Past-paper walk-through

Conservation of Linear Momentum ▪ 4.3

eXercise

Questions in this set

- 2026 Q2
- 2025 Q1
- 2024 Set 1 Q1
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- 2022 Set 2 Q2
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- 2016 Q2

Questions in this set

- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally. - At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

[Figure 1: Three side-view snapshots labeled $t = 0$, $t = t_1$, $t = t_2$, with a note "Figure not drawn to scale." At $t = 0$: a $+y$ vertical axis and $+x$ horizontal axis are shown at the origin; a small box on the ground at $x = 0$ launches with velocity v_0 at angle θ above the horizontal, following a dashed parabolic path upward. At $t = t_1$: at the top

Question 2

of the dashed parabolic trajectory, two pieces are shown separating horizontally — "Piece Q, M" moving to the left and "Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

(a) The horizontal and vertical components of a momentum vector are represented by p_x and p_y, respectively. The shaded bars in Figure 2 represent p_x and p_y of the projectile immediately after $t = 0$.

[Figure 2: "Figure 2, $t = 0$." A bar chart titled "Momentum" with vertical axis showing a dashed zero line labeled 0 and horizontal categories p_x and p_y, captioned "Complete Projectile." A shaded bar for p_x rises to a medium-large positive height above the zero line; a shaded bar for p_y rises to a smaller positive height above the zero line (shorter than the p_x bar).]

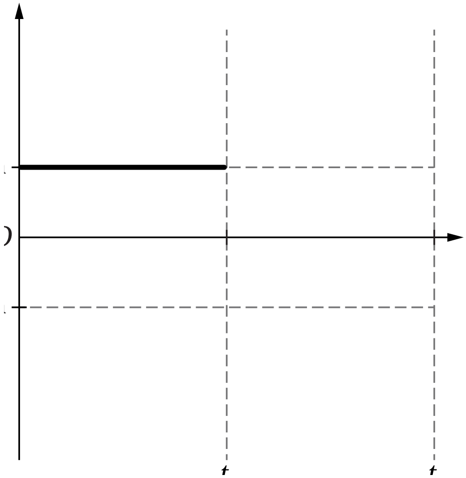
Question 2

[Figure 3: "Figure 3, $t = t_1$." Two blank bar charts titled "Momentum," each with a dashed zero line labeled 0 and horizontal categories p_x and p_y; the left one captioned "Piece Q" and the right one captioned "Piece R." Both are currently empty (no shaded bars drawn).]

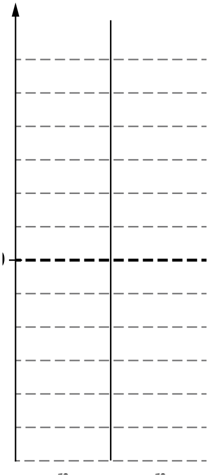
On Figure 3, **draw** shaded bars to represent p_x and p_y of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

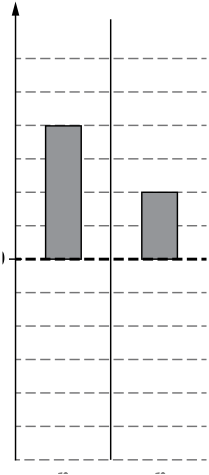
Question 2



Question 2



Question 2



Question 2

2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally. - At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

[Figure 1: Three side-view snapshots labeled $t = 0$, $t = t_1$, $t = t_2$, with a note "Figure not drawn to scale." At $t = 0$: a $+y$ vertical axis and $+x$ horizontal axis are shown at the origin; a small box on the ground at $x = 0$ launches with velocity v_0 at angle θ above the horizontal, following a dashed parabolic path upward. At $t = t_1$: at the top of the dashed parabolic trajectory, two pieces are shown separating horizontally — "Piece Q, M" moving to the left and

Question 2

"Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

(b) **Derive** an expression for x_2 in terms of v_0, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

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(c) The horizontal component of a velocity vector is represented by v_x. Figure 4 shows the horizontal component $v_{x,\text{cm}}$ of the velocity of the center of mass of the projectile as a function of t during the time interval $0 < t < t_1$.

[Figure 4: A graph with vertical axis v_x and horizontal axis t (origin O). A horizontal dashed reference line is labeled $v_{x,\text{cm}}$ above the t-axis, and another horizontal dashed reference line is labeled $-v_{x,\text{cm}}$ below the t-axis (symmetric about the axis). A solid horizontal line segment at height $v_{x,\text{cm}}$ runs from $t = 0$ to $t = t_1$, representing v_x during $0 < t < t_1$. Vertical dashed lines mark $t = t_1$ and $t = t_2$ on the horizontal axis.]

On Figure 4, **sketch** a line or curve to represent v_x as a function of t for the time interval $t_1 < t < t_2$ for each of the following.

Question 2

- Piece Q - Piece R - The center of mass of the two-piece system
Clearly **label** all lines or curves.

Question 2

2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed v_0 at an angle θ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses M and $3M$, respectively.

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Question 2

"Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

(d) Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

Indicate whether x_{new} is greater than, less than, or equal to x_2 by writing one of the following.

- $x_{\text{new}} > x_2$ - $x_{\text{new}} < x_2$ - $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

Question 1

2025 ▪ Q1

Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.
[Figure 1: Horizontal surface with a $+x$ arrow pointing right. Block 1 (mass m) on the left, moving in the $+x$-direction with speed $2v_0$ (arrow pointing right labeled $2v_0$). Block 2 (mass $6m$) on the right, moving in the $-x$-direction with speed v_0 (arrow pointing left labeled v_0).]
(a)(i) The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

Question 1

[Figure 2: A table titled "Momentum Before Collision" / "Momentum After Collision" with rows for Block 1, Block 2, and Two-Block System. Each cell shows a horizontal zero-momentum grid line labeled "0". The "Block 1, Before Collision" cell already shows an arrow starting at 0 and pointing right (+x-direction), representing the momentum of Block 1 before the collision. All other cells (Block 2 before collision; Block 1 after collision [shaded, not applicable]; Block 2 after collision [shaded, not applicable]; Two-Block System before collision; Two-Block System after collision) are blank grids to be filled in.]

Draw arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

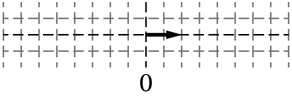
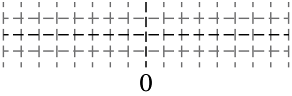
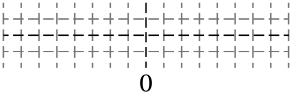
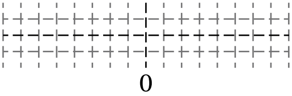
- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.

Question 1

- Represent an arrow of zero length by drawing a dot at zero.

(a)(ii) During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{\max} \sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.
Derive an expression for $F_{\max}$. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 1

	Momentum Before Collision	Momentum After Collision
Block 1		
Block 2		
Two-Block System		

Question 1

Question 1

2025 ▪ Q1

Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

[Figure 1: Horizontal surface with a $+x$ arrow pointing right. Block 1 (mass m) on the left, moving in the $+x$-direction with speed $2v_0$ (arrow pointing right labeled $2v_0$). Block 2 (mass $6m$) on the right, moving in the $-x$-direction with speed v_0 (arrow pointing left labeled v_0).]

(b) Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Question 1

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Solution 1

(a)(i) Mark scheme

- Momentum vector diagram (Figure 2). Using the scale already set by the given Block 1 arrow (2 units right, i.e. magnitude $2v_0$ for mass m, since Block 1's momentum is $m(2v_0)$): draw ONE arrow for Block 2's momentum before the collision pointing in the $-x$ direction (leftward) and 6 units long (magnitude $6mv_0$, consistent with Block 2's mass $6m$ and speed v_0) [Point A1]. Draw identical arrows – same length and direction – for the momentum of the two-block system both BEFORE and AFTER the collision, since total momentum is conserved; the length equals the vector sum of the Block 1 and Block 2 arrows: 2 units right + 6 units left = 4 units left, i.e. pointing in the $-x$ direction with magnitude $4mv_0$ (matches $m(2v_0) + 6m(-v_0) = -4mv_0 = 7m(-\frac{4}{7}v_0)$) [Point

Solution 1

A2]. Award A1 for the correctly-directed 6-unit Block 2 arrow; award A2 for two IDENTICAL two-block-system arrows (before and after) of the correct length/direction, independent of A1.

Solution 1

(a)(ii) Mark scheme

- Derive $F_{\max}$. Begin with the impulse-momentum theorem or Newton's second law: $\vec{J} = \int_{t_1}^{t_2} \vec{F}_{net}(t)\,dt = \Delta\vec{p}$, or equivalently $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$ [Point A3: multistep derivation using the integral form of impulse or the differential form of N2L]. Apply conservation of momentum to the original collision: $\sum \vec{p}_0 = \sum \vec{p}_f \Rightarrow (m)(2v_0) + (6m)(-v_0) = (m+6m)(v_f) \Rightarrow v_f = -\frac{4}{7}v_0$ [Point A4: award for stating any ONE equivalent fact – final speed of Block 1 or 2 is $\frac{4}{7}v_0$, $\Delta p_{Block2} = +\frac{18}{7}mv_0$, $\Delta p_{Block1} = -\frac{18}{7}mv_0$, $\Delta v_{Block2} = +\frac{3}{7}v_0$, or $\Delta v_{Block1} = -\frac{18}{7}v_0$]. Then $\Delta p_{Block2} = -\Delta p_{Block1} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right) = \frac{18}{7}mv_0$. Substitute the given force into the impulse integral:

Solution 1

$\Delta p_{Block2} = \int_0^{t_c} F_{\max} \sin(At)\, dt$ [Point A5: substituting F(t) into the impulse/N2L expression]. Evaluate: $\frac{18}{7} m v_0 = \dfrac{F_{\max}}{A}\left[-\cos(At)\right]_0^{t_c} = \dfrac{F_{\max}}{A}\left[1-\cos(At_c)\right]$ [Point A6: definite integral with correct limits, or an equivalent indefinite integral plus a constant of integration]. Solve: $F_{\max} = \dfrac{18}{7}\left(\dfrac{A m v_0}{1-\cos(At_c)}\right)$ [Point A7: final correct expression in terms of m, v_0, A, t_c].

Solution 1

(b) Mark scheme

- New scenario: Block 1 (mass m) slides at speed v_1 in the $+x$ direction, Block 2 (mass $6m$) slides at speed v_0 in the $-x$ direction; they collide and stick, and the combined system moves afterward at constant speed v_0. Using conservation of momentum (a multistep derivation) [Point B1]: $\sum \vec{p}_0 = \sum \vec{p}_f$. The final momentum magnitude of the combined $7m$ system is $7mv_0$ [Point B2]. $mv_1 + 6m(-v_0) = 7m(v_0) \Rightarrow mv_1 = 13mv_0 \Rightarrow v_1 = 13v_0$ [Point B3: correct final expression for v_1 in terms of v_0].

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

Question 1

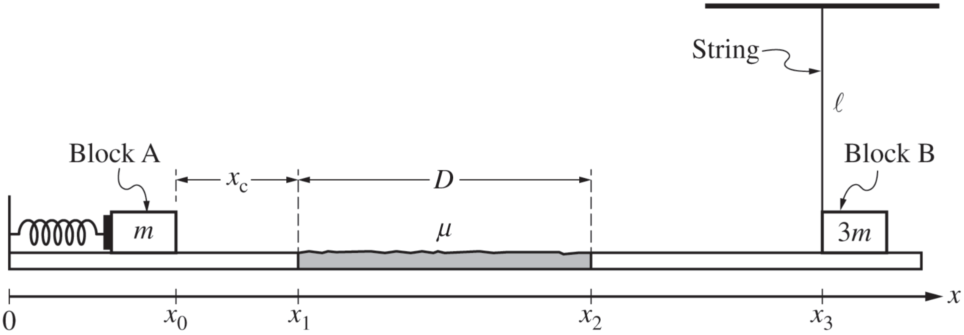
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed v of Block A at time t_1.

(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

Question 1



Note: Figure not drawn to scale.

Figure 1

Question 1

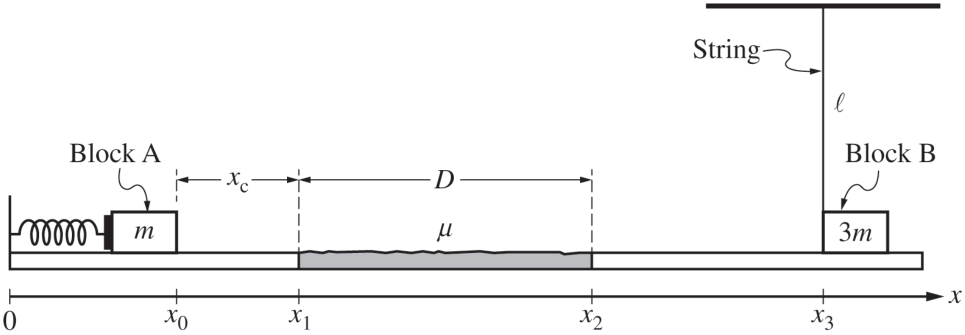


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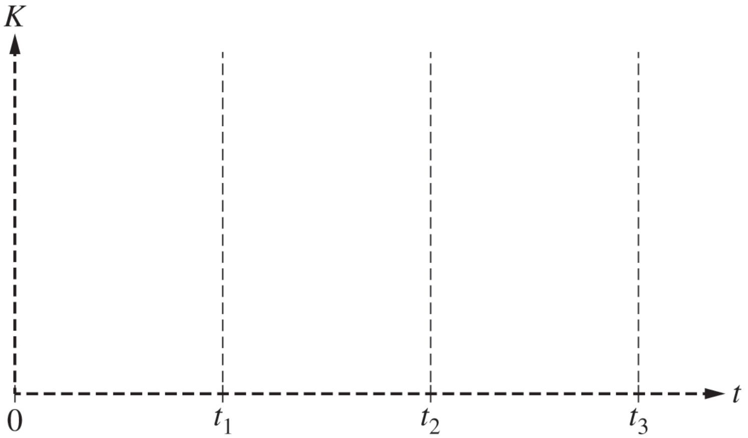
Figure 1

Question 1



Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c, friction region of length D with coefficient μ between x_1 and x_2, and Block B (mass $3m$) hanging by a string of length ℓ at x_3. Positions marked $0, x_0, x_1, x_2, x_3$ along axis x. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3.

Question 1

[Blank graph: vertical axis K, horizontal axis t with gridlines/tick marks at t_1, t_2, t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]

(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

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(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4, having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ. Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

_______ $f_{2\ell} > f_i$ _______ $f_{2\ell} < f_i$ _______ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s, so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v: $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s. Point 2: correct algebraic solution for v.

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2, after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{friction} = K_{before\ collision}$: $\frac{1}{2}m\left(x_c\sqrt{k/m}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then $v_2^2 = v^2 + 2aD = \left(x_c\sqrt{k/m}\right)^2 - 2\mu gD$, same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{before} = \Sigma p_{after}$: $mv_2 = (m + 3m)v_{A,B} = 4m\,v_{A,B}$. Solving: $v_{A,B} = \dfrac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \dfrac{1}{4}\sqrt{\dfrac{kx_c^2}{m} - 2\mu gD}$. Points: (1) energy or kinematics

Solution 1

application to find speed at x_2; (2) correct expression for friction energy loss or deceleration ($\Delta E_{friction} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{initial}v_{initial} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_\ell$ (doubling the pendulum length decreases its frequency). Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_\ell$. Points: (1) selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\mathrm{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\mathrm{max,P}}$ for Spring P with the different height $h_{\mathrm{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in

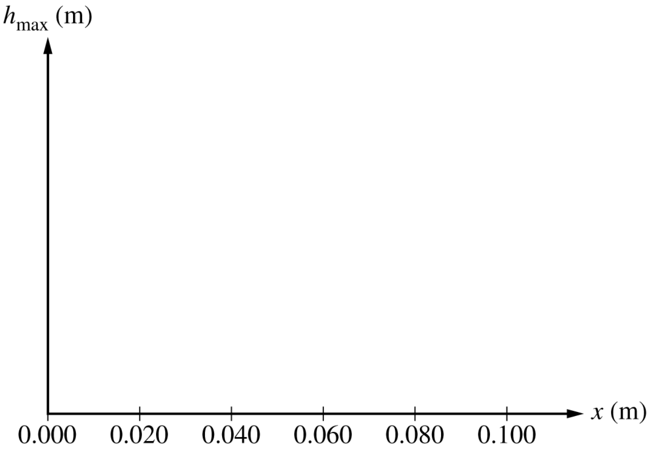
Question 1

Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]
(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following representations of the block in Trials 1 and 2, draw and label the forces (non-components) that are exerted on the block at the instant the block is released. Each force must be

Question 1

represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.
[Two grid diagrams are shown, each with a dot at the center representing the block: "Trial 1 (Spring P)" on the left and "Trial 2 (Spring Q)" on the right. Each grid is for the student to draw labeled force vectors.]

Question 1



Question 1

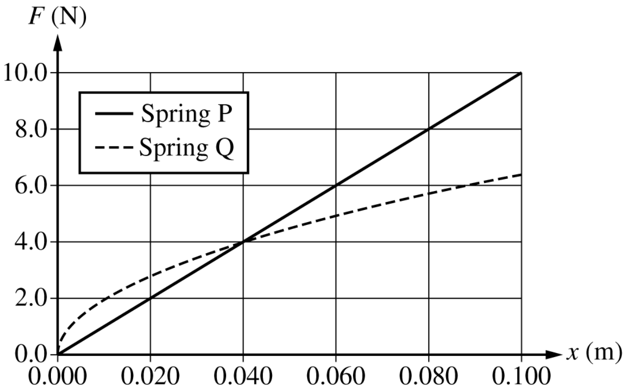
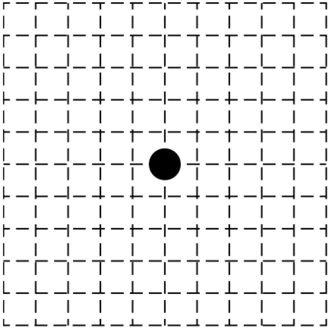


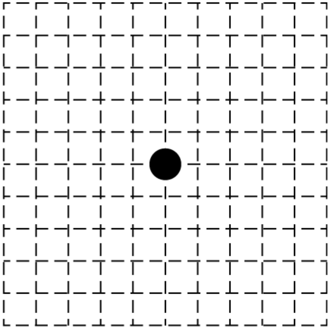
Figure 2

Question 1

Trial 1
(Spring P)



Trial 2
(Spring Q)



Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(b)(i) What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

(b)(ii) There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

(b)(iii) On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance x each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.

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[Blank axes provided: y-axis labeled $h_{\max}$ (m), x-axis labeled x (m) from 0.000 to 0.100 in increments of 0.020, for the student to sketch two curves.]

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2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.

[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(c)(i) [Figure 3: Block A rests at the top of an inclined ramp of height H, connected by a string over the top of the ramp; Block B rests on the horizontal surface at the bottom of the ramp, connected to Spring Q which is attached to a wall, with $x = 0$ marked at the natural length position of the spring.]

Spring Q is attached to the wall and at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp on the horizontal surface. The change in vertical height of Block A is $H = 0.75$ m. The student releases Block A, and it moves down the ramp and collides with Block B. After the collision, the blocks stick together and move to the right,

Question 1

compressing the spring. Frictional forces between the blocks and all surfaces are negligible.
Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.
(c)(ii) Calculate the maximum compression of the spring.

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2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.

[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D?

______ $C < D$ ______ $C > D$ ______ $C = D$

Briefly justify your answer.

Question 1

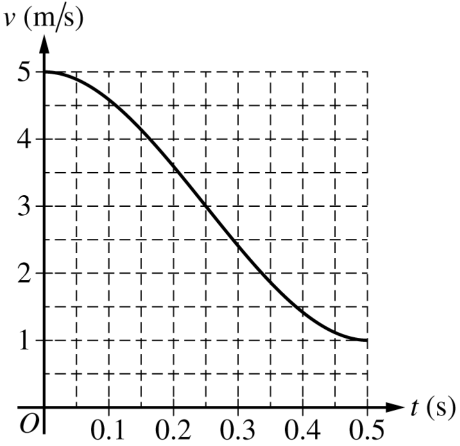
2023 Set 2 ▪ Q1

Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.
[Diagram: Cart A (with a block labeled "Foam" attached to its front) approaching Cart B along a horizontal track, both carts shown as boxes on wheels.]
(a)(i) The graph shows the velocity v of Cart A as a function of time t for the time interval when the foam and Cart B are in contact.

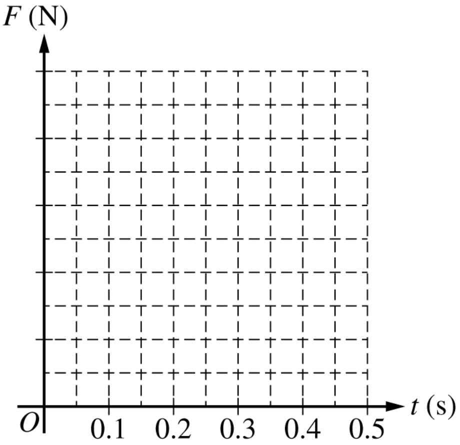
Question 1

[Graph: *v* (m/s) vs. *t* (s); y-axis from 0 to 5 m/s in increments of 1, x-axis from 0 to 0.5 s in increments of 0.1 s; curve starts at $(0, 5)$ and decreases smoothly, concave up, leveling off toward approximately $(0.5, 1)$.]
What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision?
(a)(ii) Using the information shown in the graph, determine the speed of Cart B at $t = 0.5$ s.
(a)(iii) On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.
[Grid: *v* (m/s) vs. *t* (s); y-axis from 0 to 5 m/s in increments of 1, x-axis from 0 to 0.5 s in increments of 0.1 s; the curve for Cart A is already plotted and labeled "Cart A," starting at $(0, 5)$ and decreasing to approximately $(0.5, 1)$.]

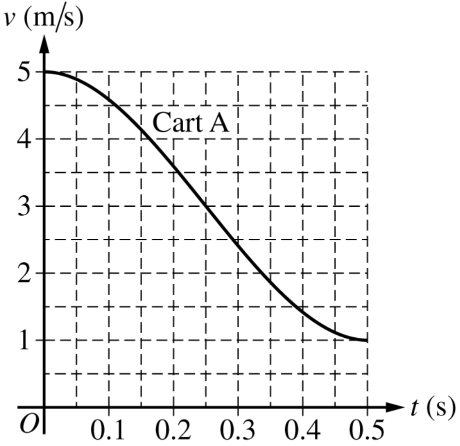
Question 1



Question 1



Question 1



Question 1

2023 Set 2 ▪ Q1

Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

[Diagram: Cart A (with a block labeled "Foam" attached to its front) approaching Cart B along a horizontal track, both carts shown as boxes on wheels.]

(b)(i) For $0 \leq t \leq 0.50$ s, the velocity v of Cart A can be described by the function $v(t) = 64t^3 - 48t^2 + 5$.

Calculate the magnitude of the maximum net force acting on Cart A during this interval.

Question 1

(b)(ii) On the following grid, draw a smooth curve of the magnitude of the force acting on Cart A as a function of time. Clearly indicate the value of the maximum force on the vertical axis.
[Grid: F (N) vs. t (s); x-axis from 0 to 0.5 s in increments of 0.1 s; y-axis unlabeled/blank for the student to fill in.]

Question 1

2023 Set 2 ▪ Q1

Scientists have created a new type of lightweight foam and are performing experiments to investigate the properties of the foam. The mass of Cart A is 1000 kg and the mass of Cart B is 2000 kg. A piece of foam with negligible mass is attached to the front of Cart A, as shown. Cart A moves with a constant speed toward Cart B, which is initially at rest. At time $t = 0$ s, the foam connected to Cart A makes contact with Cart B. The foam remains in contact with Cart B for 0.5 s, after which the carts separate and both carts move with constant velocities.

[Diagram: Cart A (with a block labeled "Foam" attached to its front) approaching Cart B along a horizontal track, both carts shown as boxes on wheels.]

(c) The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam.

Question 1

In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.
For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is F_1. For the collision without the foam, the magnitude of the average net force exerted on Cart B is F_2.
What is the relationship between the magnitude of the average net force F_1 exerted on Cart B for the collision with the foam and the magnitude of the average net force F_2 exerted on Cart B for the collision without the foam?
_____ $F_1 > F_2$ _____ $F_1 < F_2$ _____ $F_1 = F_2$
Justify your answer.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]
Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2.

Question 2

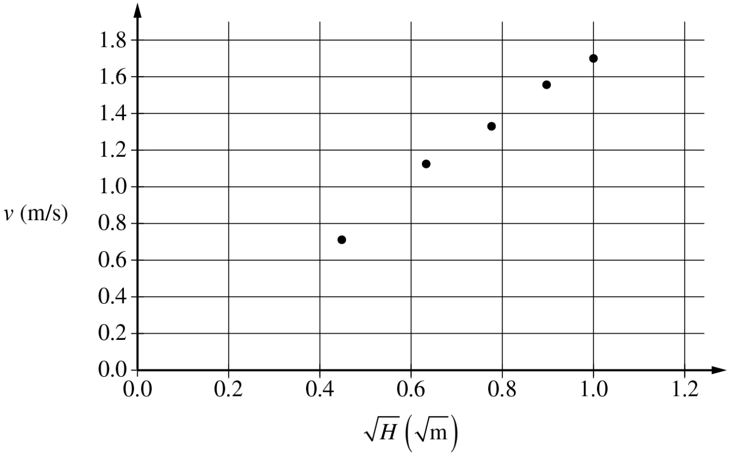
After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

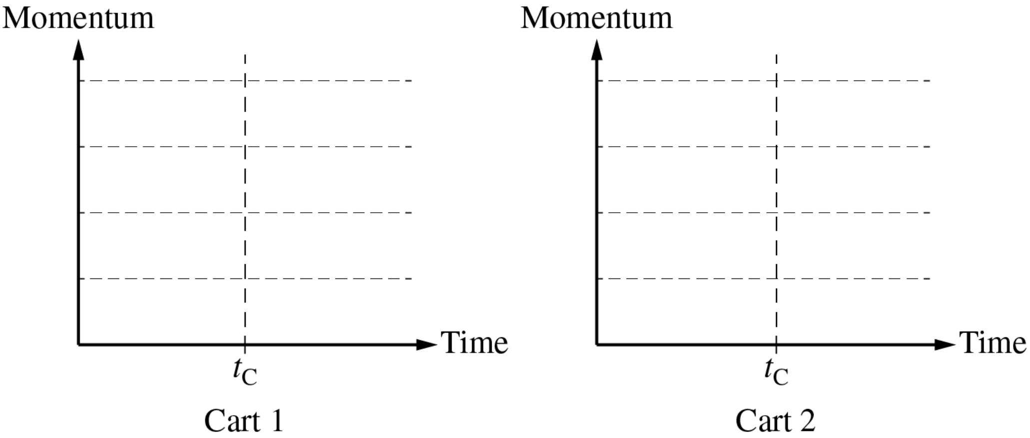
______ Greater than ______ Less than ______ Equal to

Justify your answer.

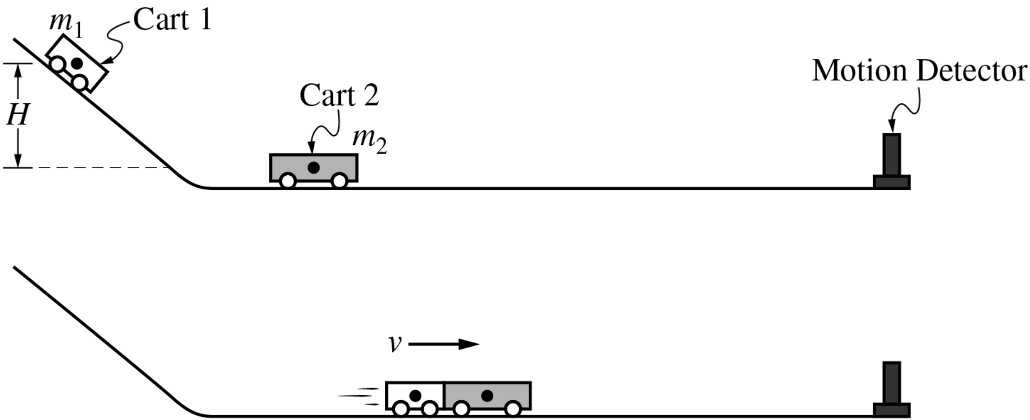
Question 2



Question 2



Question 2



Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.
[Graph 1, titled "Cart 1": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]
[Graph 2, titled "Cart 2": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

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Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(d)(i) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.

[Graph: scatter plot; vertical axis "v (m/s)" ranges from 0.0 to 1.8 in increments of 0.2; horizontal axis "$\sqrt{H}\,(\sqrt{\text{m}})$" ranges from 0.0 to 1.2 in increments of 0.2; plotted points approximately at $(0.45, 0.75)$, $(0.65, 1.05)$, $(0.65, 1.25)$, $(0.85, 1.35)$, $(1.0, 1.55)$, $(1.0, 1.65)$, showing a roughly linear increasing trend; no line drawn yet.]

Draw a line that represents the best fit to the data points shown.

(d)(ii) Use the best-fit line to calculate the mass of Cart 2.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg?

$\mathrm{m}_1' > 0.250$ kg $\quad$ $\mathrm{m}_1' < 0.250$ kg $\quad$ $\mathrm{m}_1' = 0.250$ kg

Justify your answer.

Solution 2

(a)

Mark scheme

- Answer: Equal to (1 point for selecting this with an attempted justification). Justification: By Newton's third law, the force Cart 2 exerts on Cart 1 and the force Cart 1 exerts on Cart 2 are equal in magnitude (and opposite in direction) at every instant during the collision; since both carts experience the collision over the same time interval, the impulses ($\int F\,dt$) on each cart must be equal in magnitude (1 point).

Solution 2

(b) Mark scheme

- Momentum (p) vs. time (t) graphs, before/after t_C (collision assumed instantaneous). Cart 1: for $0 < t < t_C$, a straight line increasing linearly from zero (Cart 1 accelerates down the incline); at t_C its momentum drops abruptly to a smaller constant (horizontal) value for $t > t_C$ (1 point for the $0 < t < t_C$ portion: linear increasing for Cart 1 and zero for Cart 2; 1 point for Cart 1's post-collision horizontal line being smaller in magnitude than its momentum at $t = t_C$). Cart 2: zero (flat on the axis) for $0 < t < t_C$, then jumps at t_C to a constant (horizontal) value for $t > t_C$ that is GREATER in magnitude than Cart 1's post-collision momentum (1 point). The magnitude of Cart 1's momentum loss at the collision

Solution 2

must equal the magnitude of Cart 2's momentum gain (equal and opposite changes), consistent with the equal-impulse answer in part (a) (1 point).

Solution 2

(c) Mark scheme

- Derivation of $v = \sqrt{2g}\left(\dfrac{m_1}{m_1 + m_2}\right)\sqrt{H}$. Conservation of energy (or equivalently kinematics) gives Cart 1's speed at the bottom of the incline: $m_1 gH = \frac{1}{2}m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gH}$ (1 point). Conservation of momentum for the perfectly inelastic collision: $m_1 v_1 = (m_1 + m_2)v \Rightarrow v = \dfrac{m_1}{m_1 + m_2}v_1$ (1 point). Combining: $v = \dfrac{m_1}{m_1 + m_2}\sqrt{2gH} = \sqrt{2g}\left(\dfrac{m_1}{m_1 + m_2}\right)\sqrt{H}$, as required (1 point).

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the v vs. $\sqrt{H}$ scatter plot with approximately equal numbers of data points above and below the line (through or near the origin).

Solution 2

(d)(ii) Mark scheme

- Use the best-fit line to find m_2. Calculate the slope using two well-separated points on the best-fit line, e.g. slope $= \dfrac{(1.20 - 0.60)\ \mathsf{m/s}}{(0.70 - 0.35)\sqrt{\mathsf{m}}} = 1.72\ \sqrt{\mathsf{m}}/\mathsf{s}$ (1 point). Relate the slope to the masses via the part (c) result, $slope = \dfrac{m_1}{m_1 + m_2}\sqrt{2g}$, and solve for m_2: $m_2 = \dfrac{m_1\sqrt{2g}}{slope} - m_1$ (1 point). Substituting $m_1 = 0.250$ kg and $g = 9.8\ \mathsf{m/s}^2$: $m_2 = \dfrac{(0.25\ \mathsf{kg})\sqrt{2(9.8)}}{1.72} - 0.25\ \mathsf{kg} \approx 0.39\ \mathsf{kg}$ (1 point). Any value in the range 0.30–0.60 kg is acceptable given a valid best-fit slope.

Solution 2

(e)

Mark scheme

- Answer: $m_1' < 0.250$ kg (1 point for selecting this with an attempted justification). Justification: The directly-measured mass of Cart 2 came out smaller than the value calculated in part (d). A smaller true m_2 corresponds to a smaller initial energy/momentum for the same drop height H; since the best-fit slope (and hence the computed relationship) stays the same, this discrepancy is explained if the actual mass of Cart 1 is smaller than the recorded 0.250 kg — i.e., $m_1' < 0.250$ kg (1 point).

Question 2

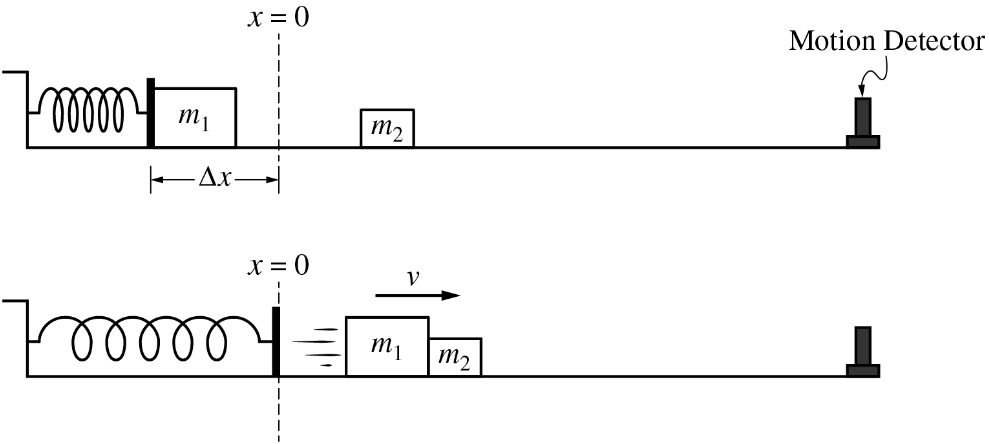
2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]
Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the

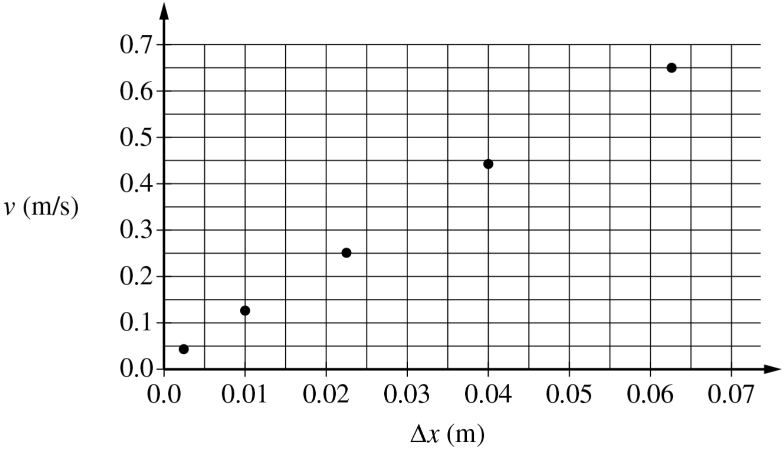
Question 2

two-block system moves with constant speed v, as shown. Frictional effects are negligible.
(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?
____ $J_S > J_2$ ____ $J_S < J_2$ ____ $J_S = J_2$
Justify your answer.

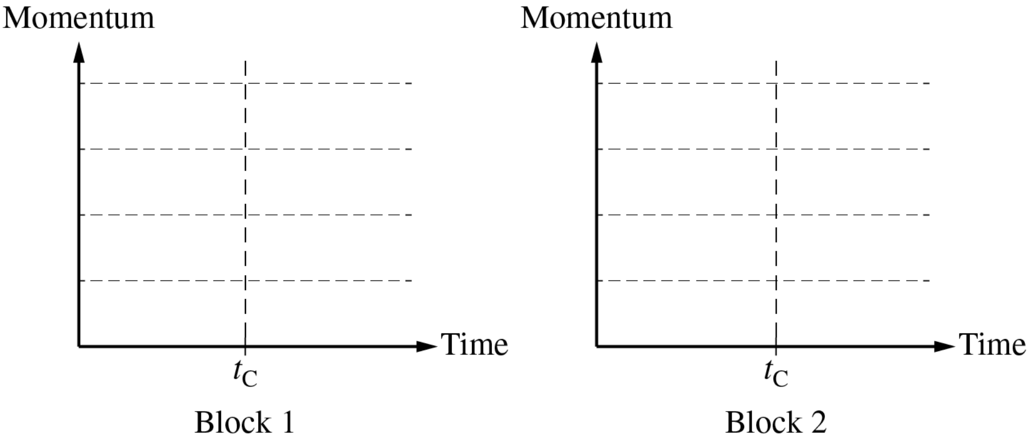
Question 2



Question 2



Question 2



Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.
[Two blank graphs labeled "Momentum" on the vertical axis and "Time" on the horizontal axis, each with a marked point t_C on the time axis and dashed gridlines; left graph labeled "Block 1", right graph labeled "Block 2", for the student to sketch momentum vs. time before and after t_C.]

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]
Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(d)(i) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant k of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity v of the two-block system is measured. They graph v as a function of Δx, as shown below. Draw a line that represents the best fit to the data points shown.
[Scatter plot: vertical axis v (m/s) from 0.0 to 0.7 in increments of 0.1; horizontal axis Δx (m) from 0.0 to 0.07 in increments of 0.01. Data points approximately at $(0.01, 0.08)$, $(0.02, 0.16)$, $(0.035, 0.28)$, $(0.05, 0.45)$, $(0.06, 0.58)$, $(0.065, 0.62)$, for the student to draw a best-fit line through.]
(d)(ii) Use the best-fit line to calculate the mass of Block 2.

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N/m, less than 150 N/m, or equal to 150 N/m ?

k' $>$ 150 N/m k' $<$ 150 N/m k' $=$ 150 N/m

Justify your answer.

Solution 2

(a) Mark scheme

- Answer: $J_S > J_2$ — select this choice with an attempt at a relevant justification (1 point), plus a fully correct justification (1 point). Example justification: By Newton's third law, the impulse Block 2 exerts on Block 1 during the collision has the same magnitude as the impulse Block 1 exerts on Block 2. Since Block 1 is still moving forward (nonzero momentum) after the collision, the impulse from the spring J_S (which accelerated Block 1 from rest up to its pre-collision speed) must be greater in magnitude than the impulse J_2 from the collision (which only reduces, not zeroes, Block 1's momentum).

Solution 2

(b) Mark scheme

- Full-credit sketch, 1 point per criterion: (1) For $0 < t < t_C$: Block 1's momentum rises along a concave-down (decreasing-slope) curve up to its maximum at $t = t_C$, while Block 2's momentum stays at zero (flat on the time axis) throughout. (2) For $t > t_C$: Block 1's momentum is a horizontal line at a value smaller in magnitude than its momentum at $t = t_C$ (it lost momentum in the collision). (3) For $t > t_C$: Block 2's momentum is a horizontal line at a value smaller in magnitude than Block 1's momentum at $t = t_C$ (Block 2 now moves, but with less momentum than Block 1 had just before the collision). (4) Blocks 1 and 2 have an equal-magnitude change in momentum at t_C — Block 1's drop equals Block 2's rise (momentum conservation in the collision).

Solution 2

(c) Mark scheme

- Use conservation of energy (spring PE converts to KE of Block 1) to find Block 1's speed at $x = 0$ — 1 point: $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m_1}}\Delta x$. Use conservation of momentum for the perfectly inelastic collision to find the speed of the combined two-block system — 1 point: $m_1 v_1 = (m_1 + m_2)v \Rightarrow v = \frac{m_1 v_1}{m_1 + m_2}$. Combine the two equations — 1 point: $v = \frac{\Delta x\sqrt{k m_1}}{m_1 + m_2}$, as required to show.

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the plotted v vs. Δx data, with approximately the same number of data points above the line as below it (need not pass exactly through any single point). 1 point for a reasonable best-fit line satisfying this balance criterion.

Solution 2

(d)(ii) Mark scheme

- Calculate the slope of the best-fit line using two well-separated points on it — 1 point, e.g. slope $= \dfrac{(0.60 - 0.02)\ \mathrm{m/s}}{(0.055 - 0.0175)\ \mathrm{m}} \approx 10.67\ \mathrm{s}^{-1}$. Correctly relate the slope to m_2 using the part (c) result $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$ — 1 point: slope $= \dfrac{\sqrt{km_1}}{m_1 + m_2} \Rightarrow m_2 = \dfrac{\sqrt{km_1}}{\mathrm{slope}} - m_1$. Compute a correct numeric mass — 1 point: $m_2 = \dfrac{\sqrt{(150\ \mathrm{N/m})(0.500\ \mathrm{kg})}}{10.67\ \mathrm{s}^{-1}} - 0.500\ \mathrm{kg} \approx 0.31\ \mathrm{kg}$ (acceptable range $0.27\ \mathrm{kg}$ to $0.37\ \mathrm{kg}$, since it depends on the student's own graph read).

Solution 2

(e) Mark scheme

- Answer: $k' > 150$ N/m — select this choice with an attempt at relevant justification (1 point), plus a fully correct justification (1 point). Example justification: A balance shows the true mass of Block 2 is greater than the value determined in part (d). From slope $= \dfrac{\sqrt{k' m_1}}{m_1 + m_2}$, the measured slope of the graph (an experimental result) stays the same, so for a larger true m_2 the spring constant k' needed to keep that same slope must be greater than the originally assumed 150 N/m.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

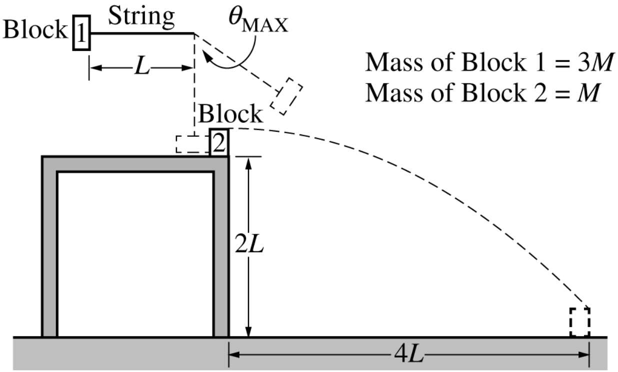
A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a

Question 2

distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

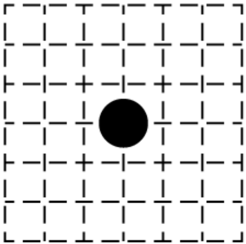
Question 2



Mass of Block 1 = $3M$
Mass of Block 2 = M

Note: Figure not drawn to scale.

Question 2



Question 2

2019 Set 1 ▪ Q2

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A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.

[A dot on a dashed grid, representing block 1, for the student to draw force vectors on.]

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 $= 3M$", "Mass of Block 2 $= M$". Note: Figure not drawn to scale.]

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(e) Calculate the speed of block 2 as it leaves the table.

Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(f) Calculate the speed of block 1 just after it collides with block 2.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 $= 3M$", "Mass of Block 2 $= M$". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Solution 2

(a)

Mark scheme

- Energy conservation for block 1's swing: $U_{g1} = K_2 \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow gL = \frac{1}{2}v^2 \Rightarrow v = \sqrt{2gL}$. Full credit even if no work is shown.

Solution 2

(b)

Mark scheme

- Draw two vectors from the dot: the weight F_W pointing straight down, and the string tension F_T pointing straight up, with the tension arrow drawn noticeably LONGER than the weight arrow (net force must be upward/centripetal). 1 pt for correctly drawing and labeling both the weight and the tension; 1 pt for drawing the tension longer than the weight. Only a maximum of 1 point can be earned if any extraneous vectors are included.

Solution 2

(c)

Mark scheme

- Use Newton's second law along the string direction, tension minus weight equals centripetal force (1 pt):
  $F_T = mg + ma_c = mg + \dfrac{mv^2}{r} = 3Mg + \dfrac{(3M)(\sqrt{2gL})^2}{L}$. Correct final answer (1 pt): $F_T = 9Mg$.

Solution 2

(d) Mark scheme

- Use a correct kinematic equation for the vertical fall from table height $2L$ (1 pt): $\Delta y = v_i t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{4(0.75\ \text{m})}{9.8\ \text{m/s}^2}}$. Correct final answer (1 pt): $t = 0.55$ s.

Solution 2

(e)

Mark scheme

- Use a correct kinematic equation for the horizontal (projectile) motion (1 pt): $\Delta x = vt \Rightarrow v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$. Substitute the time from part (d) (1 pt): $v = \dfrac{(4)(0.75\ \text{m})}{0.55\ \text{s}} = 5.45\ \text{m/s}$.

Solution 2

(f) Mark scheme

- Use conservation of momentum for the collision (1 pt): $p_i = p_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. Correctly substitute the masses (1 pt): $(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$. Substitute the speeds found in parts (a) and (e) (1 pt): $v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{3M} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{3M}$, giving $v_{1f} = \dfrac{3\sqrt{2(9.8)(0.75)} - (1)(5.45)}{3} = 2.02$ m/s (or $v_{1f} = 2.06$ m/s using $g = 10$ m/s^2).

Solution 2

(g) Mark scheme

- Use conservation of energy for block 1's rise after the collision (1 pt): $K_1 = U_{g2} \Rightarrow \frac{1}{2}mv^2 = mgh$. Correctly substitute $h = L(1-\cos\theta)$ (1 pt): $\frac{1}{2}mv^2 = mgL(1-\cos\theta) \Rightarrow \dfrac{v^2}{2gL} = 1-\cos\theta$. Substitute the speed found in part (f) (1 pt): $\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02)^2}{2(9.8)(0.75)}\right) = 43.5^\circ$ (or $\theta = 44.1^\circ$ using $g = 10\ \mathrm{m/s^2}$).

Question 2

2018 ▪ Q2

Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right) t^2 - (500\ \text{N/s})t$.

[Diagram: Cart 1 (mass $m = 0.500$ kg) with a Force Sensor attached at its right side, moving right with $v_0 = 3.00$ m/s toward a spring ($k = 130$ N/m) connected to Cart 2 (mass $m = 1.05$ kg, initially at rest, $v = 0$), all on a horizontal track.]

Question 2

[Graph: Force F (N) on the vertical axis (marked at 5, 0, -5, -10, -15, -20, -25) versus time t (s) on the horizontal axis (marked 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18). Data points start near $F = 0$ at small t, dip down to a minimum of about -20 N around $t = 0.08$–0.10 s, then rise back up toward $F = 0$ by about $t = 0.16$–0.18 s, tracing a roughly parabolic (U-shaped, inverted) curve consistent with the quadratic fit given.]

(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)(i) Calculate the speed of cart 1 after the collision.

(b)(ii) In which direction does cart 1 move after the collision?

______ Left ______ Right

______ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)(i) Calculate the speed of cart 2 after the collision.

Question 2

(c)(ii) Show that the collision between the two carts is elastic.
(d)(i) Calculate the speed of the center of mass of the two-cart-spring system.
(d)(ii) Calculate the maximum elastic potential energy stored in the spring.

Question 2

a spring constant of $k = 130\ \mathrm{N/m}$. The data for the force the spring exerts on cart 1 are sho

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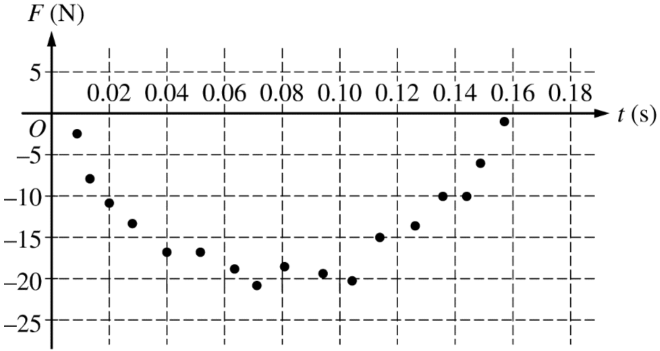
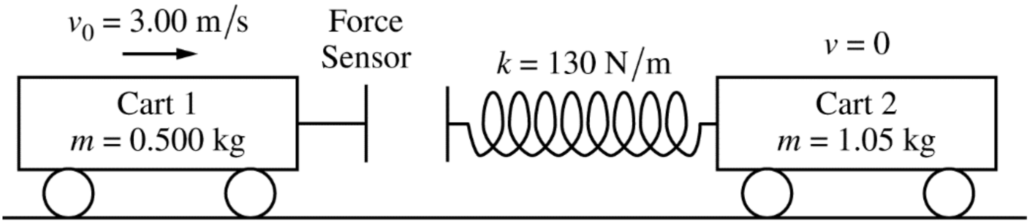


Question 2



Solution 2

(a)

Mark scheme

- $J = \int F\,dt = \int_0^{0.16} (3200t^2 - 500t)\,dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_0^{0.16} = \frac{3200}{3}(0.16)^3 - 250(0.16)^2 = -2.03$ N·s. 1 point for using the given force equation in the impulse integral; 1 point for integrating with correct limits (or constant of integration); 1 point for the correct final numeric answer $J = -2.03$ N·s.

Solution 2

(b)(i)

Mark scheme

- $J = m_1(v_{1f} - v_{1i}) \Rightarrow v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{-2.03\ \mathrm{N \cdot s}}{0.500\ \mathrm{kg}} + 3.00\ \mathrm{m/s} = -1.06\ \mathrm{m/s}.$

Solution 2

(b)(ii)

Mark scheme

- Left. Since $v_{1f} = -1.06$ m/s is negative (opposite the original direction of motion), cart 1 moves to the left after the collision.

Solution 2

(c)(i) Mark scheme

- By Newton's third law on the spring force, $-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \Rightarrow v_{2f} = \frac{-J}{m_2} = \frac{2.03\ \text{N}\cdot\text{s}}{1.05\ \text{kg}} = 1.93$ m/s. (Equivalent alternate via momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{2f} = \frac{m_1(v_{1i} - v_{1f})}{m_2} = \frac{(0.500)(3.00 - (-1.06))}{1.05} = 1.93$ m/s.) 1 point for a correct equation to find v_{2f}; 1 point for correct substitution.

Solution 2

(c)(ii)

Mark scheme

- An elastic collision requires initial KE = final KE: $K_{ii} = K_{1f} + K_{2f}$. Check: $\frac{1}{2}(0.500)(3.00)^2 = \frac{1}{2}(0.500)(-1.06)^2 + \frac{1}{2}(1.05)(1.93)^2 \Rightarrow 2.25\text{ J} \approx 2.24\text{ J}$ — equal within rounding, so the collision is elastic. 1 point for indicating that initial and final kinetic energies must be equal; 1 point for correct substitution/numeric verification.

Solution 2

(d)(i)

Mark scheme

- $m_1 v_{1i} = (m_1 + m_2) v_{cm} \Rightarrow v_{cm} = \dfrac{m_1 v_{1i}}{m_1 + m_2} = \dfrac{(0.500 \text{ kg})(3.00 \text{ m/s})}{0.500 \text{ kg} + 1.05 \text{ kg}} = 0.97$ m/s. 1 point for the correct equation for the center-of-mass speed; 1 point for correct substitution.

Solution 2

(d)(ii) Mark scheme

- Using conservation of energy: $K_i = K_f + U_{Sf} \Rightarrow U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}^2 = \frac{1}{2}(0.500)(3.00)^2 - \frac{1}{2}(1.55)(0.97)^2 = 1.52$ J. (Alternate: from $dF/dt = 0$, $t = 0.078$ s gives $|F_{max}| = 19.5$ N (accept 20-21 N from the graph); $x_{max} = F_{max}/k = 0.15$ m (accept 0.15-0.16 m); $U_S = \frac{1}{2}kx_{max}^2 = 1.46$ J, accept 1.46-1.70 J if estimated from the graph.) Scoring: 1 point for using conservation of energy to find the max elastic PE; 1 point for using the center-of-mass speed for the system's translational KE; 1 point for correct substitution/final numeric answer. Q2 also carries an overall 1-point "units" credit (correct units on all calculated answers through the question) — folded into this final part so Q2's 15 points are fully accounted for.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

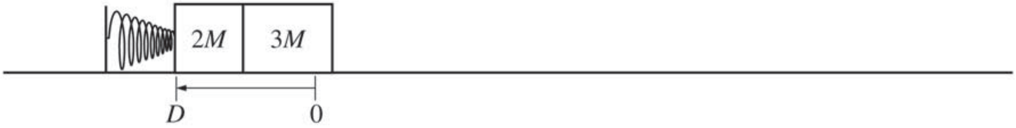
A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$" (left dot) and "Block of Mass $3M$" (right dot), for the student to draw force vectors on.]
[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled "2M" next to "3M" (the two blocks shown adjacent, having moved together). A distance D is marked from the wall/spring's compressed end to a reference position "0".]
The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

Question 2



Question 2

d pointing away from, the appropriate dot.

Block of Mass $2M$

Block of Mass $3M$

Question 2

Question 2

2016 ▪ Q2

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(b) Derive an expression for the speed of the blocks immediately after the collision.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

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(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

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(d) Derive an expression for the maximum distance D that the spring is compressed.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(e)(i) In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left _____ Right _____ The net force on the block of mass $2M$ is zero.

Question 2

Justify your answer.
(e)(ii) Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?
_____ The magnitude of the net force is greater on the block of mass $2M$. _____ The magnitude of the net force is greater on the block of mass $3M$. _____ The magnitude of the net force on each block has the same nonzero value. _____ The magnitude of the net force on each block is zero.
Justify your answer.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

______ Yes   ______ No

Justify your answer.

Solution 2

(a)

Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force F_{N1} up and weight $2Mg$ down; on the $3M$ block, normal force F_{N2} up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

Solution 2

(b)

Mark scheme

- The $3M$ block moving at v_0 collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum: $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

Solution 2

(c)

Mark scheme

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$. 1 pt for an answer consistent with the speed found in part (b).

Solution 2

(d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$: $\frac{9}{10}Mv_0^2 = \int_0^D Bx^3\,dx = \left[\frac{Bx^4}{4}\right]_0^D = \frac{BD^4}{4}$. Solving: $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to D) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18Mv_0^2/(5B)}$.

Solution 2

(e)(i)

Mark scheme

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

Solution 2

(e)(ii)

Mark scheme

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

Solution 2

(f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.

Question 3

2014 ▪ Q3

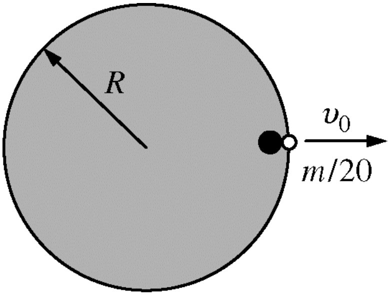
[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
[Figure, Side View: A person holding a stone stands atop a raised platform of height h above the ground, throwing the stone horizontally with initial speed v_0 (arrow labelled v_0); h is marked as the vertical distance from the platform surface to the ground.]
Mech. 3.
A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured

Question 3

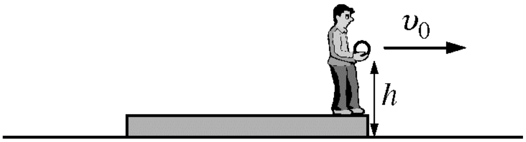
relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

Question 3

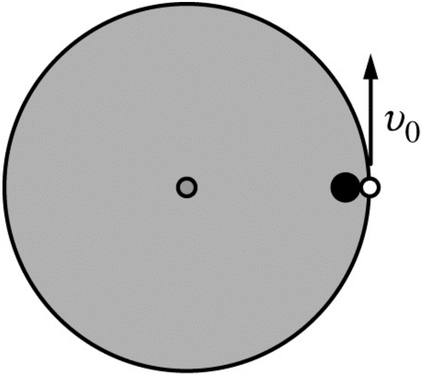


Top View



Side View

Question 3



Top View

Question 3

2014 ▪ Q3

[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

Question 3

2014 ▪ Q3

[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(c) Derive an expression for the time it will take the disk to stop sliding.

Question 3

2014 ▪ Q3

[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(d) [Figure, Top View: A large filled circle (disk) with a small open circle marking its center, and a small stone (filled circle) at the edge of the disk moving tangentially with velocity v_0 (arrow labelled v_0 pointing tangent to the circle at the edge point).]

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

Question 3

2014 ▪ Q3

[Figure, Top View: A large filled circle of radius R (labelled with an arrow from center to edge) representing a disk, with a small stone of mass $m/20$ shown just outside the edge of the disk moving away with velocity v_0 (arrow labelled v_0, $m/20$).]
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Question 3

the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.