

7.3 Representing and Analyzing SHM

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS phase 相位 • amplitude 振幅 • equilibrium 平衡位置

Objective

Build the skills to answer exam questions on **representing and analyzing SHM**.

You must be able to:

- use the position equation $x(t) = A \cos(\omega t + \varphi)$ with **amplitude** 振幅 A and **phase** 相位 φ
- find the maximum speed $v_{max} = A\omega$ (at equilibrium)
- find the maximum acceleration $a_{max} = A\omega^2$ (at the extremes)
- state that displacement and acceleration are out of phase

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The position equation

$$x(t) = A \cos(\omega t + \varphi).$$

A is the amplitude (maximum displacement) and φ the phase (starting point).

■ Maximum speed and acceleration

Differentiating gives

$$v_{max} = A\omega \text{ (at equilibrium),} \quad a_{max} = A\omega^2 \text{ (at the extremes).}$$

■ Out of step

Speed is greatest at the middle, where the displacement and acceleration are zero; acceleration is greatest at the ends, where the speed is zero.

2 Practice

Now apply the methods above.

2.1 An oscillator has $A = 0.2$ m and $\omega = 4$ rad s⁻¹. Find its maximum speed. [2]

2.2 For the same oscillator, find the maximum acceleration. [2]

2.3 Where in the motion is the speed greatest? [1]

2.4 In $x = A \cos(\omega t + \varphi)$, what does A represent? [1]

3 Exam-style questions

3.1 In SHM, the maximum speed occurs at the [1]

- **A** extremes
 - **B** equilibrium position
 - **C** turning points
 - **D** it is constant
-

3.2 The maximum acceleration in SHM equals [1]

- **A** $A\omega$
 - **B** $A\omega^2$
 - **C** A/ω
 - **D** ω^2/A
-

3.3 An oscillator has $A = 0.05$ m and $\omega = 10$ rad s⁻¹.

(a) Find the maximum speed. [2]

(b) Find the maximum acceleration.

[2]

3.4 A mass in SHM starts from rest at its far extreme.

(a) State whether to model it with cosine or sine (phase).

[1]

(b) State where its acceleration is largest.

[1]

Solutions

2.1 $v_{max} = A\omega = 0.2 \times 4 = 0.8 \text{ m s}^{-1}$.

2.2 $a_{max} = A\omega^2 = 0.2 \times 16 = 3.2 \text{ m s}^{-2}$.

2.3 At the equilibrium position.

2.4 The amplitude (the maximum displacement).

3.1 B — maximum speed is at equilibrium.

3.2 B — $a_{max} = A\omega^2$.

3.3 (a) $v_{max} = 0.05 \times 10 = 0.5 \text{ m s}^{-1}$. (b) $a_{max} = 0.05 \times 100 = 5 \text{ m s}^{-2}$.

3.4 (a) Cosine, with $\varphi = 0$. (b) At the extremes.