction 方向

Objective

Build the skills to answer exam questions on **scalars and vectors** — resolving a vector into components, finding its magnitude and direction, and adding vectors.

You must be able to:

- distinguish a **scalar** 标量 (magnitude only) from a **vector** 矢量 (magnitude **and** direction 方向)
- resolve a vector into **components** 分量: $A_x = A\cos\theta$, $A_y = A\sin\theta$
- find a vector's **magnitude** 大小 $A = \sqrt{A_x^2 + A_y^2}$ and its direction $\theta = \tan^{-1}(A_y/A_x)$
- add vectors by adding their components separately

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Scalar or vector?

A **scalar** has size only; a **vector** has size **and** direction.

scalar	**vector**
distance, speed, mass time, energy, temperature	displacement, velocity, force acceleration, momentum

■ Resolving into components

A vector of magnitude A at angle θ above the x-axis splits into

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta.$$

For $A = 10$ at $\theta = 30°$: $A_x = 10\cos 30° = 8.7$, $A_y = 10\sin 30° = 5.0$.

■ Magnitude and direction from components

Reverse the process with Pythagoras and the tangent:

$$A = \sqrt{A_x^2 + A_y^2}, \qquad \theta = \tan^{-1}\frac{A_y}{A_x}.$$

For $A_x = 3,\ A_y = 4$: $A = \sqrt{3^2 + 4^2} = 5$, and $\theta = \tan^{-1}(4/3) = 53°$.

■ **Adding vectors by components**

Add the x-components and the y-components **separately**, then recombine. Two forces $\vec{F}_1 = (3, 0)$ N and $\vec{F}_2 = (0, 4)$ N:

$$\vec{F}_{\text{net}} = (3 + 0,\ 0 + 4) = (3, 4)\ \text{N}, \qquad |\vec{F}_{\text{net}}| = 5\ \text{N}.$$

2 Practice

Now apply the methods above.

2.1 State whether each is a scalar or a vector: (a) speed, (b) velocity, (c) mass, (d) acceleration. [2]

2.2 A displacement has magnitude 20 m at 60° above the x-axis. Find its x- and y-components. [2]

2.3 A vector has components $A_x = 6$ and $A_y = 8$. Find its magnitude. [2]

2.4 Add $\vec{A} = (5, 2)$ and $\vec{B} = (-1, 3)$ by components, and give the magnitude of the resultant. [2]

3 Exam-style questions

3.1 Which quantity is a vector? [1]

- **A** speed
- **B** distance

- **C** acceleration
- **D** mass

3.2 A vector of magnitude 12 makes an angle of 30° with the x-axis. Its x-component is closest to [1]

- **A** 6.0
- **B** 10.4
- **C** 12
- **D** 0

3.3 A vector has components $A_x = 9$ and $A_y = 12$.

(a) Find the magnitude of the vector. [2]

(b) Find the angle it makes with the x-axis. [1]

3.4 Two velocity vectors are $\vec{v}_1 = (4, 0)$ m s^{-1} and $\vec{v}_2 = (0, -3)$ m s^{-1}.

(a) Find the components of the resultant. [1]

(b) Find the magnitude and direction of the resultant. [2]

Solutions

2.1 (a) scalar; (b) vector; (c) scalar; (d) vector.

2.2 $A_x = 20\cos 60° = 10$ m; $A_y = 20\sin 60° = 17$ m.

2.3 $A = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

2.4 $(5 - 1,\ 2 + 3) = (4, 5)$; magnitude $= \sqrt{4^2 + 5^2} = \sqrt{41} \approx 6.4$.

3.1 C — acceleration is a vector; the others are scalars.

3.2 B — $12\cos 30° = 10.4$.

3.3 (a) $A = \sqrt{9^2 + 12^2} = \sqrt{225} = 15$. (b) $\theta = \tan^{-1}(12/9) = 53°$.

3.4 (a) $(4, -3)$ m s^{-1}. (b) magnitude $\sqrt{4^2 + 3^2} = 5$ m s^{-1}; direction $\tan^{-1}(3/4) = 37°$ below the x-axis.

AP PHYSICS C: MECHANICS • EXERCISE SHEET

1.2 Displacement, Velocity, and Acceleration

Name: __________ Class: ________ Date: ________ **Total: 15 marks**

KEY WORDS acceleration 加速度 • velocity 速度 • integration 积分 • derivatives 导数

Objective

Build the skills to answer exam questions on **displacement, velocity, and acceleration** — the calculus links between them.

You must be able to:

- use **velocity** 速度 as the derivative of position, $v = \dfrac{dx}{dt}$
- use **acceleration** 加速度 as the derivative of velocity, $a = \dfrac{dv}{dt}$
- go the other way by **integration** 积分: $x = \int v\,dt$ and $v = \int a\,dt$
- read velocity as the slope of an x-t graph and displacement as the area under a v-t graph

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Derivatives: position to velocity to acceleration

Differentiate to move down the chain. If $x(t) = 3t^2$, then

$$v = \frac{dx}{dt} = 6t, \qquad a = \frac{dv}{dt} = 6.$$

At $t = 2$ s: $v = 12$ m s^{-1}, $a = 6$ m s^{-2}.

■ Integration: acceleration back to velocity

Integrate to move up the chain, adding the starting value. If $a = 4$ (constant) and $v_0 = 2$, then

$$v = \int a\,dt = 4t + 2.$$

■ **Slopes and areas on graphs**

- The **slope** of an x-t graph is the velocity.
- The **area** under a v-t graph is the displacement.

A v-t graph holds a steady 5 m s^{-1} for 4 s: displacement $= 5 \times 4 = 20$ m (the area).

2 Practice

Now apply the methods above.

2.1 A particle has $x(t) = 2t^3$. Find $v(t)$. [2]

2.2 A particle has $v(t) = 6t$. Find $a(t)$. [1]

2.3 A constant acceleration $a = 3$ m s^{-2} acts, with $v_0 = 5$ m s^{-1}. Write $v(t)$. [2]

2.4 A v-t graph is a constant 8 m s^{-1} for 3 s. Find the displacement. [2]

3 Exam-style questions

3.1 The velocity of an object is the ______ of its position with respect to time. [1]

- **A** integral
- **B** derivative
- **C** reciprocal
- **D** square

3.2 On a position-time graph, the velocity at a point is given by the [1]

- **A** area under the curve
- **B** slope of the tangent
- **C** height of the curve
- **D** curvature

3.3 A particle moves with $x(t) = t^3 - 6t$.

(a) Find $v(t)$. [2]

(b) Find the acceleration $a(t)$. [1]

3.4 A particle starts from rest with acceleration $a(t) = 6t$.

(a) Find $v(t)$. [2]

(b) Find the velocity at $t = 2$ s. [1]

Solutions

2.1 $v = \dfrac{dx}{dt} = 6t^2$.

2.2 $a = \dfrac{dv}{dt} = 6\ \mathrm{m\ s^{-2}}$.

2.3 $v = \int a\,dt = 3t + 5$.

2.4 Displacement = area = $8 \times 3 = 24$ m.

3.1 B — velocity is the derivative of position.

3.2 B — the slope of the tangent gives the velocity.

3.3 (a) $v = 3t^2 - 6$. (b) $a = 6t$.

3.4 (a) $v = \int 6t\,dt = 3t^2$. (b) $v(2) = 3(2)^2 = 12\ \mathrm{m\ s^{-1}}$.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

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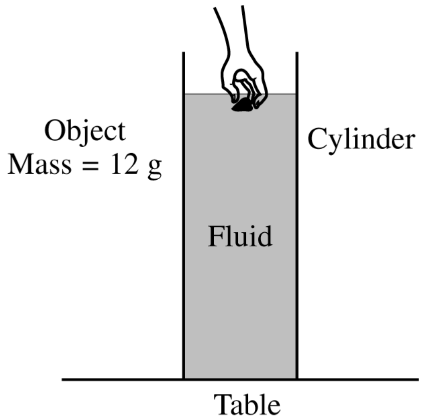
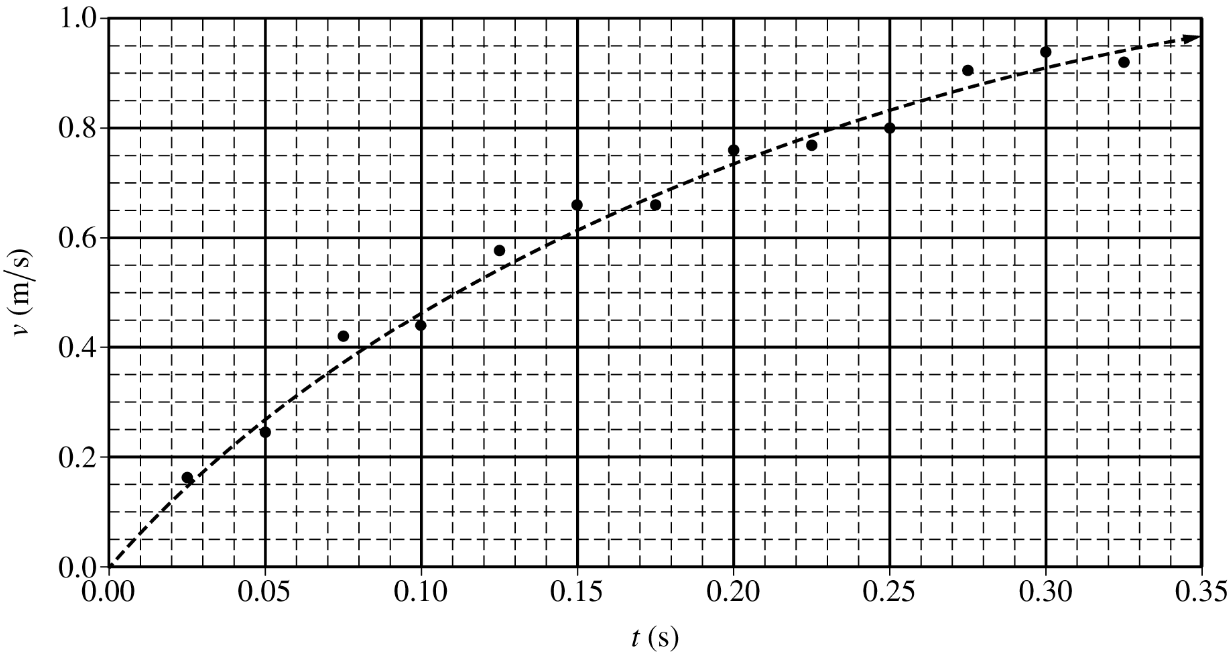


1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of m = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

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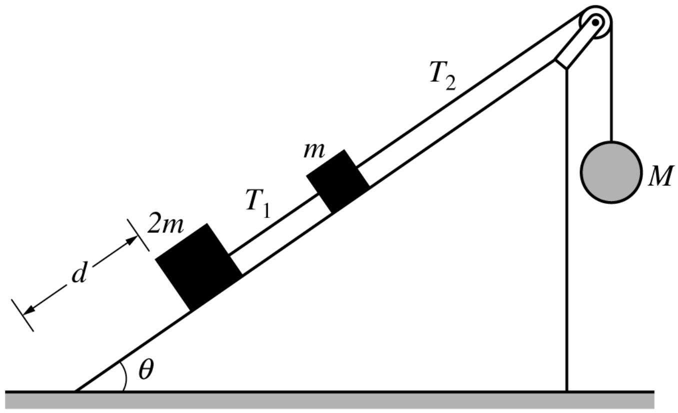


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

   (a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.

(b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

ii. The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

(c) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

ii. Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

iii. Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \mathrm{m/s^2}$, $L = 0.25\ \mathrm{m/s^4}$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

   (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

   (b) Calculate the speed of the rocket at $t = 6.0$ s.

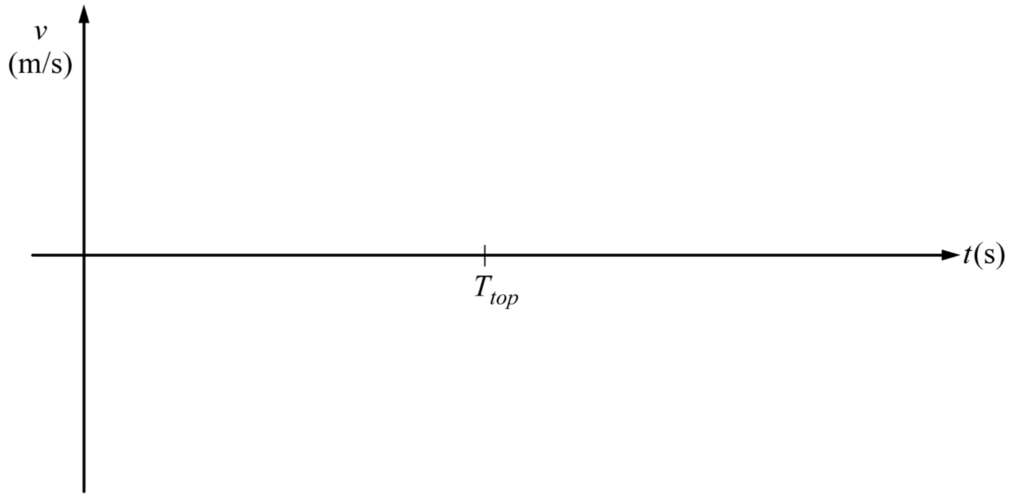
   (c)

      i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

      ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

   (d) Calculate the maximum height reached by the rocket relative to its launching point.

   (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



PHYSICS C: MECHANICS
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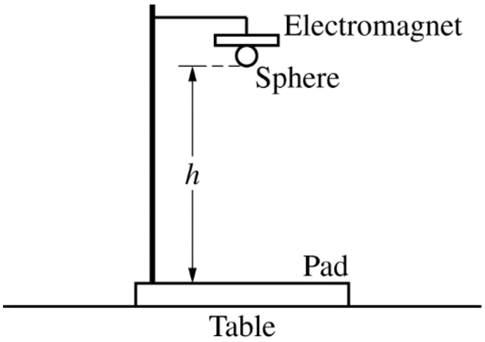


1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

   (a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

   ____ Greater than ____ Less than ____ Equal to

   Justify your answer.

   The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

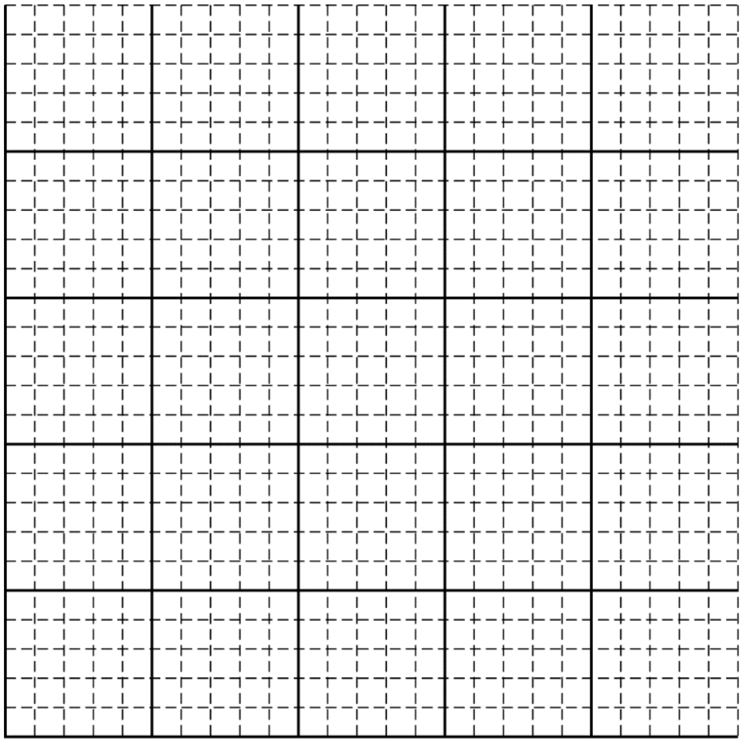
   (b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

   Vertical axis: ____________

   Horizontal axis: ____________

   Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \mathrm{m/s^2}$, $B = -0.524\ \mathrm{m/s}$, and $C = +0.080\ \mathrm{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

ii. Calculate the new experimental value for g.

iii. Using $9.81\ \mathrm{m/s^2}$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

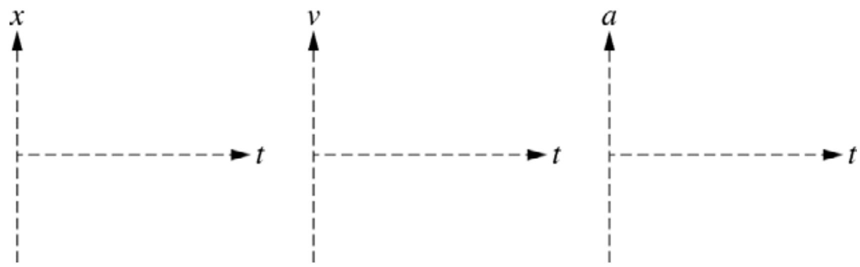
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of $\mathrm{kg/m}$.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
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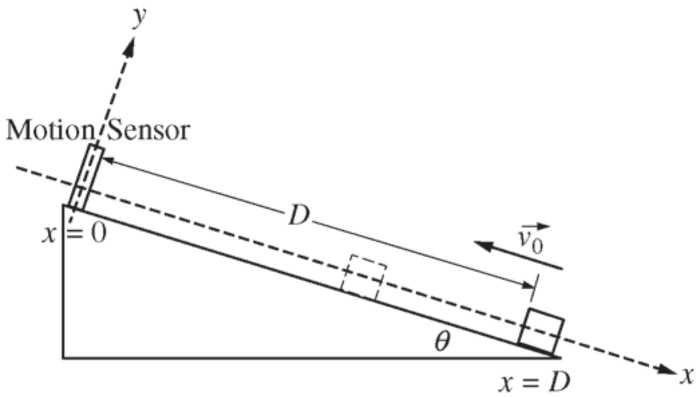


Mech.1.

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

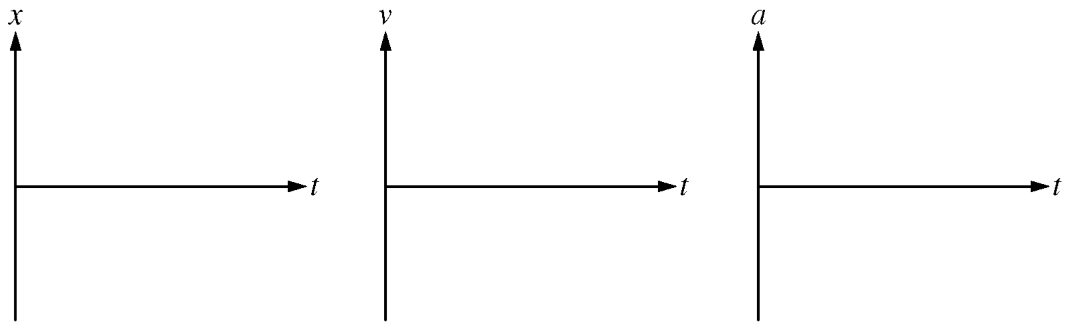
i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

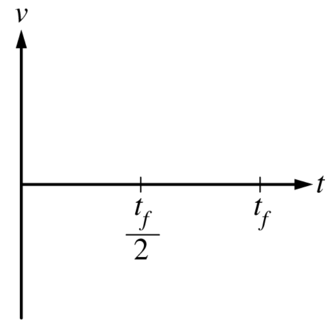
(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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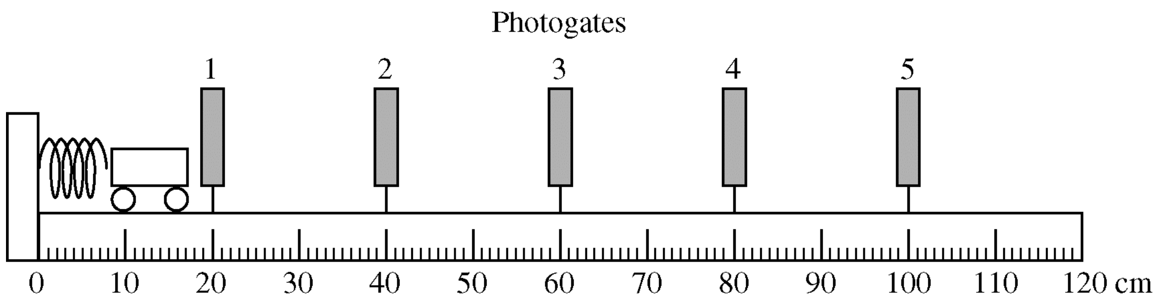


Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \mathrm{N/m^2}$ and $B = 150\ \mathrm{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

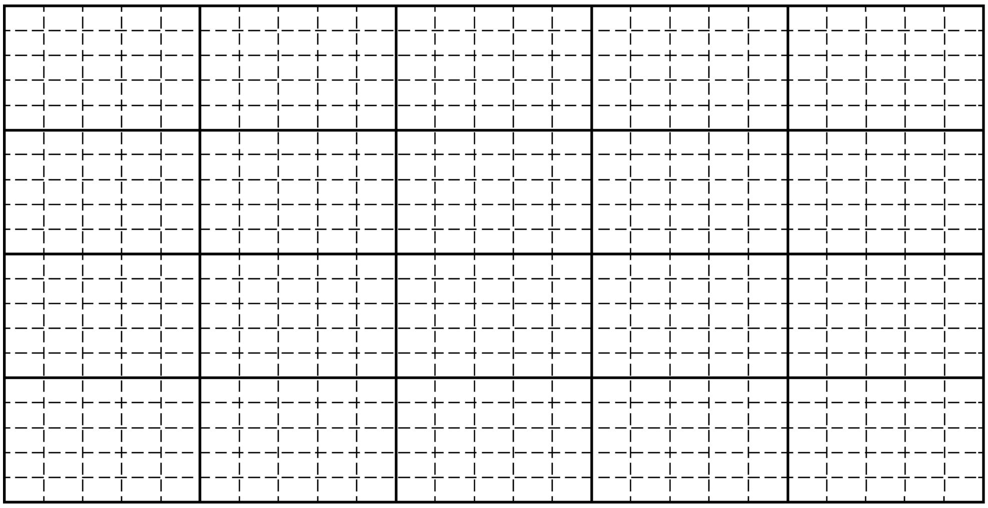
i. The speed of the cart immediately after it loses contact with the spring

ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \mathrm{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \mathrm{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

(c) On the axes below, plot the data points for the speed v of the cart as a function of position x. Clearly scale and label all axes, as appropriate.



(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

1.3 Representing Motion

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS acceleration 加速度 • velocity 速度 • displacement 位移

Objective

Build the skills to answer exam questions on **representing motion** with graphs.

You must be able to:

- read **velocity** 速度 as the slope of a position-time (x-t) graph
- read **acceleration** 加速度 as the slope of a velocity-time (v-t) graph
- read **displacement** 位移 as the area under a v-t graph
- match the shapes of x-t, v-t, and a-t graphs for the same motion

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Slopes give the next quantity down

- Slope of x-t = velocity.
- Slope of v-t = acceleration.

A straight, rising x-t line has constant slope, so constant velocity and zero acceleration.

■ Areas give the previous quantity up

- Area under v-t = displacement.
- Area under a-t = change in velocity.

A v-t triangle rising from 0 to 10 m s^{-1} over 4 s has area $\frac{1}{2}(4)(10) = 20$ m.

■ Matching graph shapes

For constant acceleration: x-t is a parabola (curving), v-t is a straight sloped line, a-t is a flat horizontal line.

2 Practice

Now apply the methods above.

2.1 What does the slope of a position-time graph represent? [1]

2.2 What does the area under a velocity-time graph represent? [1]

2.3 A v-t graph holds a constant 6 m s^{-1} for 5 s. Find the displacement. [2]

2.4 A v-t graph rises straight from 0 to 8 m s^{-1} over 2 s. Find the acceleration. [2]

3 Exam-style questions

3.1 The acceleration of an object equals the slope of its [1]

- **A** position-time graph
- **B** velocity-time graph
- **C** acceleration-time graph
- **D** force-time graph

3.2 For motion with constant acceleration, the velocity-time graph is a [1]

- **A** horizontal line
- **B** straight sloped line
- **C** parabola
- **D** circle

3.3 A v-t graph is a triangle: velocity rises straight from 0 to 12 m s^{-1} over 6 s.

(a) Find the acceleration. [2]

(b) Find the displacement (the area). [2]

3.4 An x-t graph is a horizontal straight line for 10 s.

(a) State the velocity. [1]

(b) State the acceleration. [1]

Solutions

2.1 The velocity.

2.2 The displacement.

2.3 Displacement = area = $6 \times 5 = 30$ m.

2.4 $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8}{2} = 4\ \text{m s}^{-2}$.

3.1 **B** — acceleration is the slope of the velocity-time graph.

3.2 **B** — constant acceleration gives a straight sloped v-t line.

3.3 (a) $a = 12/6 = 2\ \text{m s}^{-2}$. (b) area $= \frac{1}{2}(6)(12) = 36$ m.

3.4 (a) $v = 0$ (the position is constant). (b) $a = 0$.

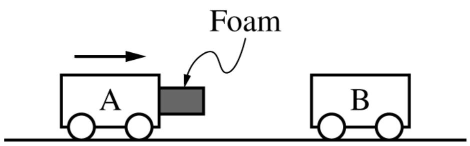
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

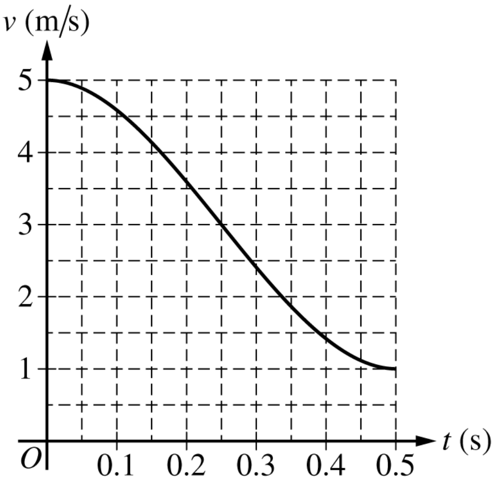
)irections: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, ,hich are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this ooklet in the spaces provided after each part.



. Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

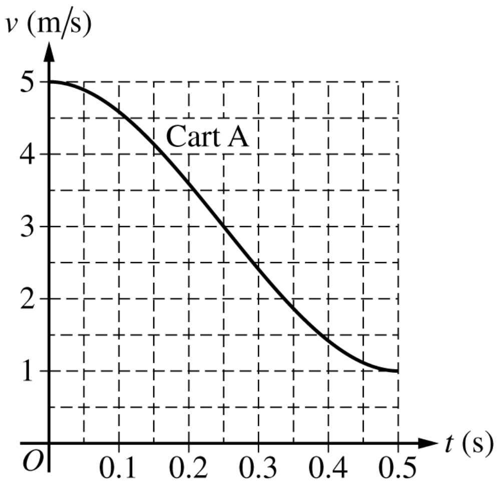


(a) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?

ii. Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.



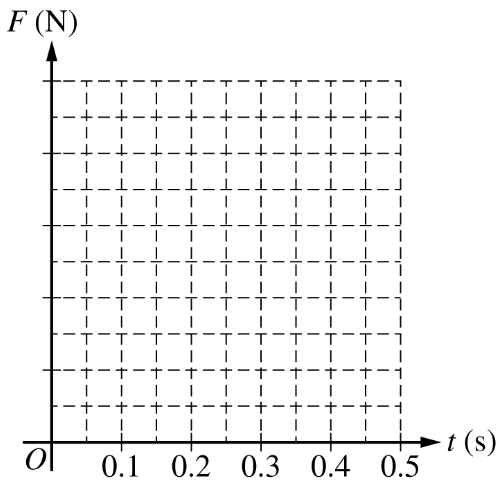
GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) For $0 \leq t \leq 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

i. Calculate the magnitude of the maximum net force acting on Cart A during this interval.

ii. On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.



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Continue your response to **QUESTION 1** on this page.

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.

(c) What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?

_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$

Justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

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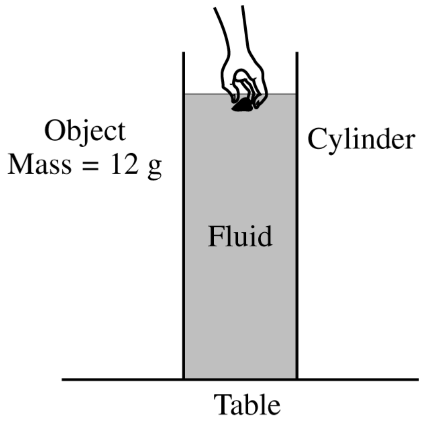
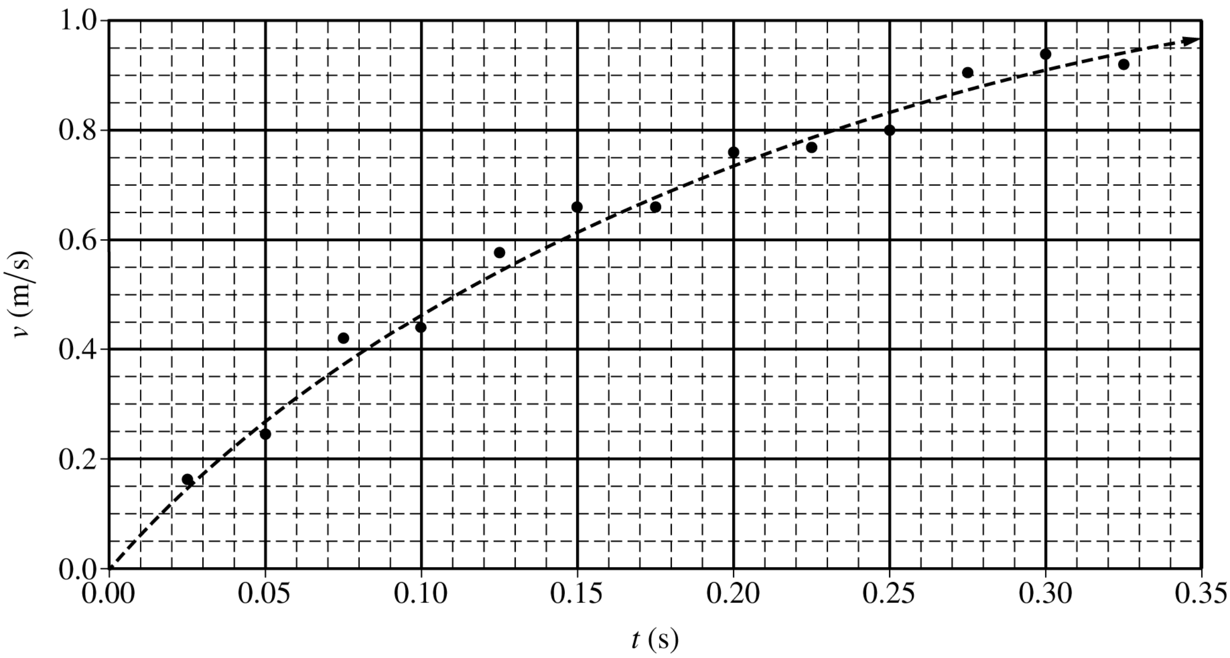


1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18\ \text{m/s}$ and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \mathrm{m/s^2}$, $L = 0.25\ \mathrm{m/s^4}$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

   (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

   (b) Calculate the speed of the rocket at $t = 6.0$ s.

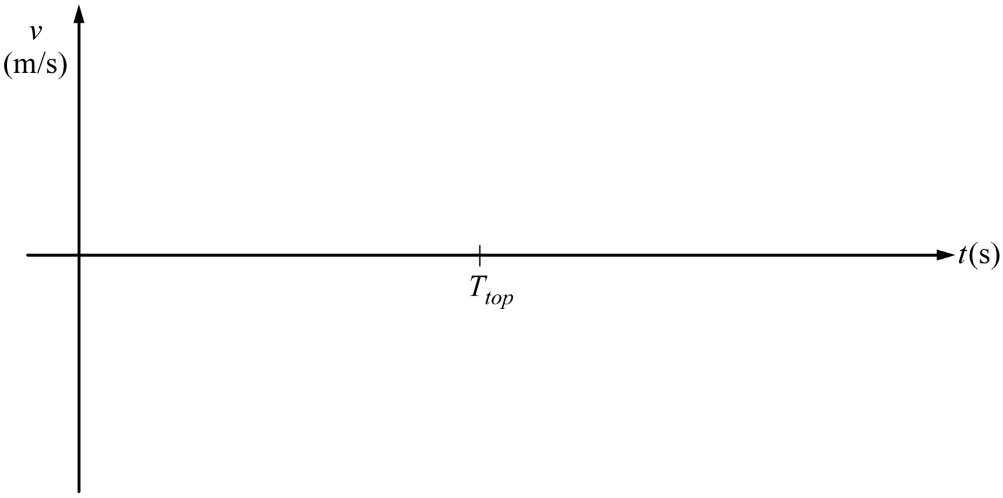
   (c)

      i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

      ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

   (d) Calculate the maximum height reached by the rocket relative to its launching point.

   (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



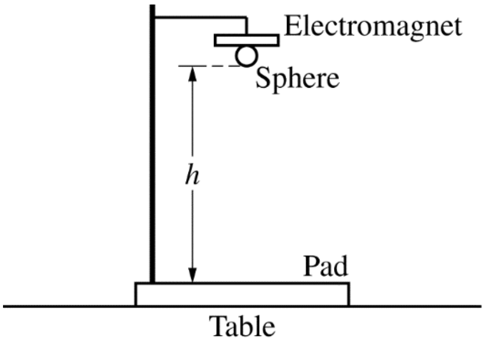
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

   (a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

   ____ Greater than ____ Less than ____ Equal to

   Justify your answer.

   The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

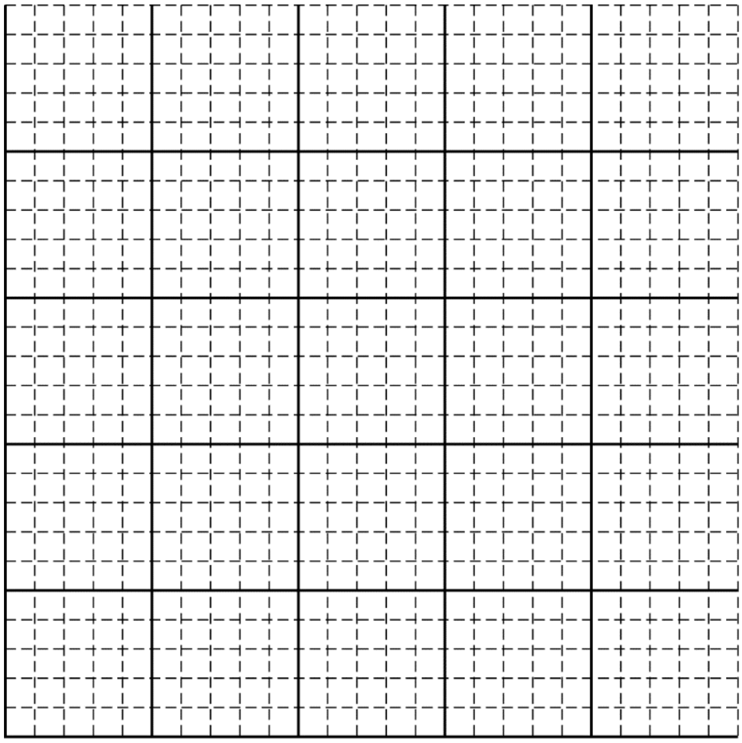
   (b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

   Vertical axis: ____________

   Horizontal axis: ____________

   Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

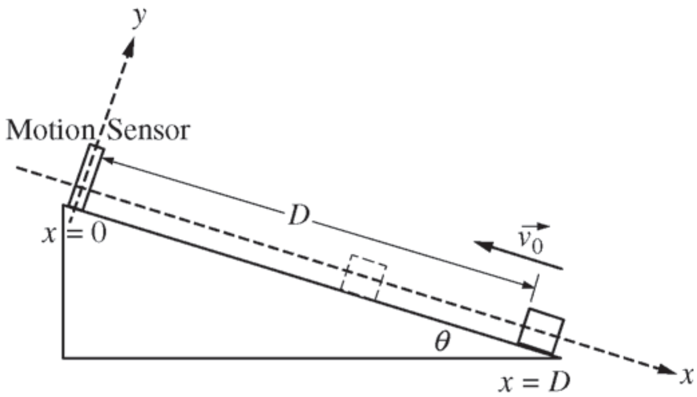
ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech.1.

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

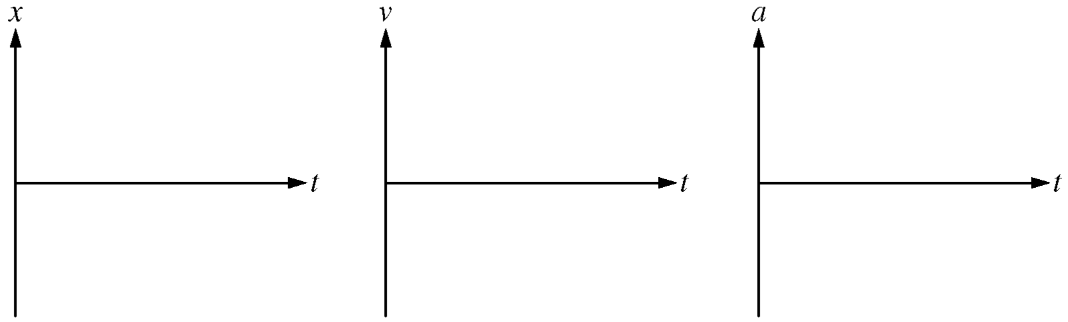
i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

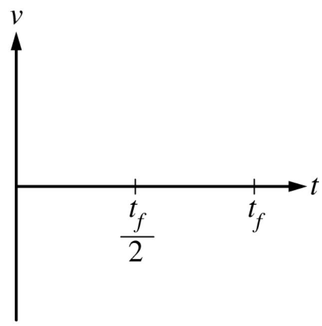
(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



1.4 Reference Frames and Relative Motion

Name: ______ Class: ______ Date: ______ **Total: 15 marks**

KEY WORDS reference frame 参考系 • relative velocity 相对速度 • relative motion 相对运动

Objective

Build the skills to answer exam questions on **reference frames and relative motion**.

You must be able to:

- describe motion relative to a chosen **reference frame 参考系**
- combine velocities: $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$
- find a **relative velocity 相对速度** by subtracting: $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
- apply this to boats in a current and objects on a moving vehicle

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Velocity is relative to a frame

A person walking 2 m s^{-1} forward on a train moving 30 m s^{-1} has a velocity relative to the ground of

$$v_{\text{ground}} = 30 + 2 = 32 \text{ m s}^{-1}.$$

Adding velocities in 1-D

Use signs for direction. Walking backward at 2 m s^{-1} on the same train: $30 + (-2) = 28$ m s^{-1} relative to the ground.

Relative velocity by subtraction

The velocity of A relative to B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$. Two cars, both measured from the ground: $v_A = +20$, $v_B = +15$. Then A relative to B is $20 - 15 = +5$ m s^{-1} (A pulls ahead).

Perpendicular velocities (a boat in a current)

A boat heads across a river at 3 m s^{-1}; the current carries it downstream at 4 m s^{-1}. The ground velocity combines by Pythagoras: $\sqrt{3^2 + 4^2} = 5$ m s^{-1}.

2 Practice

Now apply the methods above.

2.1 A passenger walks forward at 1.5 m s^{-1} on a bus moving 12 m s^{-1}. Find the passenger's velocity relative to the ground. [2]

2.2 Car A moves at +25 m s^{-1} and car B at +18 m s^{-1} (same direction). Find the velocity of A relative to B. [2]

2.3 Two cars approach head-on, each at 20 m s^{-1} relative to the ground. Find the speed of one relative to the other. [2]

2.4 A boat heads directly across a river at 6 m s^{-1} while the current flows at 8 m s^{-1}. Find the boat's speed relative to the ground. [2]

3 Exam-style questions

3.1 The velocity of an object depends on [1]

- **A** its mass
- **B** the reference frame chosen
- **C** its colour
- **D** nothing — it is absolute

3.2 Two cars move in the same direction at 30 m s^{-1} and 22 m s^{-1}. The faster car's velocity relative to the slower is [1]

- **A** 52 m s^{-1}
- **B** 8 m s^{-1}

- **C** 0
- **D** 22 m s^{-1}

3.3 A swimmer heads straight across a river at 2 m s^{-1}; the current is 1.5 m s^{-1} downstream.

(a) Find the swimmer's speed relative to the ground. [2]

(b) State why the swimmer does not land directly opposite the start. [1]

3.4 A ball is dropped inside a train moving at constant velocity.

(a) Describe its path as seen by a passenger on the train. [1]

(b) Describe its path as seen by someone standing on the ground. [1]

Solutions

2.1 $12 + 1.5 = 13.5$ m s^{-1} relative to the ground.

2.2 $25 - 18 = +7$ m s^{-1} (A ahead of B).

2.3 Take one direction positive: $20 - (-20) = 40$ m s^{-1}.

2.4 $\sqrt{6^2 + 8^2} = 10$ m s^{-1}.

3.1 B — velocity is defined relative to a reference frame.

3.2 B — $30 - 22 = 8$ m s^{-1}.

3.3 (a) $\sqrt{2^2 + 1.5^2} = 2.5$ m s^{-1}. (b) The current carries the swimmer downstream while crossing.

3.4 (a) Straight down. (b) A parabola — it keeps the train's forward velocity.

1.5 Motion in Two or Three Dimensions

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS time of flight 飞行时间 • independent 独立 • components 分量 • free fall 自由落体 • projectile motion 抛体运动

Objective

Build the skills to answer exam questions on **motion in two dimensions** — especially projectiles.

You must be able to:

- treat the horizontal and vertical motions as **independent** 独立
- use constant velocity horizontally and free-fall acceleration g vertically
- find the **time of flight** 飞行时间 from the vertical motion, then the range from the horizontal
- combine perpendicular velocity components into a resultant

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Independent axes

In projectile motion the two axes do not affect each other:

- horizontal: constant velocity, $x = v_x t$;
- vertical: free fall, $a_y = -g$.

They share only the **time** t.

■ Horizontal launch

A ball rolls off a table at $v_x = 5$ m s^{-1} from height $h = 1.25$ m. Vertical: $h = \frac{1}{2}gt^2$, so

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.25)}{10}} = 0.5 \text{ s}.$$

Horizontal range: $x = v_x t = 5 \times 0.5 = 2.5$ m.

■ **Combining velocity components**

At any instant the speed is $\sqrt{v_x^2 + v_y^2}$. If $v_x = 5$ and $v_y = 5$, speed $= \sqrt{50} \approx 7.1$ m s^{-1}.

2 Practice

Now apply the methods above.

2.1 State what the horizontal and vertical motions of a projectile share in common. [1]

2.2 A ball is thrown horizontally at 8 m s^{-1}. State its horizontal acceleration (ignore air resistance). [1]

2.3 A stone falls from rest for 2 s. Find its vertical velocity (use $g = 10$ m s^{-2}). [2]

2.4 A projectile has $v_x = 6$ m s^{-1} and $v_y = 8$ m s^{-1} at some instant. Find its speed. [2]

3 Exam-style questions

3.1 For a projectile (no air resistance), the horizontal velocity [1]

- **A** increases
- **B** decreases
- **C** stays constant
- **D** becomes zero at the top

3.2 Two balls leave a table at the same instant; one is dropped and one is thrown horizontally. They [1]

- **A** land at the same time
- **B** the dropped one lands first

- **C** the thrown one lands first
- **D** never land

3.3 A ball is launched horizontally at 10 m s^{-1} from a height of 5 m ($g = 10$ m s^{-2}).

(a) Find the time to reach the ground. [2]

(b) Find the horizontal range. [2]

3.4 A projectile's velocity components are $v_x = 12$ m s^{-1} and $v_y = -5$ m s^{-1}.

(a) Find the speed. [2]

(b) Find the angle below the horizontal. [1]

Solutions

2.1 The time of flight (both motions happen over the same time).

2.2 Zero — there is no horizontal force.

2.3 $v = gt = 10 \times 2 = 20\ \text{m s}^{-1}$.

2.4 $\sqrt{6^2 + 8^2} = 10\ \text{m s}^{-1}$.

3.1 **C** — with no horizontal force, the horizontal velocity is constant.

3.2 **A** — the vertical motions are identical, so they land together.

3.3 (a) $t = \sqrt{2h/g} = \sqrt{2(5)/10} = 1$ s. (b) $x = v_x t = 10 \times 1 = 10$ m.

3.4 (a) $\sqrt{12^2 + 5^2} = 13\ \text{m s}^{-1}$. (b) $\tan^{-1}(5/12) = 23°$ below the horizontal.

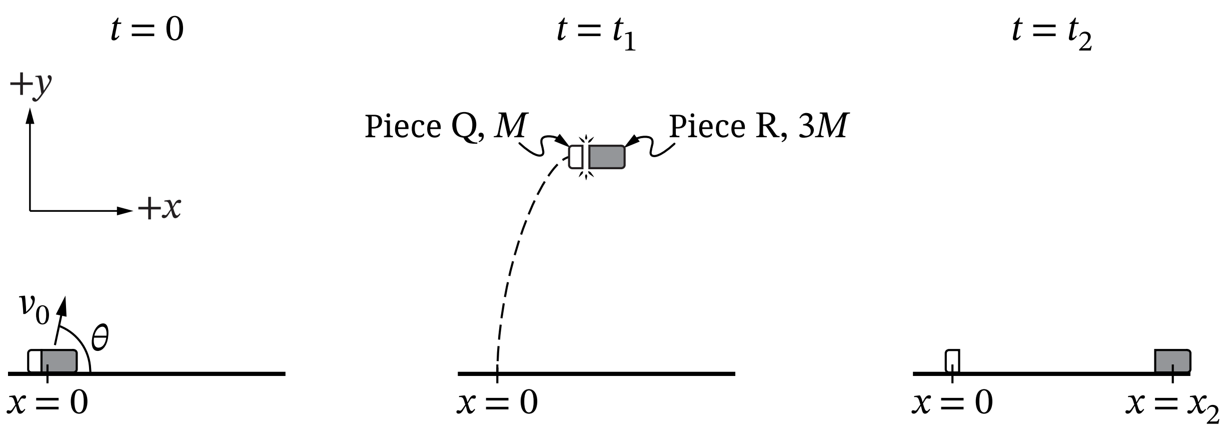
Question 2: Version J

2. A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally.
- At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

Figure 1



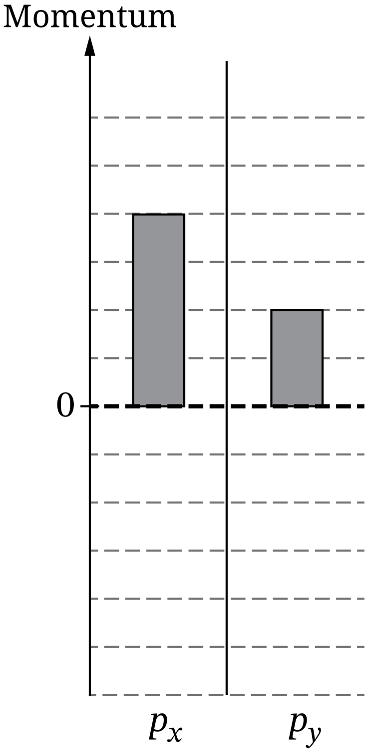
Note: Figure not drawn to scale.

Part A

The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

Figure 2

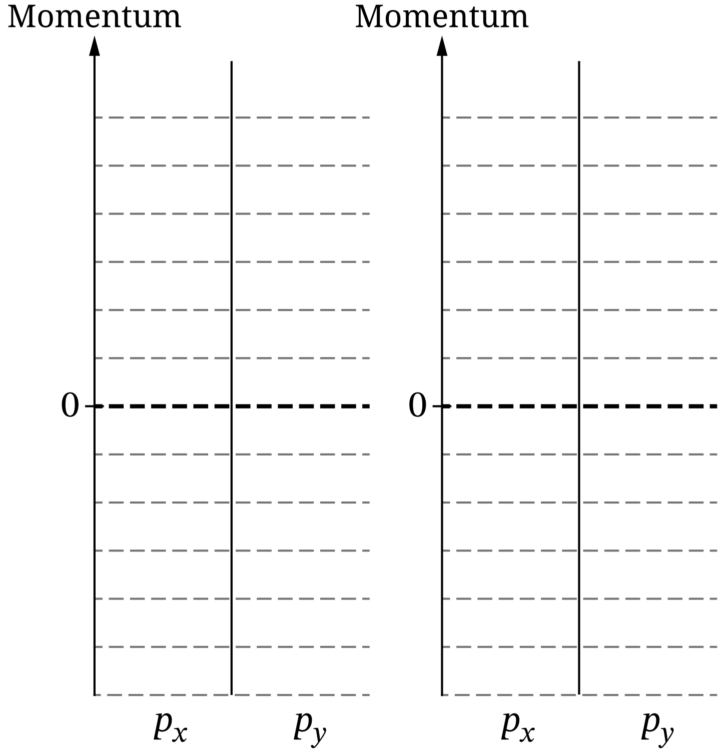
$t = 0$



Complete Projectile

Figure 3

$t = t_1$



Piece Q

Piece R

On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

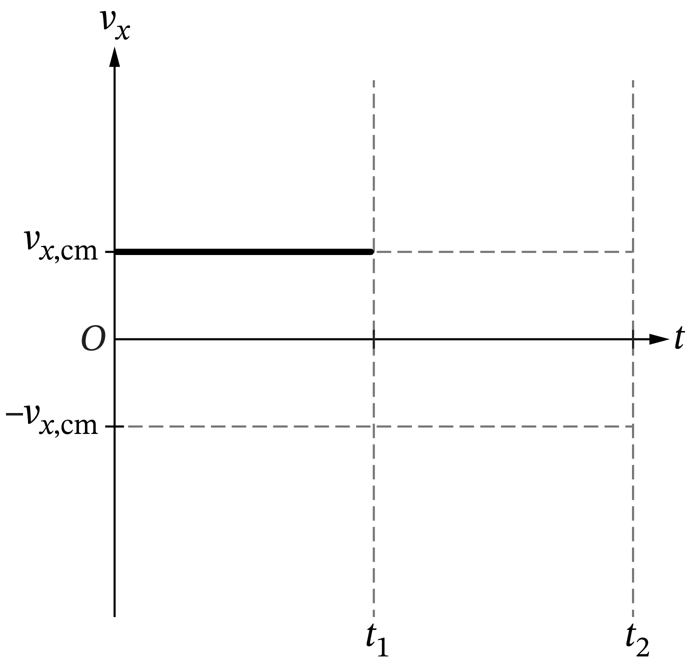
Part B

Derive an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Part C

The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\mathrm{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

Figure 4



On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

- Piece Q
- Piece R
- The center of mass of the two-piece system

Clearly **label** all lines or curves.

Part D

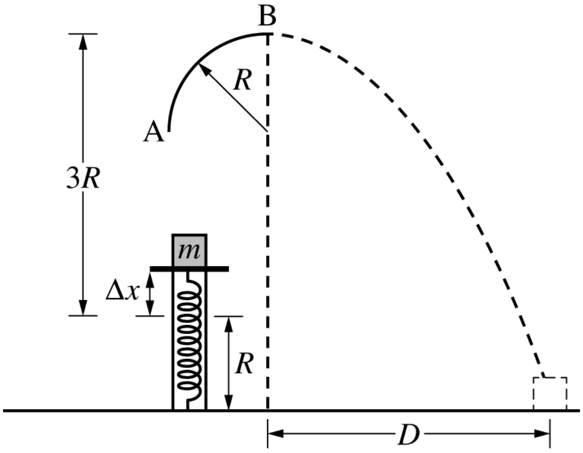
Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\mathrm{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\mathrm{new}} > x_2$
- $x_{\mathrm{new}} < x_2$
- $x_{\mathrm{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

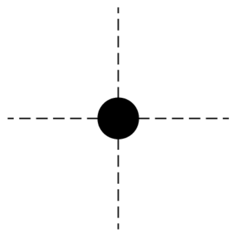
Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

GO ON TO THE NEXT PAGE.

(b)

i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

(d) Calculate the distance D that the block travels.

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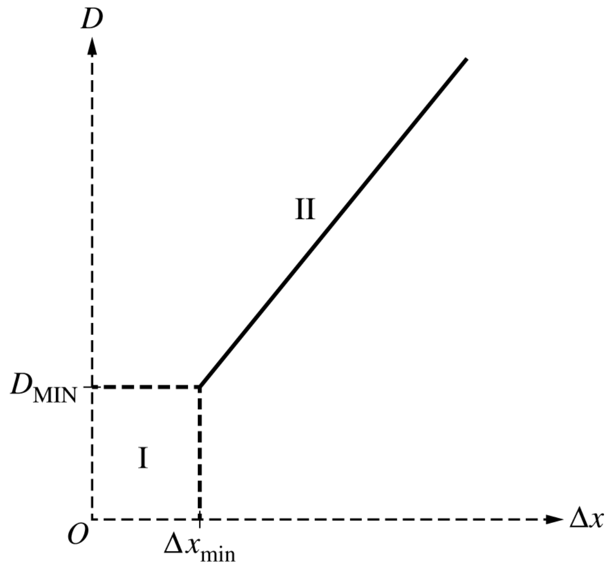
(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.

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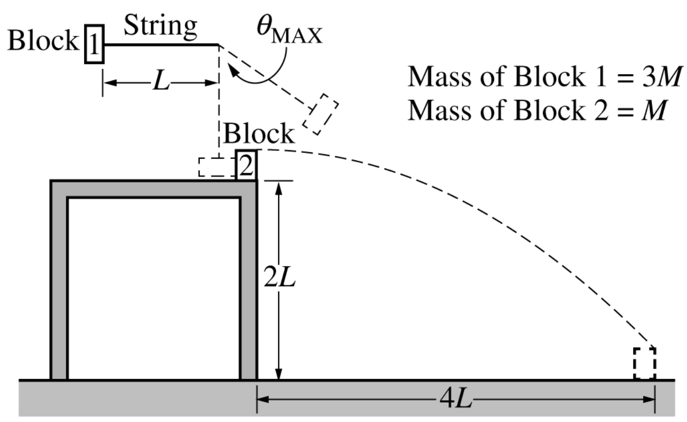
STOP

END OF EXAM

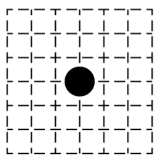


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

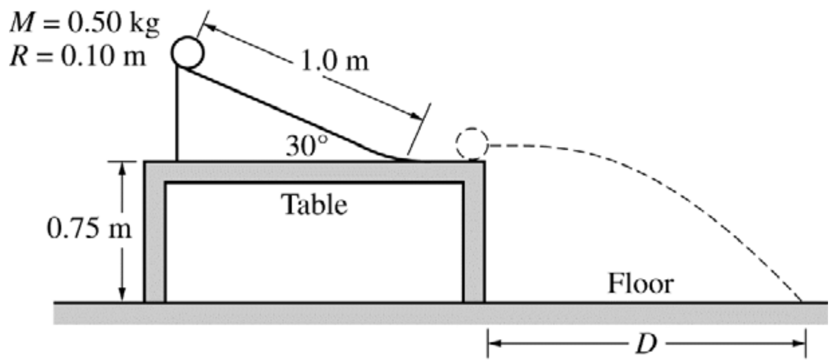
For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

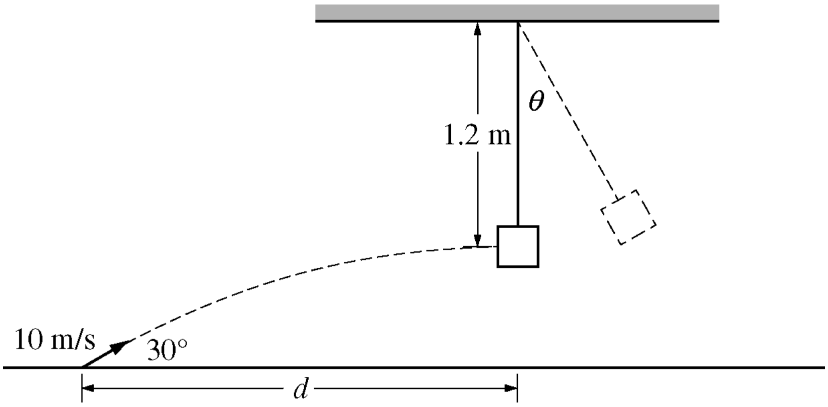
iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

(c) Calculate the speed of the block just after the dart strikes.

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

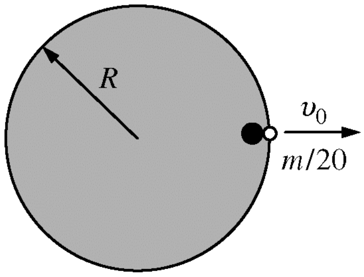
_____ Increase _____ Decrease _____ Stay the same

Justify your answer.

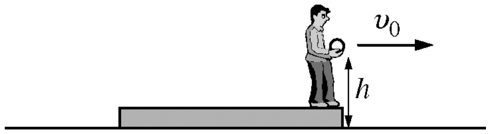
ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Justify your answer.



Top View



Side View

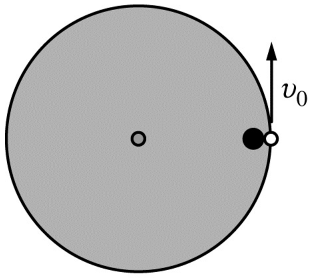
Mech. 3.

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

(c) Derive an expression for the time it will take the disk to stop sliding.



Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM