# Past-paper walk-through

## Systems and Center of Mass ▪ 2.1

eXercise

# Questions in this set

- 2026 Q2
- 2024 Set 1 Q3
- 2021 Set 1 Q3
- 2021 Set 2 Q2
- 2018 Q2

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

# Question 2

2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed $v_0$ at an angle $\theta$ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses $M$ and $3M$, respectively.

The motion of the projectile is described for the following times.

- At $t = t_1$, immediately after the projectile breaks apart, the two pieces are moving away from each other horizontally. - At $t = t_2$, Piece Q reaches the ground at $x = 0$ and Piece R reaches the ground at $x = x_2$, as shown in Figure 1.

[Figure 1: Three side-view snapshots labeled $t = 0$, $t = t_1$, $t = t_2$, with a note "Figure not drawn to scale." At $t = 0$: a $+y$ vertical axis and $+x$ horizontal axis are shown at the origin; a small box on the ground at $x = 0$ launches with velocity $v_0$ at angle $\theta$ above the horizontal, following a dashed parabolic path upward. At $t = t_1$: at the top

# Question 2

of the dashed parabolic trajectory, two pieces are shown separating horizontally — "Piece Q, $M$" moving to the left and "Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

**(a)** The horizontal and vertical components of a momentum vector are represented by $p_x$ and $p_y$, respectively. The shaded bars in Figure 2 represent $p_x$ and $p_y$ of the projectile immediately after $t = 0$.

[Figure 2: "Figure 2, $t = 0$." A bar chart titled "Momentum" with vertical axis showing a dashed zero line labeled $0$ and horizontal categories $p_x$ and $p_y$, captioned "Complete Projectile." A shaded bar for $p_x$ rises to a medium-large positive height above the zero line; a shaded bar for $p_y$ rises to a smaller positive height above the zero line (shorter than the $p_x$ bar).]

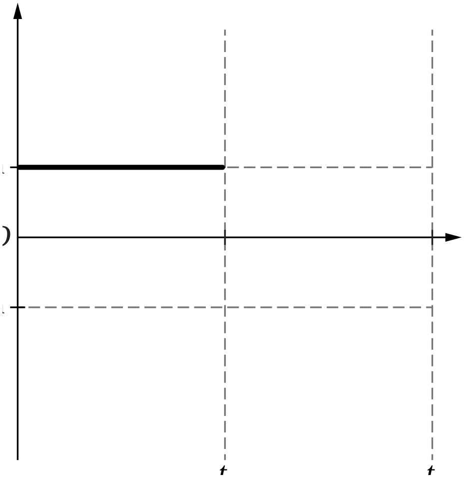
# Question 2

[Figure 3: "Figure 3, $t = t_1$." Two blank bar charts titled "Momentum," each with a dashed zero line labeled $0$ and horizontal categories $p_x$ and $p_y$; the left one captioned "Piece Q" and the right one captioned "Piece R." Both are currently empty (no shaded bars drawn).]

On Figure 3, **draw** shaded bars to represent $p_x$ and $p_y$ of Pieces Q and R at $t = t_1$.

- Start the shaded bars at the dashed line that represents zero momentum.
- Represent the magnitudes of the components of the momentum by using the heights of the shaded bars, consistent with the scale used in Figure 2.
- Represent any momentum component that is equal to zero by drawing a distinct line on the zero-momentum line.

# Question 2



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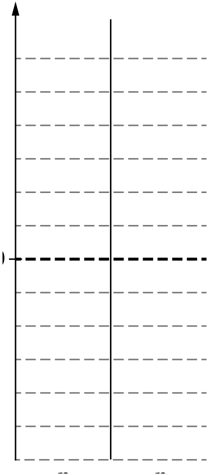
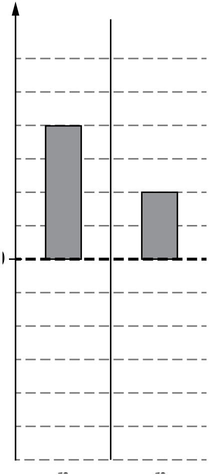


# Question 2



# Question 2

2026 ▪ Q2

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# Question 2

"Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

**(b)** **Derive** an expression for $x_2$ in terms of $v_0$, $\theta$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

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2026 ▪ Q2

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**(c)** The horizontal component of a velocity vector is represented by $v_x$. Figure 4 shows the horizontal component $v_{x,\mathrm{cm}}$ of the velocity of the center of mass of the projectile as a function of $t$ during the time interval $0 < t < t_1$.

[Figure 4: A graph with vertical axis $v_x$ and horizontal axis $t$ (origin $O$). A horizontal dashed reference line is labeled $v_{x,\mathrm{cm}}$ above the $t$-axis, and another horizontal dashed reference line is labeled $-v_{x,\mathrm{cm}}$ below the $t$-axis (symmetric about the axis). A solid horizontal line segment at height $v_{x,\mathrm{cm}}$ runs from $t = 0$ to $t = t_1$, representing $v_x$ during $0 < t < t_1$. Vertical dashed lines mark $t = t_1$ and $t = t_2$ on the horizontal axis.]

On Figure 4, **sketch** a line or curve to represent $v_x$ as a function of $t$ for the time interval $t_1 < t < t_2$ for each of the following.

# Question 2

- Piece Q - Piece R - The center of mass of the two-piece system
Clearly **label** all lines or curves.

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2026 ▪ Q2

A projectile of total mass $4M$ is launched from the ground at position $x = 0$ and time $t = 0$. The projectile is launched with an initial speed $v_0$ at an angle $\theta$ above the horizontal. When the projectile is at the highest point in its trajectory, it breaks into Pieces Q and R of masses $M$ and $3M$, respectively.

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# Question 2

"Piece R, $3M$" moving to the right, connected by a wavy break symbol; the ground below is marked $x = 0$. At $t = t_2$: on the ground, a small box (Piece Q) sits at $x = 0$ and a larger box (Piece R) sits at $x = x_2$ to the right.]

**(d)** Consider a case in which the projectile is launched at the same angle and initial speed as initially described. When the projectile breaks into Pieces Q and R, Piece Q falls straight down. In this case, Piece R reaches the ground at $x = x_{\text{new}}$.

**Indicate** whether $x_{\text{new}}$ is greater than, less than, or equal to $x_2$ by writing one of the following.

- $x_{\text{new}} > x_2$ - $x_{\text{new}} < x_2$ - $x_{\text{new}} = x_2$

Briefly **justify** your answer either by referencing a feature of the representations you drew in part A or C or by using conceptual reasoning beyond algebraic solutions.

# Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length $L$ is attached at a pivot on a vertical pole. The rod extends up and to the right at angle $\theta$ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-$S$" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length $L$ and mass $m$ is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle $\theta$ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

# Question 3

**(a)** [Figure: A schematic representation of the rod alone (no pole, no string, no block), extending diagonally with a pivot circle at the lower-left end. Points marked along the rod: P, C, and Q (in that order from the pivot outward).]
On the following representation of the rod, draw and label the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.

# Question 3

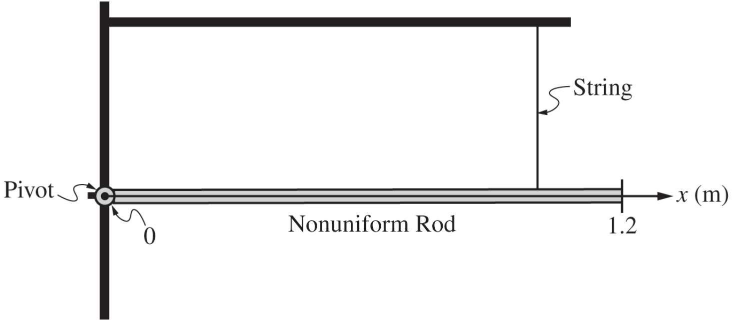



Figure 3

Note: Figure not drawn to scale.

# Question 3

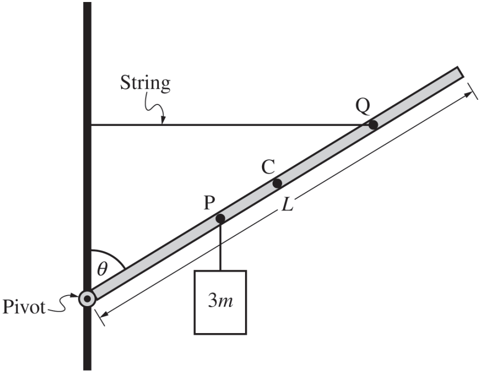



Figure 1

<u>Note</u>: Figure not drawn to scale.

# Question 3

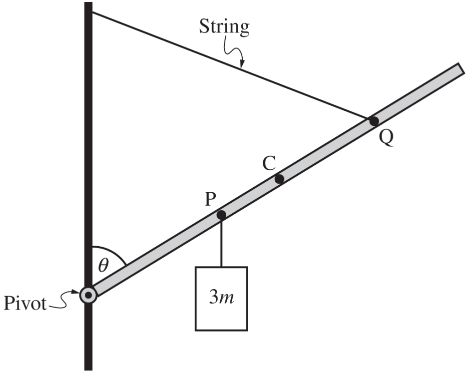



Figure 2

<u>Note</u>: Figure not drawn to scale.

# Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length $L$ is attached at a pivot on a vertical pole. The rod extends up and to the right at angle $\theta$ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-$S$" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length $L$ and mass $m$ is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle $\theta$ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

**(b)** In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot.

# Question 3

Derive an equation for the tension $F_T$ in the horizontal string in terms of $L$, $m$, $\theta$, and physical constants, as appropriate.

# Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length $L$ is attached at a pivot on a vertical pole. The rod extends up and to the right at angle $\theta$ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-$S$" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length $L$ and mass $m$ is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle $\theta$ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

**(c)** [Figure 2: Same setup as Figure 1, but the string now connects from Point Q on the rod to a higher point on the vertical pole (the string attachment point is above the

# Question 3

top of the pole shown in Figure 1), making a steeper angle with the pole. The rod still makes angle $\theta$ with the pole, with Points P, C, Q marked along the rod, and the block of mass $3m$ hanging from Point P. Pivot labeled "Pivot-$S$" at the base. Note: Figure not drawn to scale.]

The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle $\theta$ remains the same. How does the new tension $F_{T,\ new}$ compare with the original tension $F_T$ from part (b)? Justify your reasoning.

# Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length $L$ is attached at a pivot on a vertical pole. The rod extends up and to the right at angle $\theta$ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-$S$" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length $L$ and mass $m$ is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle $\theta$ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

**(d)(i)** [Figure 3: A nonuniform rod attached horizontally to a pivot on a vertical pole, extending to the right along a horizontal $x$ (m) axis from $0$ at the pivot to $1.2$ at the

# Question 3

far end labeled "Nonuniform Rod". A string labeled "String" connects from a point partway along the rod up to the top of the vertical pole, forming a right-triangle-like support. Pivot labeled "Pivot-$S$" at the base of the pole. Note: Figure not drawn to scale.]

A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of $1.2$ m and a linear mass density $\lambda(x) = A + Bx$, where $x$ is the distance from the pivot, $A = 6.0\ \text{kg/m}$, and $B = 10.0\ \text{kg/m}^2$.

Calculate the mass of the rod.

**(d)(ii)** Calculate the rotational inertia of the rod about the pivot.

# Solution 3

## (a) Mark scheme

- Draw and label: a downward force $F_{T,block}$ (or $3mg$) at point P (tension from the string holding the hanging block); a downward gravitational force $mg$ at point C (the rod's own weight, acting at its center of mass); a leftward force $F_{T,string}$ at point Q (tension from the string to the pole); and a force directed up and to the right at the pivot, $F_{pivot}$ (the pivot's reaction force), with no other (extraneous) forces drawn. Points: (1) correctly drawn/labeled downward forces at P and C; (2) correctly drawn/labeled leftward force at Q; (3) correctly drawn/labeled up-and-right force at the pivot, with no extraneous forces.

# Solution 3

## (b) Mark scheme

- Balance torques about the pivot ($\Sigma\tau = 0$), with the hanging mass $3m$ at P ($\frac{3L}{8}$ from pivot), the rod's weight $mg$ at C ($\frac{4L}{8} = \frac{L}{2}$ from pivot), and the string tension $F_T$ at Q ($\frac{6L}{8}$ from pivot, acting with perpendicular-component $F_T \cos\theta$ since the string is horizontal and the rod makes angle $\theta$ with the vertical pole): $3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$, so $\left(\frac{9}{8}\right) mg\sin\theta + \left(\frac{4}{8}\right) mg\sin\theta = \left(\frac{6}{8}\right) F_T\cos\theta$, giving $13mg\sin\theta = 6F_T\cos\theta \Rightarrow F_T = \dfrac{13}{6}mg\tan\theta$. (Scoring note: a maximum of 3 of the 4 points may be earned if sin and cos are reversed throughout for all three torque terms.) Points: (1) net torque on rod equals zero; (2) correct torque

# Solution 3

expression for the hanging mass; (3) correct torque expression for the rod's own gravitational force; (4) correct torque expression for the string tension.

# Solution 3

## (c) Mark scheme

- Because the torque that the string must supply on the rod is always the same (it must still balance the unchanged torques from the rod's weight and the hanging mass), moving the string's attachment point higher on the pole increases the angle between the string and the rod, which increases the string's perpendicular moment arm. To produce the same torque with a larger moment arm, the tension $F_{T,new}$ must be smaller. So as the angle between the string and rod increases, the force exerted on the rod by the string decreases. Points: (1) indicating the torque exerted on the rod by the string is always the same; (2) stating that as the angle between the string and rod increases, the tension force decreases.

# Solution 3

## (d)(i) Mark scheme

- The nonuniform rod has linear mass density $\lambda(x) = A + Bx$ (with $A = 6.0$ kg/m, $B = 10.0$ kg/m$^2$) over its $1.2$ m length. Total mass: $M = \int dm = \int_0^{1.2} \lambda \, dx = \int_0^{1.2} (6 + 10x)\, dx = \left(6x + \frac{10x^2}{2}\right)\Big|_0^{1.2} = 6(1.2) + 5(1.2)^2 = 7.2 + 7.2 = 14.4$ kg. Points: (1) indicating the total mass is the sum (integral) of differential masses along the rod, $M = \int dm$; (2) correctly writing $dm$ in terms of $x$ (i.e. $dm = \lambda \, dx = (6 + 10x)\, dx$); (3) a correct numeric answer with correct units, $M = 14.4$ kg.

# Solution 3

## (d)(ii) Mark scheme

- Rotational inertia about the pivot: $I = \int r^2\, dm$ with $dm = \lambda\, dx$ and $r = x$: $I = \int_0^{1.2}(A + Bx)x^2\, dx = \int_0^{1.2}(6 + 10x)x^2\, dx = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Big|_0^{1.2} = \frac{6(1.2)^3}{3} + \frac{10(1.2)^4}{4} = 3.456 + 5.184 = 8.64\ \text{kg}\cdot\text{m}^2$. Points: (1) correct substitution of $\lambda$ into an integral expression for rotational inertia; (2) correct integration; (3) a correct numeric answer with correct units, $I = 8.64\ \text{kg}\cdot\text{m}^2$.

# Question 3

2021 Set 1 ▪ Q3

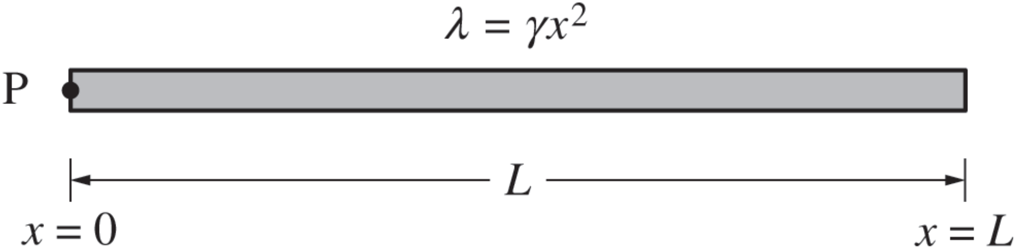
[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length $L$ is marked from $x = 0$ to $x = L$.]

A triangular rod of length $L$ and mass $M$ has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

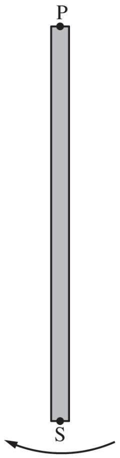
**(a)** Using integral calculus, show that the rotational inertia $I$ of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

# Question 3


# Question 3



# Question 3



# Question 3

2021 Set 1 ▪ Q3

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**(b)** Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of $L$.

# Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length $L$ is marked from $x = 0$ to $x = L$.]

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**(c)** For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

______ Greater than ______ Less than ______ Equal to

Justify your answer.

# Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length $L$ is marked from $x = 0$ to $x = L$.]

A triangular rod of length $L$ and mass $M$ has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

**(d)** The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

On the axes below, sketch graphs of the magnitude of the net torque $\tau$ on the rod and the angular speed $\omega$ of the rod as functions of time $t$ from the time the rod is released until the time its center of mass reaches its lowest point.

[Two blank axes: left axis labeled $\tau$ (vertical) vs. $t$ (horizontal); right axis labeled $\omega$ (vertical) vs. $t$ (horizontal). Both currently blank, to be sketched by the student.]

# Question 3

2021 Set 1 ▪ Q3

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**(e)** As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_______ Increasing _______ Decreasing _______ Not changing

Justify your answer.

# Question 3

2021 Set 1 ▪ Q3

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A triangular rod of length $L$ and mass $M$ has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and $x$ is the distance from point P at the left end of the rod.

**(f)** [Figure: The rod hangs vertically, pivoted at point P at the top, with point S marked at the bottom end. A curved arrow beneath S indicates the rod swinging.] The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed $v$ of point S as the rod swings through the vertical position shown.

# Solution 3

## (a) Mark scheme

- Set up the rotational-inertia integral about point P: $I = \int r^2\, dm$, with the given linear mass density $\lambda = \gamma x^2$ so $dm = \lambda\, dx = \gamma x^2\, dx$ (1 point). Substitute and integrate from $x = 0$ to $x = L$, using $\gamma = 3M/L^3$ (from normalizing the total mass $M = \int_0^L \gamma x^2\, dx = \gamma L^3/3$) (1 point):
$I = \int_0^L x^2(\gamma x^2\, dx) = \gamma \left[\frac{x^5}{5}\right]_0^L = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5}\right) = \frac{3}{5}ML^2$.

# Solution 3

## (b) Mark scheme

- Set up the center-of-mass integral: $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x\, dm}{\int dm}$ (1 point). Substitute $dm = \gamma x^2\, dx$ into the numerator (1 point), and either substitute $M$ directly for the denominator or evaluate $\int dm$ to get the rod's mass (1 point): $X_{CM} = \frac{\int_0^L x(\gamma x^2)\, dx}{M} = \frac{[\gamma x^4/4]_0^L}{M} = \frac{(3M/L^3)(L^4/4)}{M} = \frac{3}{4}L$. The center of mass is located $\frac{3}{4}L$ from point P.

# Solution 3

## (c)

### Mark scheme

- Answer: Greater than (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) more of the rod's mass is concentrated at the end opposite P (farther from the rotation axis), so the rotational inertia about P exceeds that about the center of mass; or (ii) by the parallel-axis theorem $I = I_{cm} + md^2$, an axis displaced from the center of mass ($d > 0$) always gives a larger rotational inertia than the axis through the center of mass.

# Solution 3

## (d) Mark scheme

- Sketch two graphs vs. time $t$, from release (rod horizontal) to when the center of mass reaches its lowest point (rod vertical). Torque $\tau$ vs. $t$ (1 point): starts at its maximum value at $t = 0$ and decreases along a concave-down curve to zero (torque vanishes as the rod becomes vertical, when the weight acts along the line to the pivot). Angular speed $\omega$ vs. $t$ (1 point): starts at 0 and increases along a concave-down curve that flattens, approaching a horizontal asymptote (constant $\omega$) as $t$ increases (since $\alpha = \tau / I \to 0$ as $\tau \to 0$). Consistency (1 point): $\omega$ must keep rising as long as $\tau > 0$ and level off exactly as $\tau \to 0$, matching $d\omega / dt = \tau / I$.

# Solution 3

## (e)

**Mark scheme**

- Answer: Decreases (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) from $\tau = rF\sin\theta$, as the rod swings downward the angle $\theta$ between the weight and the lever arm decreases, so the torque decreases; or (ii) the horizontal lever arm between P and the rod's center of mass continuously shrinks as the rod rotates downward, so the torque decreases. Since $I$ is constant and $\alpha = \tau/I$, angular acceleration decreases correspondingly.

# Solution 3

## (f) Mark scheme

- Use conservation of energy for the rod swinging from horizontal (at rest) to vertical: $U_i + K_i = U_f + K_f \Rightarrow U_i = K_f$ (1 point). Substitute the gravitational PE (using the center-of-mass drop $h_i = \frac{3}{4}L$ found in part (b)) and the rotational KE about P (1 point): $Mgh_i = \frac{1}{2}I\omega_f^2 \Rightarrow Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\omega_f^2$. Using $I = \frac{3}{5}ML^2$ from part (a) and $\omega_f = v/L$, where $v$ is the linear speed of point S (the free end, distance $L$ from P) (1 point): $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \Rightarrow v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}(9.8\text{ m/s}^2)(1.0\text{ m})} = 4.9\text{ m/s}$. (Given $M = 3.0$ kg, $L = 1.0$ m; note $M$ cancels out of the final expression for $v$.)

# Question 2

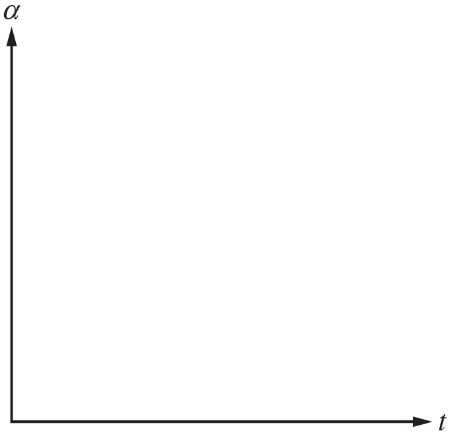
2021 Set 2 ▪ Q2

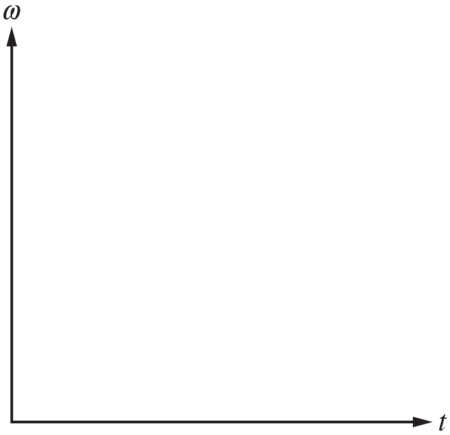
[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]
Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.
**(a)** Using integral calculus, derive an expression to show that the rotational inertia $I_A$ of object A about the pivot is given by $\frac{4}{3}ML^2$.
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an $x$-$y$ coordinate system with origin $O$ at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the $y$-axis). A horizontal rod of length $L$ extends

## Question 2

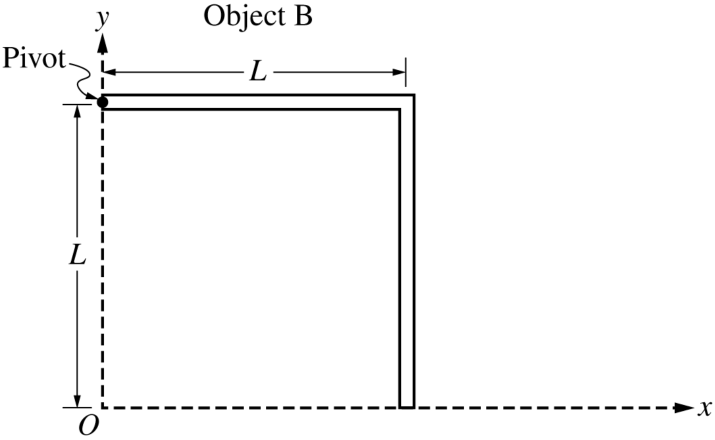
right from the pivot along the top (labelled "$L$"), and a vertical rod of length $L$ extends down from the pivot to $O$ along the $y$-axis (labelled "$L$"). The $x$-axis points right from $O$, the $y$-axis points up from $O$ through the pivot.]
Object B of total mass $M$ is formed by attaching two thin, uniform, identical rods of length $L$ at a right angle to each other. Object B is held in place, as shown above.
Express your answers in part (b) in terms of $L$.

# Question 2

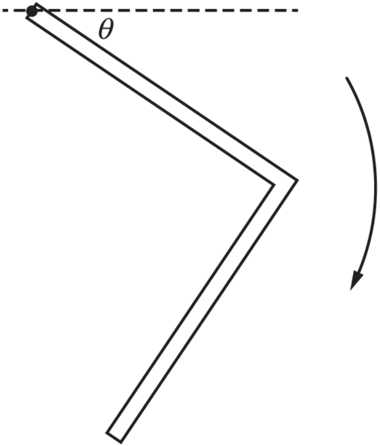


# Question 2



# Question 2



# Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

**(b)(i)** Determine the following for the given coordinate system shown in the figure.

The $x$-coordinate of the center of mass of object B

**(b)(ii)** The $y$-coordinate of the center of mass of object B

# Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

**(c)** Object B has a rotational inertia of $I_B$ about its pivot.

Is the value of $I_B$ greater than, less than, or equal to $I_A$?

_______Greater than _______Less than _______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

# Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

**(d)** On the axes below, sketch graphs of the magnitude of the angular acceleration $\alpha$ and the angular speed $\omega$ of object B as functions of time $t$ from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled $\alpha$ (vertical) vs. $t$ (horizontal); right axis labelled $\omega$ (vertical) vs. $t$ (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

# Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

**(e)** [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle $\theta$ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

# Question 2

While object B rotates from the horizontal position down through the angle $\theta$ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

Increasing Decreasing Notchanging

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length $L$ now lies along the bottom (horizontal), and the horizontal rod of length $L$ extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

# Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass $M$ and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with $M$ labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

**(f)** Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of $M$, $L$, $I_B$, and physical constants, as appropriate.

# Solution 2

## (a) Mark scheme

- Using integral calculus with $\lambda$ the linear mass density of the rod (length $2L$, mass $M$): $I = \int_{r=0}^{r=2L} \lambda r^2 \, dr = \lambda \left[\frac{r^3}{3}\right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating $\lambda$ to $M$ and $L$). This confirms $I_A = \frac{4}{3}ML^2$.

# Solution 2

## (b)(i) Mark scheme

- $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \frac{\frac{ML}{4} + \frac{ML}{2}}{M} = \frac{3}{4}L$. Final answer: $X_{CM} = \frac{3}{4}L$.

# Solution 2

## (b)(ii) Mark scheme

- $Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \frac{\frac{ML}{2} + \frac{ML}{4}}{M} = \frac{3}{4}L$. Final answer: $Y_{CM} = \frac{3}{4}L$.

# Solution 2

## (c)

### Mark scheme

- Select ‘Less than’ (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: ‘Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.’ I.e. $I_B < I_A$.

# Solution 2

## (d)

**Mark scheme**

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration $\alpha$ vs. *t* graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed $\omega$ vs. *t* graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — $\omega$ increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

# Solution 2

## (e)

**Mark scheme**

- Select 'Decreasing' (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: 'Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.'

# Solution 2

## (f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2}I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ (1 pt). Solve for $\omega$ in terms of the allowed symbols $M$, $L$, $I_B$: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt).

  Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.

# Question 2

2018 ▪ Q2

Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right) t^2 - (500 \text{ N/s})t$.

[Diagram: Cart 1 (mass $m = 0.500$ kg) with a Force Sensor attached at its right side, moving right with $v_0 = 3.00$ m/s toward a spring ($k = 130$ N/m) connected to Cart 2 (mass $m = 1.05$ kg, initially at rest, $v = 0$), all on a horizontal track.]

# Question 2

[Graph: Force $F$ (N) on the vertical axis (marked at 5, 0, $-5$, $-10$, $-15$, $-20$, $-25$) versus time $t$ (s) on the horizontal axis (marked 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18). Data points start near $F = 0$ at small $t$, dip down to a minimum of about $-20$ N around $t = 0.08$–$0.10$ s, then rise back up toward $F = 0$ by about $t = 0.16$–$0.18$ s, tracing a roughly parabolic (U-shaped, inverted) curve consistent with the quadratic fit given.]

**(a)** Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

**(b)(i)** Calculate the speed of cart 1 after the collision.

**(b)(ii)** In which direction does cart 1 move after the collision?

______ Left ______ Right

______ The direction is undefined, because the speed of cart 1 is zero after the collision.

**(c)(i)** Calculate the speed of cart 2 after the collision.

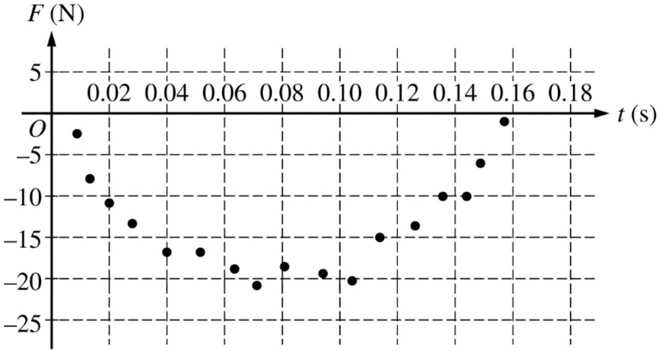
# Question 2

**(c)(ii)** Show that the collision between the two carts is elastic.
**(d)(i)** Calculate the speed of the center of mass of the two-cart-spring system.
**(d)(ii)** Calculate the maximum elastic potential energy stored in the spring.

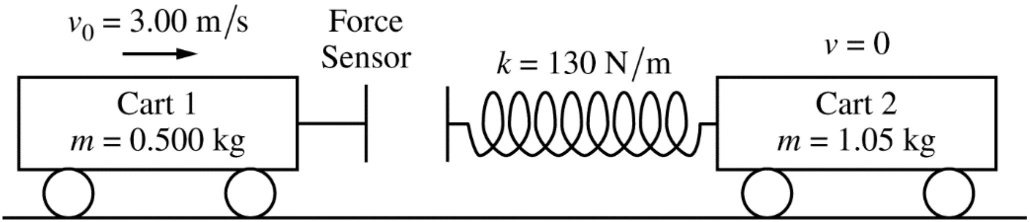
# Question 2

a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are sho

below. A student models the data as the quadratic fit $F = \left(3200\ \mathrm{N/s^2}\right)t^2 - (500\ \mathrm{N/s})t$ .



# Question 2



# Solution 2

## (a)

**Mark scheme**

- $J = \int F\,dt = \int_0^{0.16} (3200t^2 - 500t)\,dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_0^{0.16} = \frac{3200}{3}(0.16)^3 - 250(0.16)^2 = -2.03$ N·s. 1 point for using the given force equation in the impulse integral; 1 point for integrating with correct limits (or constant of integration); 1 point for the correct final numeric answer $J = -2.03$ N·s.

# Solution 2

## (b)(i)

### Mark scheme

- $J = m_1(v_{1f} - v_{1i}) \Rightarrow v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{-2.03\ \mathrm{N\cdot s}}{0.500\ \mathrm{kg}} + 3.00\ \mathrm{m/s} = -1.06\ \mathrm{m/s}.$

# Solution 2

## (b)(ii)

**Mark scheme**

- Left. Since $v_{1f} = -1.06$ m/s is negative (opposite the original direction of motion), cart 1 moves to the left after the collision.

# Solution 2

## (c)(i) Mark scheme

- By Newton's third law on the spring force, $-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \Rightarrow v_{2f} = \dfrac{-J}{m_2} = \dfrac{2.03\ \text{N}\cdot\text{s}}{1.05\ \text{kg}} = 1.93$ m/s. (Equivalent alternate via momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{2f} = \dfrac{m_1(v_{1i} - v_{1f})}{m_2} = \dfrac{(0.500)(3.00 - (-1.06))}{1.05} = 1.93$ m/s.) 1 point for a correct equation to find $v_{2f}$; 1 point for correct substitution.

# Solution 2

## (c)(ii)

**Mark scheme**

- An elastic collision requires initial KE = final KE: $K_{ii} = K_{1f} + K_{2f}$. Check: $\frac{1}{2}(0.500)(3.00)^2 = \frac{1}{2}(0.500)(-1.06)^2 + \frac{1}{2}(1.05)(1.93)^2 \;\Rightarrow\; 2.25$ J $\approx 2.24$ J — equal within rounding, so the collision is elastic. 1 point for indicating that initial and final kinetic energies must be equal; 1 point for correct substitution/numeric verification.

# Solution 2

## (d)(i)

### Mark scheme

- $m_1 v_{1i} = (m_1 + m_2) v_{cm} \;\Rightarrow\; v_{cm} = \dfrac{m_1 v_{1i}}{m_1 + m_2} = \dfrac{(0.500\ \text{kg})(3.00\ \text{m/s})}{0.500\ \text{kg} + 1.05\ \text{kg}} = 0.97$ m/s. 1 point for the correct equation for the center-of-mass speed; 1 point for correct substitution.

# Solution 2

## (d)(ii) Mark scheme

- Using conservation of energy: $K_i = K_f + U_{Sf} \Rightarrow U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}^2 = \frac{1}{2}(0.500)(3.00)^2 - \frac{1}{2}(1.55)(0.97)^2 = 1.52$ J. (Alternate: from $dF/dt = 0$, $t = 0.078$ s gives $|F_{max}| = 19.5$ N (accept 20-21 N from the graph); $x_{max} = F_{max}/k = 0.15$ m (accept 0.15-0.16 m); $U_S = \frac{1}{2}kx_{max}^2 = 1.46$ J, accept 1.46-1.70 J if estimated from the graph.) Scoring: 1 point for using conservation of energy to find the max elastic PE; 1 point for using the center-of-mass speed for the system's translational KE; 1 point for correct substitution/final numeric answer. Q2 also carries an overall 1-point "units" credit (correct units on all calculated answers through the question) — folded into this final part so Q2's 15 points are fully accounted for.