

7.4 Energy of Simple Harmonic Oscillators

Name: _____ Class: _____ Date: _____

Total: 13 marks**KEY WORDS** kinetic energy 动能 • potential energy 势能 • amplitude 振幅 • equilibrium 平衡位置

Objective

Build the skills to answer exam questions on **the energy of simple harmonic oscillators**.

You must be able to:

- use the total mechanical energy $E = \frac{1}{2}kA^2$
- describe the trade between **potential energy** 势能 $\frac{1}{2}kx^2$ and **kinetic energy** 动能 $\frac{1}{2}mv^2$
- state that E is all potential at the extremes and all kinetic at equilibrium
- explain that $E \propto A^2$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Total energy

$$E = \frac{1}{2}kA^2.$$

A spring with $k = 200 \text{ N m}^{-1}$ and amplitude 0.05 m stores $E = \frac{1}{2}(200)(0.05)^2 = 0.25 \text{ J}$.

■ The energy trade

At the extremes it is all potential; at equilibrium it is all kinetic. In between it is shared, but the total is constant.

■ Amplitude squared

Because $E \propto A^2$, doubling the amplitude quadruples the energy.

2 Practice

Now apply the methods above.

2.1 A spring with $k = 100 \text{ N m}^{-1}$ oscillates with amplitude 0.2 m. Find the total energy. [2]

2.2 At which point is the energy entirely kinetic? [1]

2.3 If the amplitude doubles, by what factor does the total energy change? [1]

2.4 A spring with $k = 100 \text{ N m}^{-1}$ is displaced 0.1 m. Find the stored potential energy. [2]

3 Exam-style questions

3.1 The total energy of a simple harmonic oscillator is [1]

- A $\frac{1}{2}kA^2$
 - B $\frac{1}{2}kA$
 - C kA^2
 - D $\frac{1}{2}mA^2$
-

3.2 Doubling the amplitude changes the total energy by a factor of [1]

- A 2
 - B 4
 - C 1/2
 - D 1
-

3.3 A spring with $k = 400 \text{ N m}^{-1}$ oscillates with amplitude 0.1 m.

(a) Find the total energy. [2]

(b) State the kinetic energy at the equilibrium position. [1]

3.4 A mass oscillates on a spring.

(a) State where the potential energy is greatest. [1]

(b) State whether the kinetic and potential energies can both be maximum at the same instant. [1]

Solutions

2.1 $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.2)^2 = 2 \text{ J}.$

2.2 At the equilibrium position.

2.3 Four times.

2.4 $U = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.1)^2 = 0.5 \text{ J}.$

3.1 A — $E = \frac{1}{2}kA^2.$

3.2 B — $E \propto A^2$, so $2^2 = 4.$

3.3 (a) $E = \frac{1}{2}(400)(0.1)^2 = 2 \text{ J}.$ (b) 2 J (all kinetic there).

3.4 (a) At the extremes (maximum x). (b) No — each is greatest where the other is zero.