

Past-paper walk-through

Translational Kinetic Energy ■ 3.1

eXercise

Questions in this set

- 2024 Set 1 Q1
- 2019 Set 2 Q2
- 2016 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

2024 Set 1 ■ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3 . Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2 . Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x -axis: 0, x_0 , x_1 , x_2 , x_3 . Note: Figure not drawn to scale.]

Question 1

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

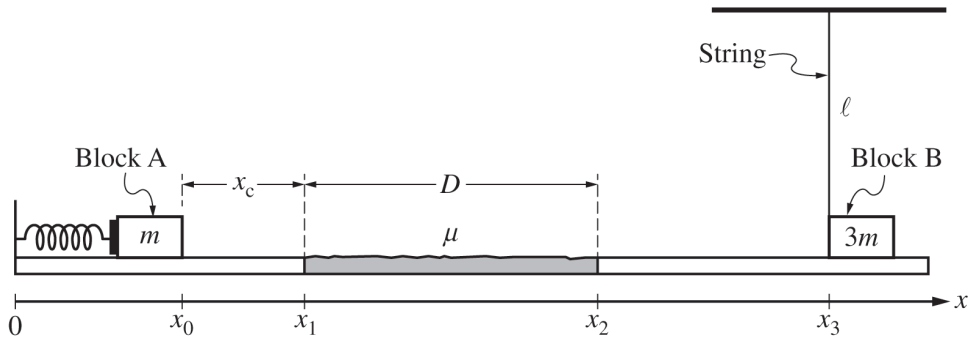
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

(a)(i) Derive an expression for the speed v of Block A at time t_1 .

(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

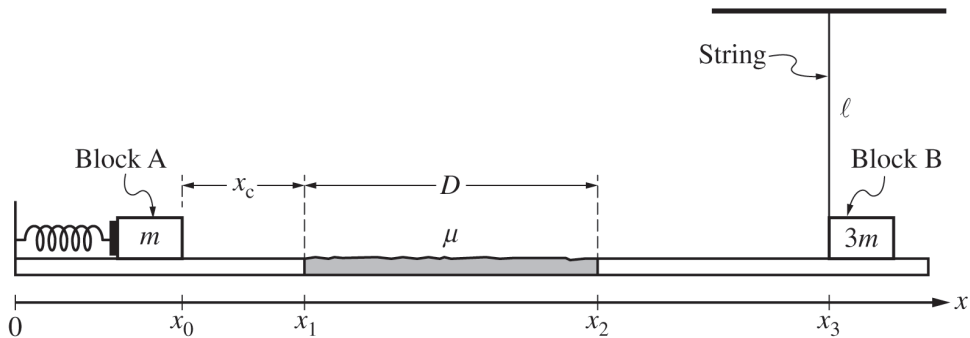
Question 1



Note: Figure not drawn to scale.

Figure 1

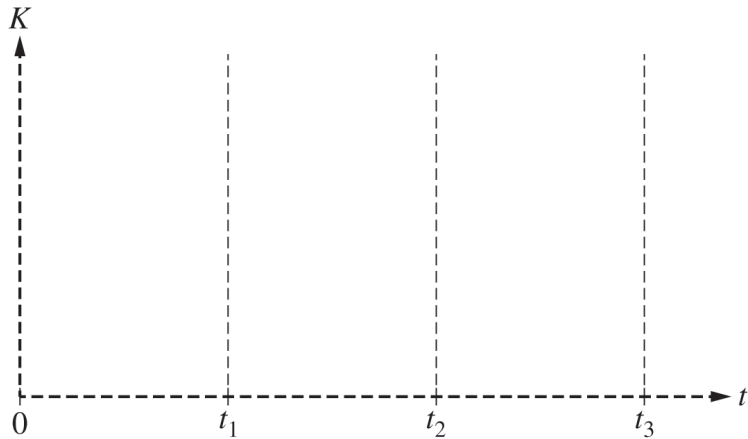
Question 1



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Figure 1

Question 1



Question 1

2024 Set 1 ■ Q1

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Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c , friction region of length D with coefficient μ between x_1 and x_2 , and Block B (mass $3m$) hanging by a string of length ℓ at x_3 . Positions marked $0, x_0, x_1, x_2, x_3$ along axis x . Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3 .

Question 1

[Blank graph: vertical axis K , horizontal axis t with gridlines/tick marks at t_1 , t_2 , t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]

(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ■ Q1

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Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4 , having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ . Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

_____ $f_{2\ell} > f_i$ _____ $f_{2\ell} < f_i$ _____ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s , so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v : $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s . Point 2: correct algebraic solution for v .

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2 , after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$:

$$\frac{1}{2}m \left(x_c \sqrt{k/m} \right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}.$$
 (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then

$$v_2^2 = v^2 + 2aD = \left(x_c \sqrt{k/m} \right)^2 - 2\mu gD,$$
 same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$:
 $mv_2 = (m + 3m)v_{A,B} = 4m v_{A,B}.$ Solving:

$$v_{A,B} = \frac{m \sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \frac{1}{4} \sqrt{\frac{kx_c^2}{m} - 2\mu gD}.$$
 Points: (1) energy or kinematics

Solution 1

application to find speed at x_2 ; (2) correct expression for friction energy loss or deceleration ($\Delta E_{\text{friction}} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{\text{initial}}v_{\text{initial}} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_{\ell}$ (doubling the pendulum length decreases its frequency).
Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_{\ell}$. Points: (1) selecting $f_{2\ell} < f_{\ell}$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

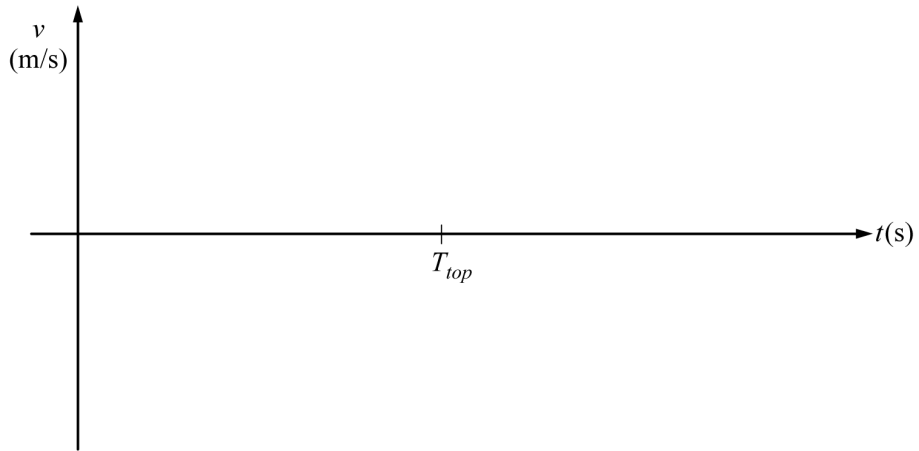
Question 2

2019 Set 2 ■ Q2

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0 \text{ m/s}^2$, $L = 0.25 \text{ m/s}^4$, and t is the time in seconds. At $t = 6.0 \text{ s}$, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0 \text{ s}$.

Question 2



Question 2

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(b) Calculate the speed of the rocket at $t = 6.0 \text{ s}$.

Question 2

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(c)(i) Calculate the kinetic energy of the rocket at $t = 6.0 \text{ s}$.

(c)(ii) Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0 \text{ s}$.

Question 2

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(d) Calculate the maximum height reached by the rocket relative to its launching point.

Question 2

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(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 2

[Graph axes: vertical axis labeled v (m/s), horizontal axis labeled t (s); the origin is at the intersection, with a tick mark labeled T_{top} on the horizontal axis to the right of the origin. The axes are otherwise unscaled and blank for the student to sketch on.]

Solution 2

(a)

Mark scheme

- $J = \int F(t) dt = \int ma(t) dt = (0.50) \int_0^6 (9.0 - 0.25t^2) dt$ (1 pt for the correct expression and substitution of $a(t)$ and m). Integrating with the correct limits:
 $J = (0.50) \left[9.0t - \frac{1}{12}t^3 \right]_0^6 = (0.50) \left[(9.0)(6) - \frac{1}{12}(6)^3 \right] = (0.50)(54 - 18)$ (1 pt for correct integration/limits). Final answer: $J = 18 \text{ N} \cdot \text{s}$.

Solution 2

(b)

Mark scheme

- Relate impulse to the change in momentum: $J = \Delta p = m(v_2 - v_1)$ (1 pt for correctly relating impulse to speed). Substitute $J = 18 \text{ N} \cdot \text{s}$ from part (a): $(18 \text{ N} \cdot \text{s}) = (0.50 \text{ kg})(v_2 - 0)$ (1 pt for correct substitution), giving $v_2 = 36 \text{ m/s}$.

Solution 2

(c)(i)

Mark scheme

- $K = \frac{1}{2}mv^2$. Substitute the mass of the rocket (1 pt) and the speed $v = 36 \text{ m/s}$ found in part (b) (1 pt): $K = \frac{1}{2}(0.50 \text{ kg})(36 \text{ m/s})^2 = 324 \text{ J}$.

Solution 2

(c)(ii)

Mark scheme

- Integrate the acceleration twice to obtain position. First:

$$v(t) = \int a(t) dt = \int (9.0 - 0.25t^2) dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$$
 (1 pt for integrating with correct limits/constant of integration, since $v_0 = 0$). Then integrate again:

$$\Delta y = \int_0^6 v(t) dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4 \right]_0^6 = \frac{9}{2}(36) - \frac{1}{48}(1296) = 135 \text{ m}$$
 (1 pt for the second integration with correct limits). Substitute into

$$\Delta U_g = mg\Delta y = (0.50 \text{ kg})(9.8 \text{ m/s}^2)(135 \text{ m}) \approx 660 \text{ J}$$
 (1 pt for substituting into the potential-energy equation).

Solution 2

(d) Mark scheme

- Use $v_2^2 = v_1^2 + 2a(y_2 - y_1)$ with $a = g = -9.8 \text{ m/s}^2$ after burnout and $v_2 = 0$ at maximum height: $y_2 - y_1 = -\frac{v_1^2}{2a}$ (1 pt for using $a = g$ in a correct kinematics equation). Substitute the speed $v_1 = 36 \text{ m/s}$ from part (b) (1 pt):

$$y_2 - y_1 = -\frac{(36 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} \approx 66 \text{ m.}$$
Add the height already gained by $t = 6.0 \text{ s}$, $\Delta y = 135 \text{ m}$ from part (c)(ii) (1 pt for substituting the height from part (c)): $y_2 = 66 \text{ m} + 135 \text{ m} = 201 \text{ m}$ (199.8 m if $g = 10 \text{ m/s}^2$). [Equivalent alternate solutions using energy conservation, $mg\Delta y = \frac{1}{2}mv^2$, or $K_1 + U_1 = U_{top}$ with parts (c)(i)/(c)(ii): $324 + 660 = (0.5)(9.8)\Delta y$, also earn full credit.]

Solution 2

(e) Mark scheme

- Sketch: for $0 \leq t \leq 6.0$ s, an initial concave-down curve starting at the origin $(0, 0)$ that rises to a maximum of $v = 36$ m/s at $t = 6.0$ s (1 pt, for an initial concave-down curve starting at the origin, consistent with the decreasing-but-positive acceleration $a = 9.0 - 0.25t^2$). The curve then transitions, before T_{top} , into a straight line with constant negative slope $-g$ (1 pt, for a transition before T_{top} into a straight line of negative slope). Label the maximum value 36 m/s (consistent with part (b)) with the line crossing the time axis exactly at $t = T_{top}$, continuing to negative v as the rocket falls back to the ground (1 pt, for labeling the maximum consistent with part (b) and crossing the axis at T_{top}).

Question 2

2016 ■ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown. **(a)** On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$ " (left dot) and "Block of Mass $3M$ " (right dot), for the student to draw force vectors on.]

[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled " $2M$ " next to " $3M$ " (the two blocks shown adjacent, having moved together). A distance D is marked from the wall/spring's compressed end to a reference position " 0 ".]

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D , as shown above. Express your answers to parts (b), (c), and (d) in terms of M , B , v_0 , and physical constants, as appropriate.

Question 2



Question 2

a pointing away from, the appropriate dot.

Block of Mass $2M$



Block of Mass $3M$



Question 2



Question 2

2016 ■ Q2

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(b) Derive an expression for the speed of the blocks immediately after the collision.

Question 2

2016 ■ Q2

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(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

Question 2

2016 ■ Q2

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(d) Derive an expression for the maximum distance D that the spring is compressed.

Question 2

2016 ■ Q2

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(e)(i) In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left _____ Right _____ The net force on the block of mass $2M$ is zero.

Question 2

Justify your answer.

(e)(ii) Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

_____ The magnitude of the net force is greater on the block of mass $2M$. _____ The magnitude of the net force is greater on the block of mass $3M$. _____ The magnitude of the net force on each block has the same nonzero value. _____ The magnitude of the net force on each block is zero.

Justify your answer.

Question 2

2016 ■ Q2

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A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

_____ Yes _____ No

Justify your answer.

Solution 2

(a)

Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force F_{N1} up and weight $2Mg$ down; on the $3M$ block, normal force F_{N2} up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

Solution 2

(b)

Mark scheme

- The $3M$ block moving at v_0 collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum:
 $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

Solution 2

(c)

Mark scheme

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M) \left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$.
1 pt for an answer consistent with the speed found in part (b).

Solution 2

(d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$:

$$\frac{9}{10} Mv_0^2 = \int_0^D Bx^3 dx = \left[\frac{Bx^4}{4} \right]_0^D = \frac{BD^4}{4}.$$
 Solving: $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to D) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18Mv_0^2/(5B)}$.

Solution 2

(e)(i)

Mark scheme

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

Solution 2

(e)(ii)

Mark scheme

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

Solution 2

(f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.