Exercise walk-through

Rolling ▪ 6.5

eXercise

You must be able to

- use the **rolling without slipping 无滑滚动** condition: $v = \omega r$ (and $a = \alpha r$)
- state that a rolling object's total kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- explain that the contact point is instantaneously at rest
- reason about which shape wins a downhill race

Method — The no-slip condition

When a wheel rolls without slipping, its speed and spin are locked:

$$v = \omega r.$$

A wheel of radius 0.3 m spinning at 10 rad s^{-1} rolls forward at $v = 3$ m s^{-1}.

Method — Total kinetic energy

A rolling object carries both translational and rotational KE:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Method — The downhill race

Sharing energy between motion and spin, a shape with a **smaller** rotational inertia (per mr^2) keeps more for speed and reaches the bottom first: solid sphere before cylinder before ring.

Practice 2.1 ▪ 2 marks

A wheel of radius 0.4 m rolls without slipping at $\omega = 5$ rad s^{-1}. Find its forward speed.

Solution 2.1

Method: The no-slip condition

Worked steps

$v = \omega r = 5 \times 0.4 = 2$ m s^{-1} M1 A1.

Practice 2.2 ▪ 1 mark

State the speed of the point where a rolling wheel touches the ground.

Solution 2.2

Method: The no-slip condition

Worked steps

Zero (instantaneously at rest) **B1**.

Practice 2.3 ▪ 2 marks

Write the two terms in the total kinetic energy of a rolling object.

Solution 2.3

Method: Total kinetic energy

Worked steps

$\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational) **B1** **B1**.

Practice 2.4 ▪ 2 marks

A rolling ball of radius 0.1 m moves at 2 m s^{-1}. Find its angular velocity.

Solution 2.4

Method: The no-slip condition

Worked steps

$\omega = v/r = 2/0.1 = 20$ rad s^{-1} **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

For rolling without slipping, the link between speed and spin is

A $v = \omega / r$

B $v = \omega r$

C $v = r / \omega$

D $v = \omega^2 r$

Solution 3.1

Method: The no-slip condition

A $v = \omega / r$

B $v = \omega r$ ✓

C $v = r / \omega$

D $v = \omega^2 r$

Worked steps

B — $v = \omega r$.

Exam-style 3.2 ▪ 1 mark

Released together down the same ramp, which reaches the bottom first?

A a hollow ring

B a solid cylinder

C a solid sphere

D they tie

Solution 3.2

Method: The downhill race

A a hollow ring

B a solid cylinder

C a solid sphere ✓

D they tie

Worked steps

C — the solid sphere (smallest rotational inertia per mr^2) wins.

Exam-style 3.3 ▪ 1 mark

A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.
(a) State which arrives first.
(b) Explain why, in terms of how the energy is shared.

Solution 3.3

Method: The downhill race

Worked steps

(a) The solid disc **B1**. (b) The hoop puts more energy into spin (larger I), leaving less for speed, so it is slower **M1** **A1**.

Exam-style 3.4 ▪ 2 marks

A ball of radius 0.2 m rolls without slipping at 6 m s^{-1}.
(a) Find its angular velocity.
(b) State what happens to v if it begins to skid (slip).

Solution 3.4

Method: The no-slip condition

Worked steps

(a) $\omega = v/r = 6/0.2 = 30$ rad s^{-1} **M1** **A1**. (b) $v = \omega r$ no longer holds — the speeds decouple **B1**.