Exercise walk-through

Motion of Orbiting Satellites ▪ 6.6

eXercise

You must be able to

- explain that **gravity 引力** provides the centripetal force for a circular orbit
- derive the orbital speed: $v = \sqrt{\frac{GM}{r}}$
- use **Kepler's third law 开普勒第三定律**: $T^2 \propto r^3$
- state that a satellite's own mass cancels out

Method — Gravity is the centripetal force

For a circular orbit,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}.$$

Lower orbits are faster; higher orbits are slower.

Method — Kepler's third law

The period and radius are linked by

$$T^2 = \frac{4\pi^2}{GM} r^3,$$

so $T^2 \propto r^3$ – a bigger orbit takes much longer.

Method — The mass cancels

The satellite's mass m cancels, so orbital speed and period do not depend on it.

Practice 2.1 ▪ 1 mark

What provides the centripetal force that holds a satellite in orbit?

Solution 2.1

Method: Gravity is the centripetal force

Worked steps

Gravity B1.

Practice 2.2 ▪ 1 mark

A satellite moves to a lower orbit. State whether its speed increases or decreases.

Solution 2.2

Worked steps

It increases (lower orbits are faster) **B1**.

Practice 2.3 ▪ 2 marks

Compute the orbital speed where $GM/r = 4.0 \times 10^7$ m^2 s^{-2}.

Solution 2.3

Method: The mass cancels

Worked steps

$v = \sqrt{4.0 \times 10^7} \approx 6300$ m s^{-1} **M1** **A1**.

Practice 2.4 ▪ 1 mark

State whether a heavier satellite at the same radius orbits faster, slower, or the same.

Solution 2.4

Method: Gravity is the centripetal force

Worked steps

The same (mass cancels) **B1**.

Exam-style 3.1 ▪ 1 mark

The orbital speed of a satellite depends on

A its own mass

B the central mass and the orbital radius

C its colour

D the launch angle

Solution 3.1

Method: The mass cancels

A its own mass

B the central mass and the orbital radius ✓

C its colour

D the launch angle

Worked steps

B — the central mass and the orbital radius.

Exam-style 3.2 ▪ 1 mark

According to Kepler's third law, a larger orbit has a

A shorter period

B longer period

C the same period

D zero period

Solution 3.2

Method: Kepler's third law

A shorter period

B longer period ✓

C the same period

D zero period

Worked steps

B — a larger orbit has a longer period.

Exam-style 3.3 ▪ 1 mark

Satellite B orbits at twice the radius of satellite A (same planet).
(a) State how B's orbital speed compares with A's (qualitatively).
(b) Using $T^2 \propto r^3$, state how B's period compares with A's.

Solution 3.3

Method: The mass cancels

Worked steps

(a) B is slower (larger r gives smaller v) **B1**. (b) $T^2 \propto r^3$, so $T_B/T_A = 2^{3/2} \approx 2.8$ times longer **M1** **A1**.

Exam-style 3.4 ▪ 2 marks

A satellite orbits where $GM/r = 2.5 \times 10^7$ m^2 s^{-2}.

(a) Find its orbital speed.

(b) State whether doubling the satellite's mass changes this speed.

Solution 3.4

Method: The mass cancels

Worked steps

(a) $v = \sqrt{2.5 \times 10^7} \approx 5000$ m s^{-1} **M1** **A1**. (b) No — mass cancels, so the speed is unchanged **B1**.