Past-paper walk-through

Rotational Inertia ▪ 5.4

eXercise

Questions in this set

- 2024 Set 1 Q3
- 2024 Set 2 Q3
- 2023 Set 1 Q2
- 2023 Set 2 Q3
- 2022 Set 2 Q3
- 2021 Set 1 Q3
- 2021 Set 2 Q2
- 2019 Set 1 Q3
- 2019 Set 2 Q3
- 2018 Q3

Questions in this set

- 2016 Q3
- 2015 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

Question 3

(a) [Figure: A schematic representation of the rod alone (no pole, no string, no block), extending diagonally with a pivot circle at the lower-left end. Points marked along the rod: P, C, and Q (in that order from the pivot outward).]
On the following representation of the rod, draw and label the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.

Question 3

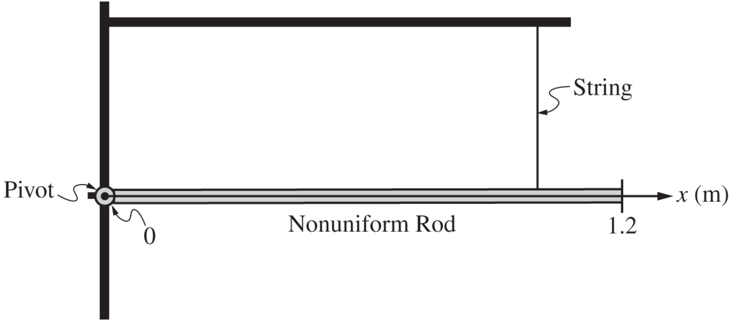


Figure 3

<u>Note:</u> Figure not drawn to scale.

Question 3

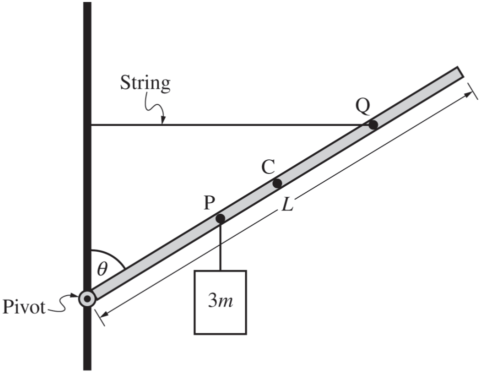


Figure 1

Note: Figure not drawn to scale.

Question 3

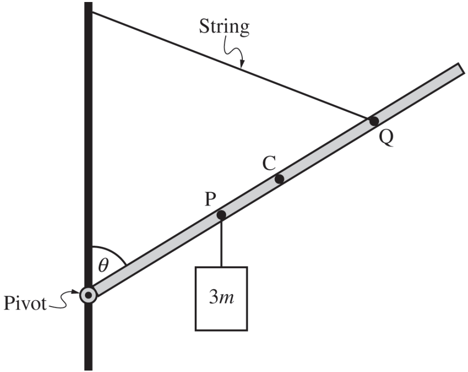


Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot.

Question 3

Derive an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(c) [Figure 2: Same setup as Figure 1, but the string now connects from Point Q on the rod to a higher point on the vertical pole (the string attachment point is above the

Question 3

top of the pole shown in Figure 1), making a steeper angle with the pole. The rod still makes angle θ with the pole, with Points P, C, Q marked along the rod, and the block of mass $3m$ hanging from Point P. Pivot labeled "Pivot-S" at the base. Note: Figure not drawn to scale.]

The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\ new}$ compare with the original tension F_T from part (b)? Justify your reasoning.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(d)(i) [Figure 3: A nonuniform rod attached horizontally to a pivot on a vertical pole, extending to the right along a horizontal x (m) axis from 0 at the pivot to 1.2 at the

Question 3

far end labeled "Nonuniform Rod". A string labeled "String" connects from a point partway along the rod up to the top of the vertical pole, forming a right-triangle-like support. Pivot labeled "Pivot-S" at the base of the pole. Note: Figure not drawn to scale.]

A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

Calculate the mass of the rod.

(d)(ii) Calculate the rotational inertia of the rod about the pivot.

Solution 3

(a) Mark scheme

- Draw and label: a downward force $F_{T,block}$ (or $3mg$) at point P (tension from the string holding the hanging block); a downward gravitational force mg at point C (the rod's own weight, acting at its center of mass); a leftward force $F_{T,string}$ at point Q (tension from the string to the pole); and a force directed up and to the right at the pivot, F_{pivot} (the pivot's reaction force), with no other (extraneous) forces drawn. Points: (1) correctly drawn/labeled downward forces at P and C; (2) correctly drawn/labeled leftward force at Q; (3) correctly drawn/labeled up-and-right force at the pivot, with no extraneous forces.

Solution 3

(b) Mark scheme

- Balance torques about the pivot ($\Sigma\tau = 0$), with the hanging mass $3m$ at P ($\frac{3L}{8}$ from pivot), the rod's weight mg at C ($\frac{4L}{8} = \frac{L}{2}$ from pivot), and the string tension F_T at Q ($\frac{6L}{8}$ from pivot, acting with perpendicular-component $F_T \cos\theta$ since the string is horizontal and the rod makes angle θ with the vertical pole): $3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$, so $\left(\frac{9}{8}\right) mg\sin\theta + \left(\frac{4}{8}\right) mg\sin\theta = \left(\frac{6}{8}\right) F_T\cos\theta$, giving $13mg\sin\theta = 6F_T\cos\theta \Rightarrow F_T = \dfrac{13}{6} mg\tan\theta$. (Scoring note: a maximum of 3 of the 4 points may be earned if sin and cos are reversed throughout for all three torque terms.) Points: (1) net torque on rod equals zero; (2) correct torque

Solution 3

expression for the hanging mass; (3) correct torque expression for the rod's own gravitational force; (4) correct torque expression for the string tension.

Solution 3

(c) Mark scheme

- Because the torque that the string must supply on the rod is always the same (it must still balance the unchanged torques from the rod's weight and the hanging mass), moving the string's attachment point higher on the pole increases the angle between the string and the rod, which increases the string's perpendicular moment arm. To produce the same torque with a larger moment arm, the tension $F_{T,new}$ must be smaller. So as the angle between the string and rod increases, the force exerted on the rod by the string decreases. Points: (1) indicating the torque exerted on the rod by the string is always the same; (2) stating that as the angle between the string and rod increases, the tension force decreases.

Solution 3

(d)(i) Mark scheme

- The nonuniform rod has linear mass density $\lambda(x) = A + Bx$ (with $A = 6.0$ kg/m, $B = 10.0$ kg/m^2) over its 1.2 m length. Total mass: $M = \int dm = \int_0^{1.2} \lambda\, dx = \int_0^{1.2} (6 + 10x)\, dx = \left(6x + \frac{10x^2}{2}\right)\Big|_0^{1.2} = 6(1.2) + 5(1.2)^2 = 7.2 + 7.2 = 14.4$ kg. Points: (1) indicating the total mass is the sum (integral) of differential masses along the rod, $M = \int dm$; (2) correctly writing dm in terms of x (i.e. $dm = \lambda\, dx = (6 + 10x)\, dx$); (3) a correct numeric answer with correct units, $M = 14.4$ kg.

Solution 3

(d)(ii) Mark scheme

- Rotational inertia about the pivot: $I = \int r^2\, dm$ with $dm = \lambda\, dx$ and $r = x$: $I = \int_0^{1.2} (A + Bx)x^2\, dx = \int_0^{1.2} (6 + 10x)x^2\, dx = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Big|_0^{1.2} = \frac{6(1.2)^3}{3} + \frac{10(1.2)^4}{4} = 3.456 + 5.184 = 8.64$ kg · m^2. Points: (1) correct substitution of λ into an integral expression for rotational inertia; (2) correct integration; (3) a correct numeric answer with correct units, $I = 8.64$ kg · m^2.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]
Note: Figure not drawn to scale.
A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal

Question 3

string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, draw and label the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.

[Figure: A shaded disk viewed face-on, with a dot at the center marking the axle, and point A marked on the upper-right edge of the disk (blank, for the student to draw force arrows on).]

Question 3

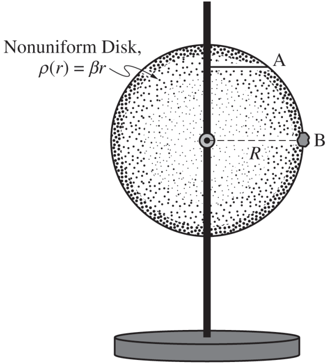


Figure 3

Note: Figure not drawn to scale.

Question 3

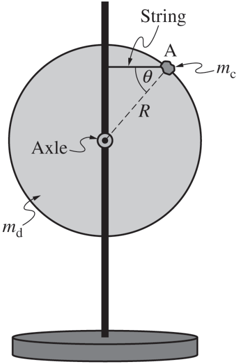


Figure 1

Note: Figure not drawn to scale.

Question 3

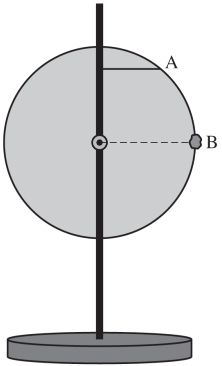


Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(b) Derive an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(c) [Figure 2: The same disk-axle-pole setup, now with point A marked on the upper edge of the disk and point B marked at the right edge of the disk, horizontally in line with the axle; a dashed horizontal line connects the axle to point B. Note: Figure not drawn to scale.]

The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,new}$ compare with tension F_T from part (b)? Justify your reasoning.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(d)(i) [Figure 3: A nonuniform disk (shown with radial shading/stippling denoting varying density) mounted on the same axle-pole setup, labeled "Nonuniform Disk, $\rho(r) = \beta r$". Point A is marked on the upper edge of the disk, and point B is marked at the right edge of the disk, horizontally in line with the axle (dashed line from axle to B). The disk radius is labeled R. Note: Figure not drawn to scale.]

A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0$ kg/m^3. Calculate the rotational inertia of the disk about the axle.

Question 3

(d)(ii) The string connecting the disk to the pole is cut. Calculate the magnitude of the initial angular acceleration of the clay-disk system.

Solution 3

(a) Mark scheme

- Force diagram on the clay-disk system with three distinct, correctly placed/labeled external forces (each arrow starting on, and pointing away from, its point of application): (1) a downward gravitational force on the disk, $m_d g$ (or a single downward gravitational force on the clay-disk system as a whole), drawn at the disk's center / appropriate location; (2) a downward gravitational force on the lump of clay, $m_c g$, drawn at Point A; (3) a leftward tension force F_T exerted by the string, drawn at Point A; and (4) a force directed up-and-to-the-right exerted by the axle, F_{axle}, drawn at the axle – with no other (extraneous) forces shown.

Solution 3

(b) Mark scheme

- Net torque about the axle is zero (the disk is in rotational equilibrium): $\Sigma\tau = 0$, i.e. $\tau_{clay} - \tau_{string} = 0$, where only the weight of the clay and the string tension exert torque about the axle. Torque from the clay's weight: $\tau_{clay} = Rm_c g\cos\theta$. Torque from the string tension: $\tau_{string} = RF_T \sin\theta$. Setting them equal: $Rm_c g\cos\theta = RF_T\sin\theta \Rightarrow F_T = \dfrac{m_c g\cos\theta}{\sin\theta} = m_c g\cot\theta$. Points: 1 for a multi-step derivation indicating the net torque is zero, 1 for indicating only the clay's weight and the string tension exert torque on the system about the axle, 1 for a correct torque expression from the clay's weight ($\tau_{clay} = Rm_c g\cos\theta$), 1 for a correct torque expression from the string tension ($\tau_{string} = RF_T\sin\theta$). (A maximum of 3

Solution 3

of these 4 points can be earned if sin and cos are consistently reversed in both torque expressions.)

Solution 3

(c)

Mark scheme

- $F_{T,new} > F_T$ (the tension is greater when the clay is at Point B). Justification: there is a direct relationship between the torque exerted by the clay's weight and the string tension (from part (b), $F_T \propto \tau_{clay}$). At Point B (horizontally in line with the axle) the component of the clay's weight perpendicular to the radial direction – equivalently the lever arm for the weight – is larger than at Point A, so the torque from the clay's weight is greater at B. Since the net torque must still be zero, the tension needed to balance it, $F_{T,new}$, must also be greater than F_T.

Solution 3

(d)(i) Mark scheme

- Rotational inertia of the nonuniform disk ($\rho(r) = \beta r$, radius $R = 0.3$ m) found by integration, treating a thin ring of radius r and thickness dr as $dm = \rho(2\pi r)dr = \beta r(2\pi r)dr = 2\pi\beta r^2\,dr$:

$$I = \int r^2\,dm = \int_0^{0.3\text{ m}} r^2\rho(2\pi r)\,dr = \int_0^{0.3\text{ m}} 2\pi\beta r^4\,dr = \frac{2\pi\beta R^5}{5}.$$

With $\beta = 4.0\text{ kg/m}^3$ and $R = 0.3$ m: $I = \dfrac{2\pi(4.0\text{ kg/m}^3)(0.3\text{ m})^5}{5} = 0.012\text{ kg}{\cdot}\text{m}^2$.
Points: 1 for setting up integration to calculate rotational inertia, 1 for either

Solution 3

substituting $\rho(2\pi r)\,dr$ for dm or indicating the correct integration limits, 1 for the correct final answer $I = 0.012$ kg·m^2, including units.

Solution 3

(d)(ii) Mark scheme

- After the string is cut, apply the rotational form of Newton's second law, $\Sigma\tau = I\alpha$, to the clay-disk system: the only remaining torque about the axle is from the clay's weight, $\tau_{net} = Rm_c g$ (clay still at Point A). The system's rotational inertia is the sum of the disk's and the clay's rotational inertia, $I_{system} = I_{disk} + I_{clay}$, where $I_{clay} = m_c R^2$ (point mass at radius R) and $I_{disk} = 0.012$ kg·m^2 from part (d)(i). So

$$\alpha = \frac{\tau_{net}}{I_{system}} = \frac{Rm_c g}{I_{disk} + I_{clay}} = \frac{(0.3\text{ m})(0.60\text{ kg})(9.8\text{ m/s}^2)}{(0.012\text{ kg}\cdot\text{m}^2) + (0.60\text{ kg})(0.3\text{ m})^2} = 26.7\text{ rad/s}^2.$$

Solution 3

Points: 1 for using the rotational form of Newton's second law, 1 for a correct expression for the net torque exerted on the clay-disk system, 1 for indicating that the system's rotational inertia is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$.

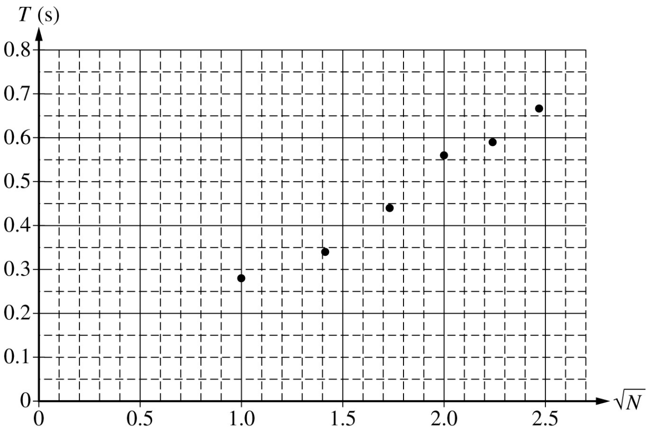
The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

Question 2

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N. Express your answer in terms of M, R, κ, N, and physical constants, as appropriate.

Question 2



Question 2

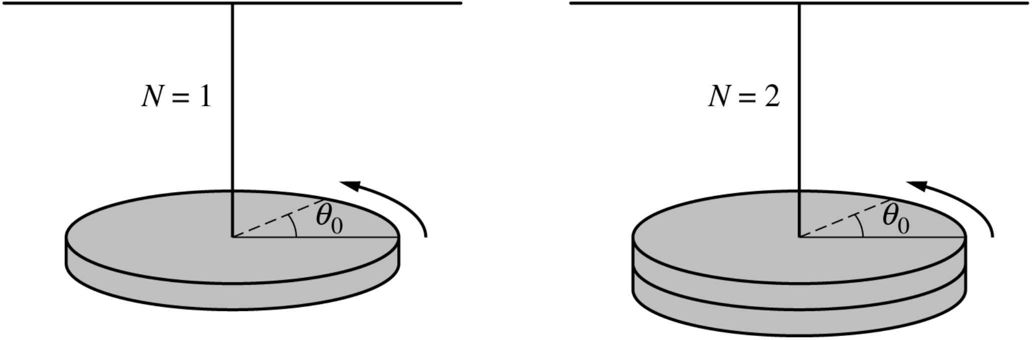


Figure 1 Figure 2

Question 2

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical

Question 2

disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2}\kappa(\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy K_{max} of the torsional pendulum as a function of N for $N \geq 1$.
[Blank axes provided: y-axis labeled K_{max}, x-axis labeled N, for the student to sketch a curve.]

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical

Question 2

disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(c)(i) [Figure: Graph of T (s) on the y-axis from 0 to 0.8 in increments of 0.1, versus $\sqrt{N}$ on the x-axis from 0 to 2.5 in increments of 0.5. Scattered data points are plotted showing an approximately linear increasing trend, roughly from $(\sqrt{N} \approx 0.7, T \approx 0.25)$ up to $(\sqrt{N} \approx 2.3, T \approx 0.65)$, for the student to draw a best-fit line through.]
The student plots the data for T as a function of $\sqrt{N}$, as shown.
Draw the best-fit line for the data.

(c)(ii) The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N $\cdot$ m. Using the graph, calculate the mass M of a single disk.

(c)(iii) The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single

Question 2

source of experimental error that could result in the observed difference in the value of M. Justify your answer.

Question 2

2023 Set 1 ▪ Q2

[Figure 1: A torsional pendulum with a single uniform disk ($N = 1$) suspended from a light wire, showing an angular displacement θ_0 from the untwisted position. Figure 2: The same torsional pendulum with a second identical disk attached below the first ($N = 2$), also displaced by angle θ_0.]

A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical

Question 2

disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

(d)(i) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i)?

_______ greater than _______ less than _______ the same as

Justify your answer.

(d)(ii) When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_{U}. When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_{U} and $\omega_{\text{non-U}}$?

Question 2

$\omega_{\text{U}} > \omega_{\text{non-U}}$ $\omega_{\text{U}} < \omega_{\text{non-U}}$ $\omega_{\text{U}} = \omega_{\text{non-U}}$
Briefly justify your answer.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.
[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]
[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

Question 3

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \dfrac{1}{18}ML^2$.

Question 3

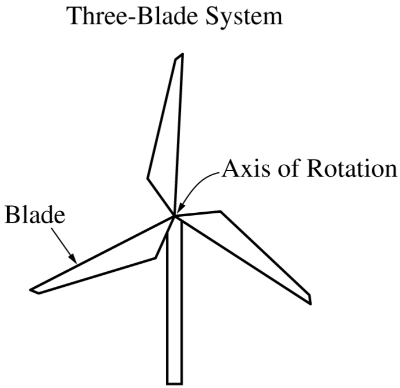


Figure 1

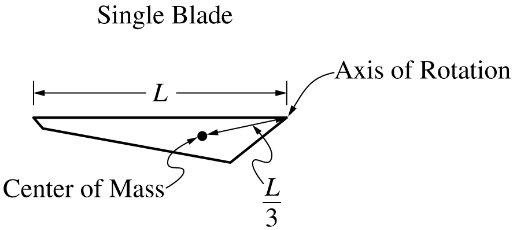


Figure 2

Question 3

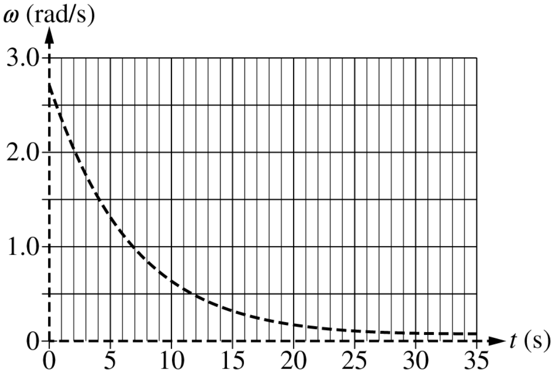


Figure 4

Question 3

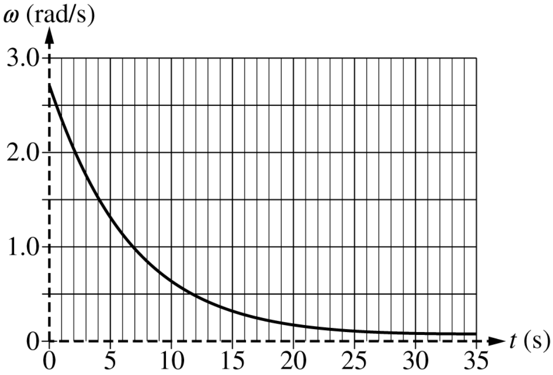


Figure 3

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(c)(i) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

Question 3

[Figure 3: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; curve starts at $(0, 2.6)$ and decays exponentially toward 0 as t increases, with a dashed vertical line at $t = 0$ marking the initial value.]
Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.
(c)(ii) Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
(c)(iii) Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the

Question 3

equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.
[Figure 4: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; a dashed curve shows the original $\omega_0 e^{-\beta_0 t}$ decay from $(0, 2.6)$ toward 0.]

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.
[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]
[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow away from the board, and should represent the location at which that force acts.
[A blank horizontal rectangle representing the board, with a dashed vertical centerline and dashed horizontal centerline, for the student to draw and label force vectors acting on the board.]

Question 3

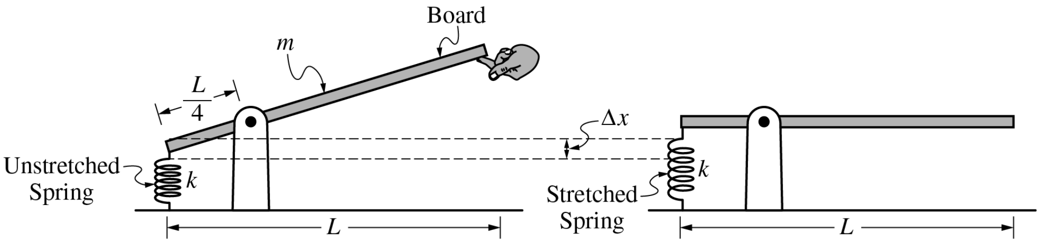
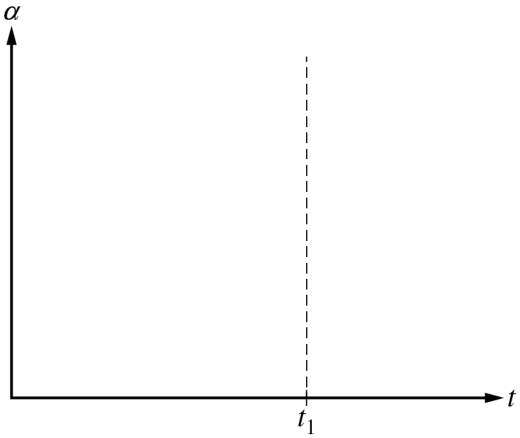


Figure 1

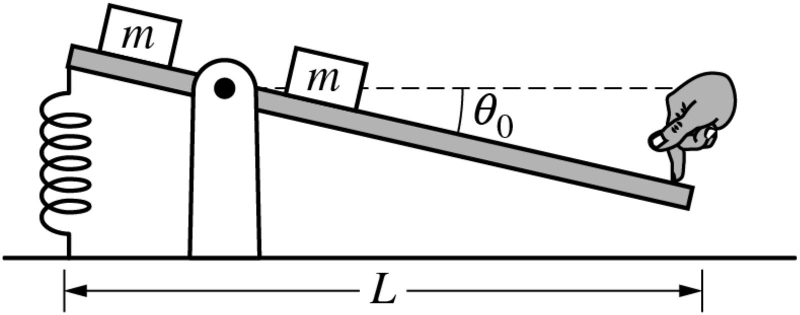
Figure 2

Note: Figures not drawn to scale.

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.
[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]
[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(c)(i) [Figure: A board pushed down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest. Note: Figure not drawn to scale.]
A student pushes down the board on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.
At time $t = 0$, the board is released. At time t_1, the board first crosses the horizontal. Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.
[A blank graph with vertical axis α and horizontal axis t, with a marked point t_1 on the time axis, for the student to sketch angular acceleration vs. time.]

Question 3

(c)(ii) Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(d) [Figure: The same board-spring-pivot setup, with two blocks of equal mass *m* attached to the board equal distances from the pivot point, as shown. Note: Figure not drawn to scale.]
Two blocks of equal mass *m* are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. The new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?
$\alpha' > \alpha_0$ $\alpha' < \alpha_0$ $\alpha' = \alpha_0$
Justify your answer.

Solution 3

(a) Mark scheme

- Free-body diagram on the board rectangle, 1 point each: (1) Spring force F_{spring} (or F_S, "spring force") drawn as an arrow pointing downward at the location where the spring attaches (the right end of the board, $L/4$ from the pivot). (2) Force of gravity F_g (or mg, F_G, W, "grav force" — not "G"/"g" alone) drawn pointing downward at the center of the board (its center of mass). (3) Pivot force F_{pivot} (or F_N, F_n, N, "normal force") drawn pointing upward at the pivot location (the fixed support at the board's center). Each force must be correctly located and labeled; extraneous forces cap the score at 2 points.

Solution 3

(b) Mark scheme

- Indicate the sum of torques about the pivot equals zero (equilibrium) — 1 point: $\Sigma\tau = 0$. Substitute mg for the gravitational force and $k\Delta x$ for the spring force into the torque equation, using the given lever arms ($L/4$ each) — 1 point: $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$. Solve for a correct expression for Δx — 1 point: $\Delta x = \frac{mg}{k}$ (earned regardless of an overall sign).

Solution 3

(c)(i)

Mark scheme

- Sketch of α vs. t from $t = 0$ to $t = t_1$, 1 point each: (1) The curve begins at a maximum (positive) value at $t = 0$ and monotonically decreases to a minimum of zero at $t = t_1$ (when the board reaches horizontal/equilibrium, so the net torque and hence α is zero there). (2) The curve is concave down over the entire interval $0 < t < t_1$ (not a straight line or concave-up decrease). Behavior of the sketch beyond t_1 is not scored.

Solution 3

(c)(ii) Mark scheme

- Write the rotational form of Newton's second law with the spring and gravitational torques in opposite directions — 1 point: $\tau_{spring} - \tau_g = I\alpha$ (i.e. $I_B\alpha_0$). Include $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque (the correct perpendicular lever-arm factor for the small angle θ_0) — 1 point: $F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$. Substitute mg for the force of gravity and $k\Delta x_2$ for the spring force — 1 point: $d_{spring}k\Delta x_2(\cos\theta_0) - d_g\, mg(\cos\theta_0) = I_B\alpha_0$. Substitute the correct lever arms $d_{spring} = d_g = L/4$ — 1 point: $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$, giving $\alpha_0 = \frac{L}{4I_B}\left(k\Delta x_2\cos\theta_0 - mg\cos\theta_0\right)$.

Solution 3

Solution 3

(d) Mark scheme

- Answer: $\alpha' < \alpha_0$ — select this choice with an attempt at a relevant justification (1 point). Justification must indicate the net torque on the system does not change — 1 point (the spring and gravity on the board still exert the same torques at the same lever arms; the two added equal masses sit at equal distances on opposite sides of the pivot, so their gravitational torques cancel and add no net torque). Justification must also indicate the rotational inertia of the system increases — 1 point (the two added point masses increase the total moment of inertia I about the pivot). Example full justification: The net torque on the system is unchanged (the spring and the board's own weight exert the same torques as before, and the additional torques from the two added masses cancel

Solution 3

since they sit at equal distances on opposite sides of the pivot), but the rotational inertia of the system is now larger. Since $\alpha = \tau_{net}/I$, with τ_{net} unchanged and I larger, the angular acceleration is now smaller: $\alpha' < \alpha_0$.

Question 3

2021 Set 1 ▪ Q3

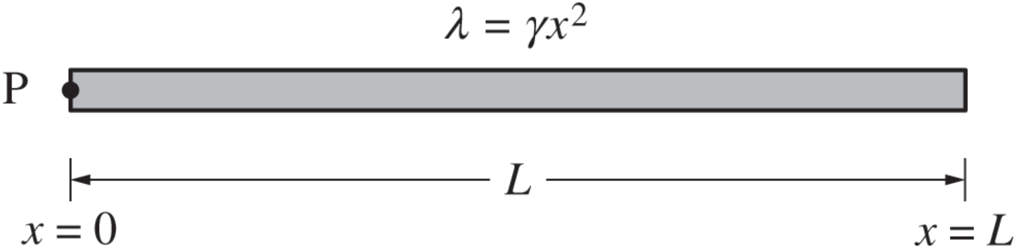
[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

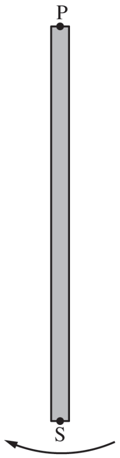
(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

Question 3

Question 3



Question 3



Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(d) The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

[Two blank axes: left axis labeled τ (vertical) vs. t (horizontal); right axis labeled ω (vertical) vs. t (horizontal). Both currently blank, to be sketched by the student.]

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_______ Increasing _______ Decreasing _______ Not changing

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(f) [Figure: The rod hangs vertically, pivoted at point P at the top, with point S marked at the bottom end. A curved arrow beneath S indicates the rod swinging.] The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

Solution 3

(a) Mark scheme

- Set up the rotational-inertia integral about point P: $I = \int r^2\, dm$, with the given linear mass density $\lambda = \gamma x^2$ so $dm = \lambda\, dx = \gamma x^2\, dx$ (1 point). Substitute and integrate from $x = 0$ to $x = L$, using $\gamma = 3M/L^3$ (from normalizing the total mass $M = \int_0^L \gamma x^2\, dx = \gamma L^3/3$) (1 point):
  $I = \int_0^L x^2(\gamma x^2\, dx) = \gamma \left[\frac{x^5}{5}\right]_0^L = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5}\right) = \frac{3}{5}ML^2.$

Solution 3

(b) Mark scheme

- Set up the center-of-mass integral: $X_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} = \dfrac{\int x\,dm}{\int dm}$ (1 point).
  Substitute $dm = \gamma x^2\,dx$ into the numerator (1 point), and either substitute M directly for the denominator or evaluate $\int dm$ to get the rod's mass (1 point):
  $X_{CM} = \dfrac{\int_0^L x(\gamma x^2)\,dx}{M} = \dfrac{[\gamma x^4/4]_0^L}{M} = \dfrac{(3M/L^3)(L^4/4)}{M} = \dfrac{3}{4}L$. The center of mass is located $\frac{3}{4}L$ from point P.

Solution 3

(c)

Mark scheme

- Answer: Greater than (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) more of the rod's mass is concentrated at the end opposite P (farther from the rotation axis), so the rotational inertia about P exceeds that about the center of mass; or (ii) by the parallel-axis theorem $I = I_{cm} + md^2$, an axis displaced from the center of mass ($d > 0$) always gives a larger rotational inertia than the axis through the center of mass.

Solution 3

(d) Mark scheme

- Sketch two graphs vs. time t, from release (rod horizontal) to when the center of mass reaches its lowest point (rod vertical). Torque τ vs. t (1 point): starts at its maximum value at $t = 0$ and decreases along a concave-down curve to zero (torque vanishes as the rod becomes vertical, when the weight acts along the line to the pivot). Angular speed ω vs. t (1 point): starts at 0 and increases along a concave-down curve that flattens, approaching a horizontal asymptote (constant ω) as t increases (since $\alpha = \tau/I \to 0$ as $\tau \to 0$). Consistency (1 point): ω must keep rising as long as $\tau > 0$ and level off exactly as $\tau \to 0$, matching $d\omega/dt = \tau/I$.

Solution 3

(e)

Mark scheme

- Answer: Decreases (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) from $\tau = rF\sin\theta$, as the rod swings downward the angle θ between the weight and the lever arm decreases, so the torque decreases; or (ii) the horizontal lever arm between P and the rod's center of mass continuously shrinks as the rod rotates downward, so the torque decreases. Since I is constant and $\alpha = \tau/I$, angular acceleration decreases correspondingly.

Solution 3

(f) Mark scheme

- Use conservation of energy for the rod swinging from horizontal (at rest) to vertical: $U_i + K_i = U_f + K_f \Rightarrow U_i = K_f$ (1 point). Substitute the gravitational PE (using the center-of-mass drop $h_i = \frac{3}{4}L$ found in part (b)) and the rotational KE about P (1 point): $Mgh_i = \frac{1}{2}I\omega_f^2 \Rightarrow Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\omega_f^2$. Using $I = \frac{3}{5}ML^2$ from part (a) and $\omega_f = v/L$, where v is the linear speed of point S (the free end, distance L from P) (1 point): $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \Rightarrow v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}(9.8\ \text{m/s}^2)(1.0\ \text{m})} = 4.9\ \text{m/s}$. (Given $M = 3.0$ kg, $L = 1.0$ m; note M cancels out of the final expression for v.)

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]
Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.
(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an x-y coordinate system with origin O at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the y-axis). A horizontal rod of length L extends

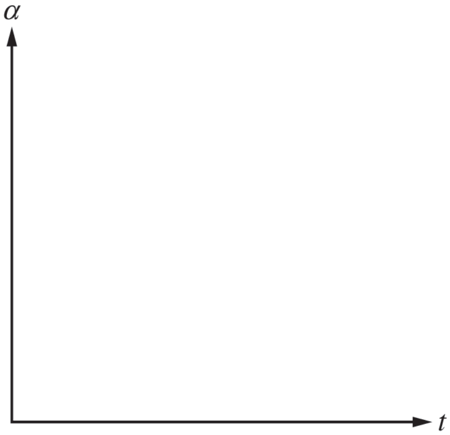
Question 2

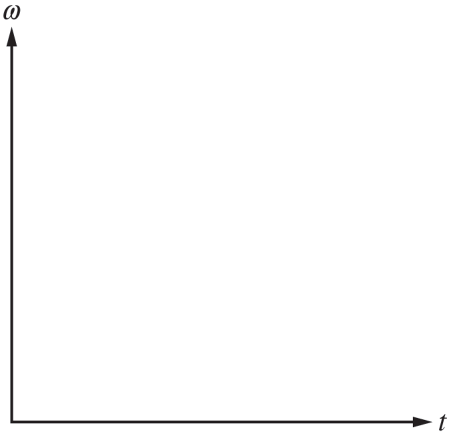
right from the pivot along the top (labelled "L"), and a vertical rod of length L extends down from the pivot to O along the y-axis (labelled "L"). The x-axis points right from O, the y-axis points up from O through the pivot.]

Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above.

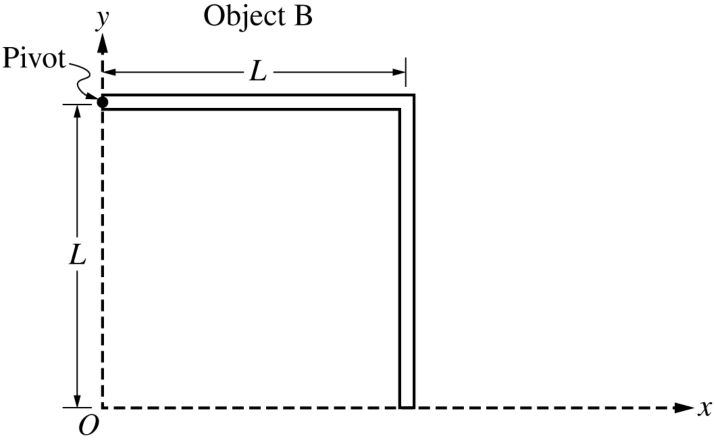
Express your answers in part (b) in terms of L.

Question 2

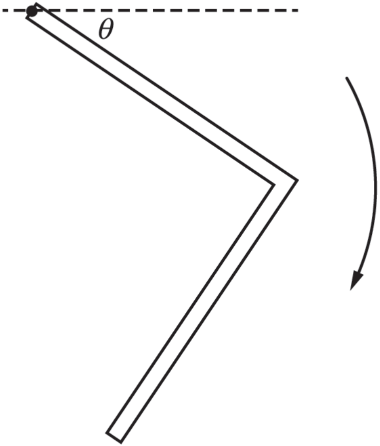




Question 2



Question 2



Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(b)(i) Determine the following for the given coordinate system shown in the figure.

The x-coordinate of the center of mass of object B

(b)(ii) The y-coordinate of the center of mass of object B

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(c) Object B has a rotational inertia of I_B about its pivot.

Is the value of I_B greater than, less than, or equal to I_A?

______Greater than ______Less than ______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled α (vertical) vs. t (horizontal); right axis labelled ω (vertical) vs. t (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(e) [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle θ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

Question 2

While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

Increasing *Decreasing* *Notchanging*

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length L now lies along the bottom (horizontal), and the horizontal rod of length L extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Solution 2

(a) Mark scheme

- Using integral calculus with λ the linear mass density of the rod (length $2L$, mass M): $I = \int_{r=0}^{r=2L} \lambda r^2 \, dr = \lambda \left[\frac{r^3}{3} \right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating λ to M and L). This confirms $I_A = \frac{4}{3}ML^2$.

Solution 2

(b)(i) Mark scheme

- $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \frac{\frac{ML}{4} + \frac{ML}{2}}{M} = \frac{3}{4}L$. Final answer: $X_{CM} = \frac{3}{4}L$.

Solution 2

(b)(ii) Mark scheme

- $Y_{CM} = \dfrac{\sum m_i y_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{2} + \frac{ML}{4}}{M} = \dfrac{3}{4}L.$ Final answer: $Y_{CM} = \dfrac{3}{4}L.$

Solution 2

(c)

Mark scheme

- Select 'Less than' (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: 'Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.' I.e. $I_B < I_A$.

Solution 2

(d)

Mark scheme

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration α vs. t graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed ω vs. t graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — ω increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

Solution 2

(e)

Mark scheme

- Select ‘Decreasing’ (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: ‘Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.’

Solution 2

(f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2}I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ (1 pt). Solve for ω in terms of the allowed symbols M, L, I_B: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt).

  Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.

Question 3

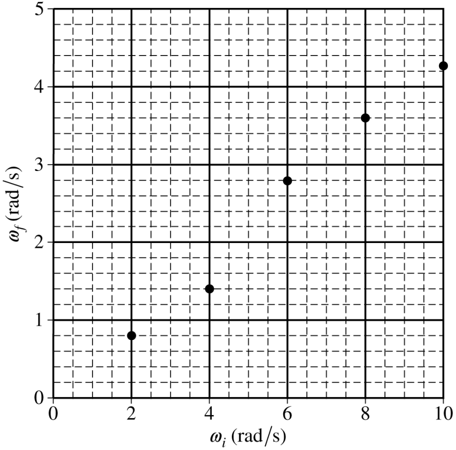
2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]
A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

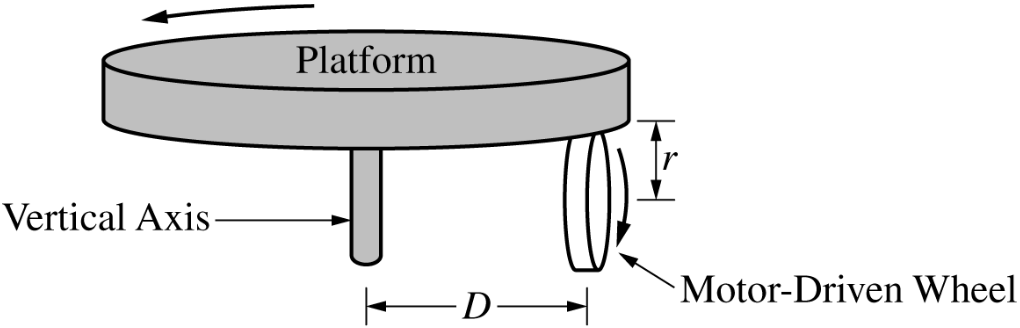
Question 3

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

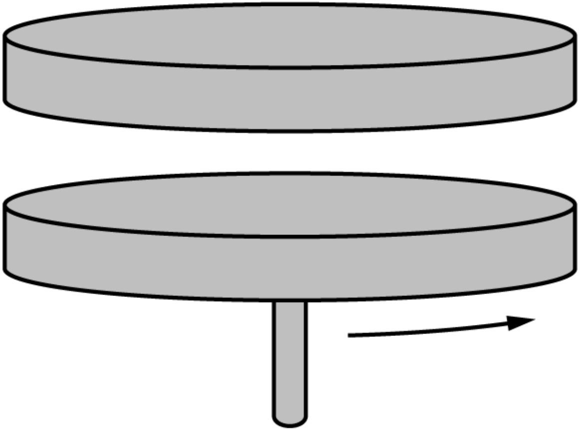
Question 3



Question 3



Question 3



Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(c) Derive an expression for the angular speed ω_W of the wheel when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(d) [Figure: the platform shown alone from a 3D perspective, spinning (arrow indicating rotation direction), with the motor-driven wheel removed.]
When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.
Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(e)(i) A student now uses the rotating platform ($I_P = 3.1$ kg·m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.
[Graph: ω_f (rad/s) vs. ω_i (rad/s); y-axis from 0 to 5 in increments of 1, x-axis from 0 to 10 in increments of 2. Data points roughly follow a straight line through the origin, e.g. near (2, 0.9), (4, 1.4), (6, 2.6), (8, 3.6), (10, 4.2).]
On the graph on the previous page, draw a best-fit line for the data.
(e)(ii) Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

______ $K_f < K_i$ ______ $K_f = K_i$ ______ $K_f > K_i$

Justify your answer.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

______ Greater than   ______ Less than   ______ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Substitute into the rotational form of Newton's second law (1 pt): $\tau = I\alpha \Rightarrow FD = I_P\alpha \Rightarrow \alpha = \dfrac{FD}{I_P}$. Substitute into a rotational kinematic equation to find the angular speed (1 pt): $\omega_2 = \omega_1 + \alpha\Delta t = 0 + \dfrac{FD}{I_P}\Delta t$, giving $\omega_P = \dfrac{FD\Delta t}{I_P}$.

Solution 3

(b)

Mark scheme

- Use the rotational kinetic energy equation (1 pt): $K = \frac{1}{2} I\omega_P^2 = \frac{1}{2}(I)\left(\frac{FD\Delta t}{I}\right)^2$. Give an answer consistent with part (a) (1 pt): $K = \frac{(FD\Delta t)^2}{2I}$.

Solution 3

(c)

Mark scheme

- Indicate that the linear speed of the platform's edge equals the linear speed of the wheel's edge at the point of contact (1 pt): $v_P = v_W$ or $(r\omega)_P = (r\omega)_W$. Correctly relate the linear speeds to the angular speeds (1 pt):
  $D\omega_P = r\omega_W \Rightarrow \omega_W = \dfrac{D\omega_P}{r}$.

Solution 3

(d)

Mark scheme

- Use conservation of angular momentum for the disk landing on the platform (1 pt): $L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$. Correctly substitute (the disk has the same rotational inertia I_P as the platform, so the combined inertia doubles) (1 pt): $I_P\omega_P = (2I_P)\omega_f \Rightarrow \omega_f = \frac{1}{2}\omega_P$.

Solution 3

(e)(i)

Mark scheme

- Draw a reasonable straight best-fit line through the plotted ω_f vs. ω_i data points on the given graph (1 pt for an appropriate best-fit line).

Solution 3

(e)(ii)

Mark scheme

- Use conservation of angular momentum to derive an expression containing I_U (1 pt): $L_i = L_f \Rightarrow I_P\omega_i = (I_P + I_U)\omega_f \Rightarrow I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$. Substitute two points read from the best-fit line (1 pt): $I_U = (3.1\ \mathsf{kg}\cdot\mathrm{m}^2)\left(\frac{4.0 - 1.0\ \mathsf{rad/s}}{9.3 - 2.4\ \mathsf{rad/s}}\right) - (3.1\ \mathsf{kg}\cdot\mathrm{m}^2) = 4.1\ \mathsf{kg}\cdot\mathrm{m}^2$ (the point (0,0) may be used implicitly if the best-fit line passes through the origin).

Solution 3

(f)

Mark scheme

- Select $K_f < K_i$ with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the two disks end up rotating together at the same final angular speed, this is an inelastic collision, so kinetic energy is lost during the collision — therefore $K_f < K_i$.

Solution 3

(g)

Mark scheme

- Select 'Greater than' with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the object's center of mass is off the platform's axis, the parallel-axis theorem $I = I_{CM} + Mh^2$ applies, so the true (actual) rotational inertia about the axis through the center of mass is smaller than what was measured — the experimental value obtained in part (e) is therefore greater than the actual value, increased by Mh^2.

Question 3

2019 Set 2 ▪ Q3

[Figure: A point P at height h above the floor, at the top of a straight incline going down and to the right, with a small circle labeled m, r at P representing the rolling sphere. The incline descends to a circular vertical loop of radius R (labeled with a radius line from the center to the edge of the loop), with point A marked at the top of the loop. After the loop, the track continues to the right and up as a straight incline. A vertical dashed line with arrows shows the height h measured from point P down to the level of the bottom of the loop/floor. Note: Figure not drawn to scale.]

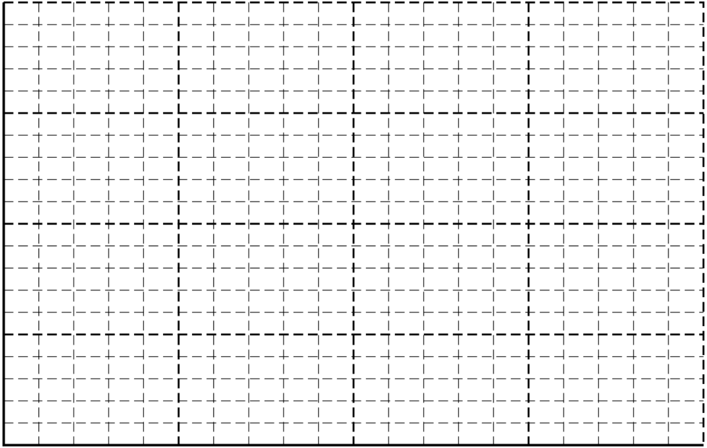
The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius

Question 3

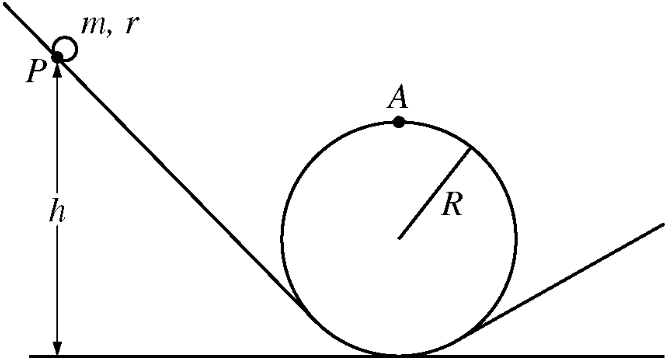
$R \gg r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20