Exercise walk-through

Defining Simple Harmonic Motion (SHM) ▪ 7.1

eXercise

You must be able to

- state the defining condition of **simple harmonic motion** 简谐运动: $F = -kx$
- explain the minus sign (the **restoring force** 回复力 points back toward equilibrium)
- use $a = -\omega^2 x$ with $\omega^2 = k/m$
- identify where the acceleration is largest and where it is zero

Method — The defining condition

SHM happens when the restoring force is proportional to the displacement and points back toward equilibrium:

$$F = -kx.$$

Method — The acceleration form

Dividing by mass,

$$a = -\frac{k}{m}x = -\omega^2 x, \qquad \omega^2 = \frac{k}{m}.$$

Acceleration is proportional to displacement but opposite in direction.

Method — Where it is largest

The acceleration is greatest at the extremes (largest x) and zero at equilibrium ($x = 0$) – the opposite of the speed.

Practice 2.1 ▪ 2 marks

A spring with $k = 20$ N m^{-1} is stretched 0.15 m. Find the restoring force.

Solution 2.1

Method: The defining condition

Worked steps

$|F| = kx = 20 \times 0.15 = 3$ N M1 A1.

Practice 2.2 ▪ 1 mark

State the meaning of the minus sign in $F = -kx$.

Solution 2.2

Worked steps

The force opposes the displacement (points back to equilibrium) B1.

Practice 2.3 ▪ 2 marks

For SHM with $\omega^2 = 25$ s^{-2} at $x = 0.2$ m, find the acceleration magnitude.

Solution 2.3

Method: The acceleration form

Worked steps

$|a| = \omega^2 x = 25 \times 0.2 = 5$ m s^{-2} M1 A1.

Practice 2.4 ▪ 1 mark

Where in the motion is the acceleration largest?

Solution 2.4

Method: Where it is largest

Worked steps

At the extremes (largest x) B1.

Exam-style 3.1 ▪ 1 mark

A motion is simple harmonic when the restoring force is

A constant

B proportional to displacement and opposite in direction

C proportional to speed

D zero

Solution 3.1

Method: The defining condition

A constant

B proportional to displacement and opposite in direction ✓

C proportional to speed

D zero

Worked steps

B — proportional to displacement and opposite in direction.

Exam-style 3.2 ▪ 1 mark

In SHM, the acceleration is zero at the

A extremes

B equilibrium position

C turning points

D it is never zero

Solution 3.2

Method: Where it is largest

A extremes

B equilibrium position ✓

C turning points

D it is never zero

Worked steps

B — at equilibrium, $x = 0$ so $a = 0$.

Exam-style 3.3 ▪ 2 marks

A 0.5 kg mass on a spring ($k = 8$ N m^{-1}) is pulled 0.1 m and released.
(a) Find the restoring force at release.
(b) Find the acceleration at release.

Solution 3.3

Worked steps

(a) $F = -kx = -8 \times 0.1 = -0.8$ N (magnitude 0.8 N) M1 A1. (b) $\omega^2 = k/m = 16$; $a = -\omega^2 x = -1.6$ m s^{-2} M1 A1.

Exam-style 3.4 ▪ 1 mark

A pendulum swings with a small amplitude.
(a) State whether its motion is SHM.
(b) State the condition (angle) required for this to hold.

Solution 3.4

Worked steps

(a) Yes (approximately) B1. (b) Small angles (roughly under 15°) B1.