Past-paper walk-through

Displacement, Velocity, and Acceleration ▪ 1.2

eXercise

Questions in this set

- 2019 Set 1 Q1
- 2019 Set 2 Q1
- 2019 Set 2 Q2
- 2018 Q1
- 2017 Q2
- 2015 Q1
- 2014 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2019 Set 1 ▪ Q1

In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.
[Figure above the question: a cylinder of fluid on a table, with hands releasing an object labeled "Object, Mass = 12 g" at the top of the "Cylinder" containing "Fluid", resting on a "Table".]
[Graph: v (m/s) vs. t (s); y-axis from 0.0 to 1.0 in increments of 0.2, x-axis from 0.00 to 0.35 in increments of 0.05. Data points (dots) roughly follow a dashed best-fit curve that rises steeply from the origin, e.g. passing near (0.00, 0.0), (0.025, 0.17), (0.05, 0.25), (0.075, 0.42), (0.10, 0.47), (0.125, 0.58), (0.15, 0.66), (0.175, 0.65),

Question 1

(0.20, 0.76), (0.225, 0.78), (0.25, 0.80), (0.275, 0.90), (0.30, 0.93), (0.325, 0.92); the curve rises quickly at first then levels off, approaching an asymptote near $v \approx 1.0$ m/s.]

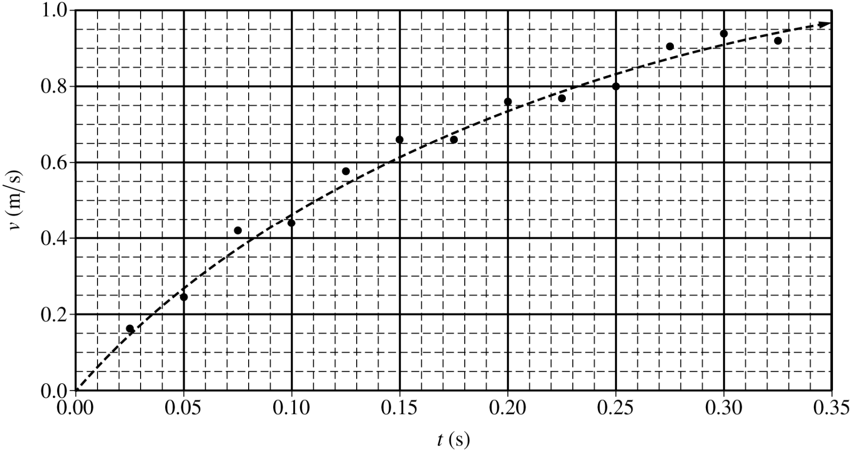
(a)(i) Does the speed of the object increase, decrease, or remain the same?

______ Increase   ______ Decrease   ______ Remain the same

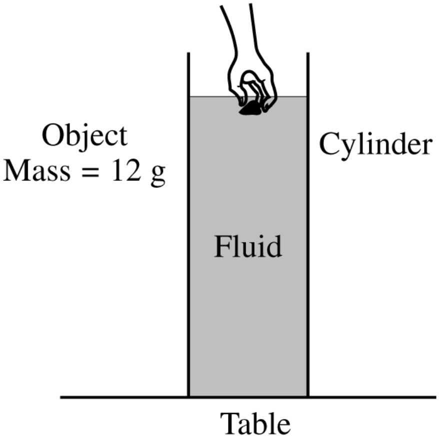
(a)(ii) In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

(a)(iii) Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

Question 1



Question 1



Question 1

2019 Set 1 ▪ Q1

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Question 1

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(b)(i) The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s^{-1}. Use the above equation to derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

(b)(ii) Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

Question 1

2019 Set 1 ▪ Q1

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Question 1

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(c)(i) The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed. Determine the constant speed of the object.
Justify your answer.

(c)(ii) Determine the force exerted by the fluid on the object at this time.
Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The speed **Increases** (full credit for selecting 'Increase'). The object is falling and speeding up as it accelerates through the fluid.

Solution 1

(a)(ii)

Mark scheme

- The acceleration points downward (1 pt), and its magnitude decreases as the object falls (1 pt). Example: Because the object is moving downward and speeding up, the acceleration must be downward. Because the slope of the graph of speed vs. time is decreasing, the magnitude of the acceleration must be decreasing.

Solution 1

(a)(iii)

Mark scheme

- Draw a tangent line to the curve at $t = 0.20$ s and compute its slope using two points on that tangent line (1 pt): $a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$. Correct answer using points from the tangent line (1 pt): $a \approx 2.22\ \text{m/s}^2$ (accept values close to this from a reasonable tangent line).

Solution 1

(b)(i) Mark scheme

- Recognize that vertical displacement is the integral of velocity (1 pt): $\Delta y = \int v\,dt = \int A(1 - e^{-Bt})\,dt$. Evaluate with appropriate limits or constant of integration (1 pt): $\Delta y = \int_0^t A(1 - e^{-Bt'})\,dt' = A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$. Correct final answer (1 pt): $\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = 1.18\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$. Credit is given for leaving the answer in terms of A and B or for plugging in the given numeric values.

Solution 1

(b)(ii)

Mark scheme

- Attempt the derivative of the speed equation (1 pt): $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left[A(1-e^{-Bt})\right]$. Correct expression for acceleration (1 pt): $a = ABe^{-Bt}$. Multiply by the mass to get net force (1 pt): $F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$, i.e. $F = (0.012)(1.18)(5)e^{-5t} = 0.071\,e^{-5t}$ N. Credit is given for leaving the answer in terms of A, B, m or for plugging in the given numeric values.

Solution 1

(c)(i)

Mark scheme

- State that the constant (terminal) speed is $v = A$ (1 pt), found by letting $t \to \infty$ in the fit equation (1 pt). After a long time the falling object reaches a terminal constant speed in the fluid; setting $t \to \infty$ in $v = A(1 - e^{-Bt})$ gives the constant speed $v = A = 1.18$ m/s.

Solution 1

(c)(ii)

Mark scheme

- Indicate that the net force is zero at constant speed (1 pt), and that the resistive (fluid) force therefore equals the weight of the object (1 pt). Since the object moves at constant speed, the net force is zero; the only vertical forces are gravity and the fluid's resistive force, so the resistive force equals the weight: $F = mg = (0.012\ \text{kg})(9.8\ \text{m/s}^2) = 0.12\ \text{N}$.

Question 1

2019 Set 2 ▪ Q1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

[Figure 1: An inclined plane rising to the right at angle θ from the horizontal, with a vertical wall/support on the right holding a pulley at the top. On the incline, two blocks are shown in contact, connected by a light string: a lower block labeled $2m$ and, distance d up the slope from the bottom of the incline to the lower block, an upper block labeled m connected to the lower block by tension T_1. A second string from block m, with tension T_2, runs up the incline and over the pulley at the top, then down vertically to a hanging sphere of mass M.]

Question 1

(a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.
[Two dots are shown for the student to draw free-body diagrams on: one labeled m, positioned on a dashed line representing the incline surface (inclined at angle θ), and one labeled M, positioned in open space to represent the hanging sphere.]

Question 1

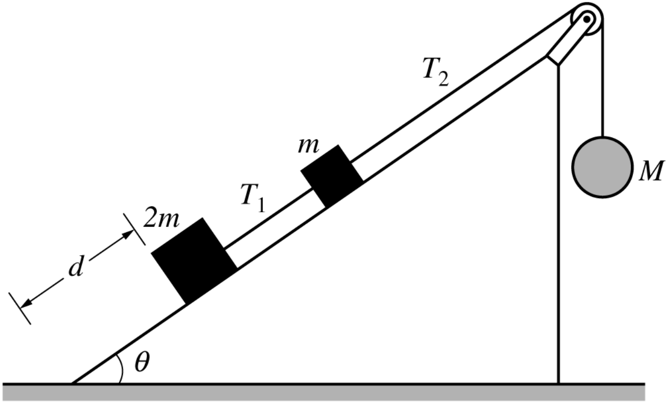

Figure 1

Question 1

Question 1

2019 Set 2 ▪ Q1

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Question 1

(b)(i) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).
The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.
(b)(ii) The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

Question 1

2019 Set 2 ▪ Q1

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Question 1

(c)(i) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.
Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

_____ Increase _____ Decrease to a nonzero value
_____ Decrease to zero _____ Stay the same

(c)(ii) Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(c)(iii) Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

Question 1

2019 Set 2 ▪ Q1

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(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is

Question 1

μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

Question 1

2019 Set 2 ▪ Q1

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(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d

Question 1

from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

Solution 1

(a)

Mark scheme

- Free-body diagrams (3 points total, max 2 if extraneous vectors are drawn). On block m: draw F_N (normal force, perpendicular to and away from the incline surface) and mg (weight, straight down) — 1 point; draw T_1 (tension from the string to block $2m$, directed down the incline) and T_2 (tension from the string over the pulley, directed up the incline) — 1 point. On sphere M: draw T_2 (tension, upward) and Mg (weight, downward) — 1 point. Each force must be a single arrow starting on and pointing away from the dot.

Solution 1

(b)(i)

Mark scheme

- Treat blocks m and $2m$ as one system of total mass $3m$ in equilibrium ($a = 0$): $F_{net} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g\sin\theta = 0$ (1 pt for an attempt at a correct Newton's-second-law statement for the two blocks). Solving gives the final answer $T_2 = 3mg\sin\theta$ (1 pt for the correct answer with supporting work).

Solution 1

(b)(ii)

Mark scheme

- Apply Newton's second law to the whole system (all three masses, $a = 0$): $F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$, giving $M = 3m\sin\theta$ (1 pt). [Equivalent alternate solution: from part (b)(i), $T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g = 3m\sin\theta$ — also earns the 1 point.]

Solution 1

(c)(i)

Mark scheme

- Correct choice: tension T_1 decreases to zero (1 pt). Once the sphere M lands, the string over the pulley (carrying T_2) goes slack because it can no longer pull the blocks. With no tension pulling them and the incline frictionless, both blocks m and $2m$ independently experience the same gravitational acceleration component $g \sin\theta$ down the incline, so no tension is needed to keep them moving together, and $T_1 \to 0$.

Solution 1

(c)(ii)

Mark scheme

- Correct choice: up the ramp (1 pt). Velocity cannot change instantaneously (only acceleration can). Immediately after the sphere lands, the blocks are still moving in the same direction as the instant before — up the incline, since they had been dragged upward as M descended on the other side of the pulley.

Solution 1

(c)(iii)

Mark scheme

- Correct choice: down the ramp (1 pt). With $T_2 = 0$ once the sphere lands, the only net force along the frictionless incline is the gravity component, which points down the incline, so the acceleration is directed down the ramp — this decelerates the upward motion and will eventually reverse it.

Solution 1

(d)

Mark scheme

- Newton's second law for the 3-mass system on the verge of sliding down, with static friction at its maximum value acting up the incline on each block: $F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$ (1 pt, attempt at a correct statement for the system). Substitute $f_{s1} = \mu_s F_{N1} = \mu_s mg\cos\theta$ and $f_{s2} = \mu_s F_{N2} = \mu_s(2mg\cos\theta)$: $3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$ (1 pt, attempting to substitute the friction forces). Final answer: $M = 3m(\sin\theta - \mu_s\cos\theta)$ (1 pt, correct answer with supporting work).

Solution 1

(e)

Mark scheme

- Newton's second law for the two blocks (total mass $3m$) sliding down with kinetic friction: $F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$ (1 pt, attempt at a correct statement), which gives $a = g(\sin\theta - \mu_k\cos\theta)$. Use kinematics with $v_1 = 0$ over distance d: $v_2^2 = v_1^2 + 2ad$ (1 pt, correct kinematics equation) $= 2g(\sin\theta - \mu_k\cos\theta)d$. Final answer: $v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$ (1 pt, correct answer with supporting work). [Equivalent alternate solution via conservation of energy, $U_1 + K_1 = U_2 + K_2 + E_{lost}$, gives the same result and earns full credit.]

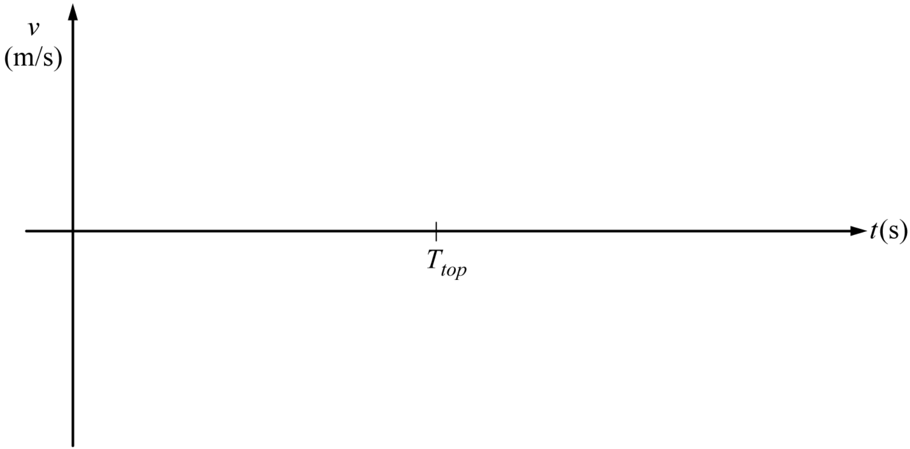
Question 2

2019 Set 2 ▪ Q2

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0$ m/s^2, $L = 0.25$ m/s^4, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

Question 2



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(b) Calculate the speed of the rocket at $t = 6.0$ s.

Question 2

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(c)(i) Calculate the kinetic energy of the rocket at $t = 6.0$ s.

(c)(ii) Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

Question 2

2019 Set 2 ▪ Q2

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(d) Calculate the maximum height reached by the rocket relative to its launching point.

Question 2

2019 Set 2 ▪ Q2

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(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 2

[Graph axes: vertical axis labeled v (m/s), horizontal axis labeled t(s); the origin is at the intersection, with a tick mark labeled T_{top} on the horizontal axis to the right of the origin. The axes are otherwise unscaled and blank for the student to sketch on.]

Solution 2

(a)

Mark scheme

- $J = \int F(t)\,dt = \int ma(t)\,dt = (0.50)\int_0^6 (9.0 - 0.25t^2)\,dt$ (1 pt for the correct expression and substitution of $a(t)$ and m). Integrating with the correct limits: $J = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_0^6 = (0.50)\left[(9.0)(6) - \frac{1}{12}(6)^3\right] = (0.50)(54 - 18)$ (1 pt for correct integration/limits). Final answer: $J = 18\ \mathrm{N \cdot s}$.

Solution 2

(b)

Mark scheme

- Relate impulse to the change in momentum: $J = \Delta p = m(v_2 - v_1)$ (1 pt for correctly relating impulse to speed). Substitute $J = 18$ N · s from part (a): $(18 \text{ N} \cdot \text{s}) = (0.50 \text{ kg})(v_2 - 0)$ (1 pt for correct substitution), giving $v_2 = 36$ m/s.

Solution 2

(c)(i)

Mark scheme

- $K = \frac{1}{2}mv^2$. Substitute the mass of the rocket (1 pt) and the speed $v = 36$ m/s found in part (b) (1 pt): $K = \frac{1}{2}(0.50\text{ kg})(36\text{ m/s})^2 = 324$ J.

Solution 2

(c)(ii)

Mark scheme

- Integrate the acceleration twice to obtain position. First: $v(t) = \int a(t)\,dt = \int (9.0 - 0.25t^2)\,dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$ (1 pt for integrating with correct limits/constant of integration, since $v_0 = 0$). Then integrate again: $\Delta y = \int_0^6 v(t)\,dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_0^6 = \frac{9}{2}(36) - \frac{1}{48}(1296) = 135$ m (1 pt for the second integration with correct limits). Substitute into $\Delta U_g = mg\Delta y = (0.50\text{ kg})(9.8\text{ m/s}^2)(135\text{ m}) \approx 660$ J (1 pt for substituting into the potential-energy equation).

Solution 2

(d) Mark scheme

- Use $v_2^2 = v_1^2 + 2a(y_2 - y_1)$ with $a = g = -9.8\ \mathrm{m/s^2}$ after burnout and $v_2 = 0$ at maximum height: $y_2 - y_1 = -\frac{v_1^2}{2a}$ (1 pt for using $a = g$ in a correct kinematics equation). Substitute the speed $v_1 = 36\ \mathrm{m/s}$ from part (b) (1 pt): $y_2 - y_1 = -\frac{(36\ \mathrm{m/s})^2}{2(-9.8\ \mathrm{m/s^2})} \approx 66\ \mathrm{m}$. Add the height already gained by $t = 6.0\ \mathrm{s}$, $\Delta y = 135\ \mathrm{m}$ from part (c)(ii) (1 pt for substituting the height from part (c)): $y_2 = 66\ \mathrm{m} + 135\ \mathrm{m} = 201\ \mathrm{m}$ (199.8 m if $g = 10\ \mathrm{m/s^2}$). [Equivalent alternate solutions using energy conservation, $mg\Delta y = \frac{1}{2}mv^2$, or $K_1 + U_1 = U_{top}$ with parts (c)(i)/(c)(ii): $324 + 660 = (0.5)(9.8)\Delta y$, also earn full credit.]

Solution 2

(e) Mark scheme

- Sketch: for $0 \le t \le 6.0$ s, an initial concave-down curve starting at the origin $(0,0)$ that rises to a maximum of $v = 36$ m/s at $t = 6.0$ s (1 pt, for an initial concave-down curve starting at the origin, consistent with the decreasing-but-positive acceleration $a = 9.0 - 0.25t^2$). The curve then transitions, before T_{top}, into a straight line with constant negative slope $-g$ (1 pt, for a transition before T_{top} into a straight line of negative slope). Label the maximum value 36 m/s (consistent with part (b)) with the line crossing the time axis exactly at $t = T_{top}$, continuing to negative v as the rocket falls back to the ground (1 pt, for labeling the maximum consistent with part (b) and crossing the axis at T_{top}).

Question 1

2018 ▪ Q1

A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

[Diagram: A vertical stand on a table. An electromagnet is mounted at the top of the stand, holding a sphere directly beneath it. The distance from the sphere down to a pad on the table is labeled h. Labels: "Electromagnet", "Sphere", "h" (vertical arrow), "Pad", "Table".]

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated

Question 1

from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

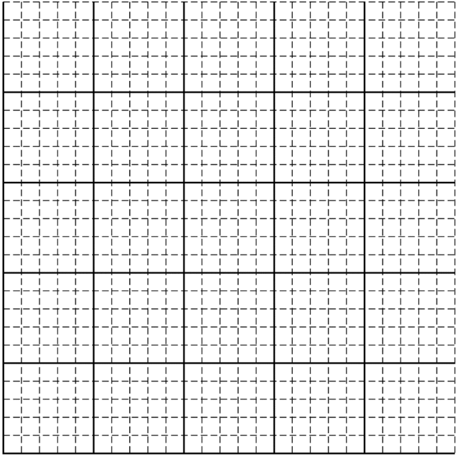
$G^{reaterthan}{}_{L^{essthan}}{}_{E^{qualto}}$

Justify your answer.

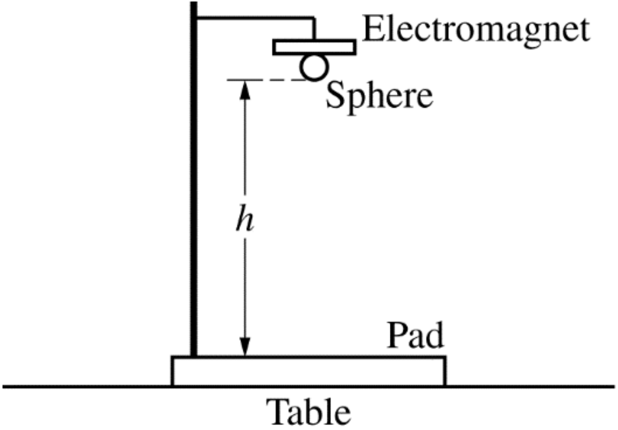
The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

Question 1



Question 1



Question 1

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

Question 1

2018 ▪ Q1

A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

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(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ________________

Horizontal axis: ________________

Question 1

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

Question 1

2018 ▪ Q1

A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

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(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.

Question 1

[Graph: A blank grid (approximately 6 columns $\times$ 16 rows of small squares) provided for the student to plot data and draw a best-fit straight line. No axes, scale, or data are pre-printed.]

Question 1

2018 ▪ Q1

A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

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(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where

Question 1

$A = 5.75$ m/s^2, $B = -0.524$ m/s, and $C = +0.080$ m. Vertically down is the positive direction.

Question 1

2018 ▪ Q1

A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

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(e)(i) Using the student's equation above, derive an expression for the velocity of the sphere as a function of time.

(e)(ii) Using the student's equation above, calculate the new experimental value for g.

Question 1

(e)(iii) Using 9.81 m/s^2 as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.
(e)(iv) Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

Solution 1

(a) Mark scheme

- Less than. Justification: because the electromagnet actually released the sphere after the timer had already begun, the measured time is longer than the true fall time for that height. Since $g = 2h/\Delta t^2$ (i.e. $a \propto 1/t^2$), a longer measured Δt makes the calculated value of g smaller than the true value, so the computed g is LESS THAN the actual g. (1 point for selecting "Less than" with an attempt at a relevant justification; 1 point for a correct justification, e.g. "the measured time to fall the same distance is larger, so the acceleration must be less" or "the time is larger and $a \propto 1/t^2$, so g must decrease.")

Solution 1

(b)

Mark scheme

- Vertical axis: h. Horizontal axis: $(\Delta t)^2$. Since $h = \frac{1}{2}g(\Delta t)^2$, plotting h vs. $(\Delta t)^2$ gives a straight line through the origin with slope $g/2$. Full credit is also given if the axes are reversed (horizontal h, vertical $(\Delta t)^2$, slope $= 2/g$) or for any other combination of quantities that yields a straight line usable to find g.

Solution 1

(c) Mark scheme

- Using the part (b) quantities, compute $(\Delta t)^2$ for each row: $h = 0.10$ m $\rightarrow (\Delta t)^2 = 0.011$ s^2; $h = 0.20 \rightarrow 0.045$; $h = 0.60 \rightarrow 0.117$; $h = 0.80 \rightarrow 0.161$; $h = 1.00 \rightarrow 0.203$ s^2. Plot h (vertical) vs $(\Delta t)^2$ (horizontal) and draw a best-fit straight line through the points (the official answer graph is linear, passing near the origin, rising from about (0.01,0.10) to (0.20,1.00)). Scoring: 1 point for a scale that uses more than half the grid; 1 point for correctly labeling axes with the part-(b) variables and units; 1 point for correctly plotting the data consistent with part (b); 1 point for drawing a straight line consistent with the plotted data.

Solution 1

(d)

Mark scheme

- Using two points ON THE BEST-FIT LINE (not raw data points): slope $= \frac{\Delta h}{\Delta(\Delta t)^2} = \frac{(0.80 - 0.20)\text{ m}}{(0.160 - 0.039)\text{ s}^2} \approx 4.96\text{ m/s}^2$ (linear regression on all data gives 4.83 m/s^2). Since slope $= \frac{1}{2}g$, $g = 2 \times$ slope. So $g = 2(4.96) = 9.92$ m/s^2 (linear regression: 9.66 m/s^2). 1 point for using points on the line (not the raw data) to compute the slope; 1 point for correctly relating the slope to g.

Solution 1

(e)(i)

Mark scheme

- Matching $y = At^2 + Bt + C$ to the kinematic form $y = y_0 + v_i t + \frac{1}{2}at^2$ and differentiating: $v(t) = \dfrac{dy}{dt} = 2At + B$. (Credit is also earned for substituting the given numbers: $v(t) = (11.5\ \text{m/s}^2)t - 0.524\ \text{m/s}$.)

Solution 1

(e)(ii)

Mark scheme

- Comparing $y = y_0 + v_i t + \frac{1}{2}at^2$ to $y = At^2 + Bt + C$ gives $A = \frac{1}{2}a$, so $a = 2A$. Here the acceleration is g (free fall), so $g = 2A = 2(5.75\ \mathrm{m/s^2}) = 11.5\ \mathrm{m/s^2}$. 1 point for correctly relating the given equation to a correct kinematic equation; 1 point for correctly relating the equation to the value of g (i.e. $g = 2A$).

Solution 1

(e)(iii)

Mark scheme

- $\%\text{error} = \dfrac{|\text{acc} - \text{exp}|}{\text{acc}} \times 100\% = \dfrac{|11.5 - 9.81|\ \text{m/s}^2}{9.81\ \text{m/s}^2} \times 100\% = 17.2\%$. Credit is earned whether the percent error is expressed as positive or negative.

Solution 1

(e)(iv) Mark scheme

- From the fitted equation: $v_i = B = -0.524$ m/s, $a = 2A = 11.5$ m/s^2, $y_0 = C = 0.080$ m, so $\Delta y = 1.40 - 0.080 = 1.32$ m. Using $v^2 = v_i^2 + 2a\Delta y$: $v = \sqrt{(-0.524)^2 + 2(11.5)(1.32)} = 5.54$ m/s. 1 point for relating the equation's coefficients to the kinematic variables v_i, a, y_0; 1 point for correctly using an appropriate kinematics equation to find v. (Equivalent full-credit alternates: solve the quadratic $5.75t^2 - 0.524t - 1.32 = 0$ for $t = 0.53$ s and substitute into part (e)(i)'s $v(t)$; or use conservation of energy $mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}mv^2$ — both give $v = 5.54$ m/s.)

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]
A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed of the block just before it collides with the spring.

Question 2

(a)(ii) Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

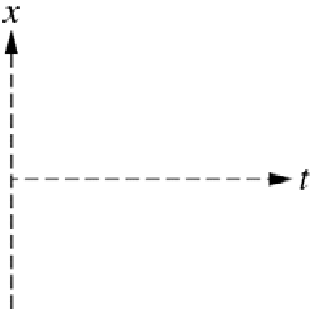
_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 2

Note: Figure not drawn to scale.

Question 2



Question 2

2017 ▪ Q2

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A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(b) Derive an expression for the maximum compression of the spring.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

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(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

Question 2

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface.
The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

Question 2

(d)(i) Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.
(d)(ii) Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

Question 2

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.
[Three blank axes side by side, each with a vertical axis and horizontal axis labeled t: the first labeled x, the second labeled v, the third labeled a — for sketching the respective graphs.]

Solution 2

(a)(i)

Mark scheme

- Using conservation of energy for the block sliding down the frictionless incline: loss in gravitational PE = gain in KE, $U_g = K$, i.e. $mgh = \frac{1}{2}mv^2$. Solving: $v = \sqrt{2gh}$. 1 point for correctly applying conservation of energy ($U_g = K$), 1 point for the correct final answer $v = \sqrt{2gh}$.

Solution 2

(a)(ii) Mark scheme

- Correct answer: Greater than. Since $v = \sqrt{2gh}$ is proportional to $\sqrt{h}$, reducing the height by a factor of 2 reduces the speed by a factor of $\sqrt{2} \approx 1.41$ (not by a factor of 2). So the speed halfway down the incline is $v/\sqrt{2} \approx 0.71v$, which is greater than half the bottom speed $(0.5v)$. 1 point for correctly selecting 'Greater than' AND giving this correct justification – an incorrect selection cannot earn credit even with valid-looking reasoning.

Solution 2

(b)

Mark scheme

- Using conservation of energy ($U_g = U_s$) for the block compressing the spring, consistent with part (a): $mgh = \frac{1}{2}kx_{max}^2$. Solving for the maximum compression: $x_{max} = \sqrt{\frac{2mgh}{k}}$. 1 point for this correct answer, consistent with the part (a) approach.

Solution 2

(c) Mark scheme

- Recognize that the block's contact with the spring, from first touching it to maximum compression, is one-quarter of a simple-harmonic-motion cycle, with period $T = 2\pi\sqrt{m/k}$. So $t = \frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$. 1 point for identifying the simple-harmonic-motion (quarter-period) approach, 1 point for the correct final expression.

Solution 2

(d)(i) Mark scheme

- Newton's second law for the block on the horizontal surface, with the resistive force opposing the motion: $F_{net} = ma$, and $F_{net} = -\beta v^2$ (negative because it decelerates the block), giving $-\beta v^2 = ma$. Writing acceleration as dv/dt gives the differential equation $-\beta v^2 = m\dfrac{dv}{dt}$. 1 point for correctly applying Newton's second law ($F_{net} = ma$), 1 point for correctly indicating that F_{net} is opposite the direction of motion, 1 point for expressing the result as a differential equation in dv/dt.

Solution 2

(d)(ii) Mark scheme

- Separate variables in $-\beta v^2 = m\,dv/dt$ to get $-\frac{\beta}{m}dt = \frac{1}{v^2}dv$. Integrating both sides: $\int -\frac{\beta}{m}dt = \int \frac{1}{v^2}dv$ gives $-\frac{\beta}{m}[t] = \left[-\frac{1}{v}\right]$. Applying the limits $t : 0 \to t$ and $v : v_0 \to v(t)$: $-\frac{\beta t}{m} = -\frac{1}{v(t)} - \left(-\frac{1}{v_0}\right) = -\frac{1}{v(t)} + \frac{1}{v_0}$. Rearranging gives $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, as required. 1 point for correctly separating variables, 1 point for correctly integrating, 1 point for correctly applying the limits/constant of integration to reach the given result.

Solution 2

(e) Mark scheme

- Since the resistive force continually decelerates the block (from the $1/v(t) = 1/v_0 + \beta t/m$ result, $v(t)$ decreases toward 0 but never reaches it in finite time, and the deceleration itself decreases), the three sketches are: position $x(t)$ – concave down, increasing and leveling off toward (approaching but not reaching) a horizontal asymptote; velocity $v(t)$ – concave up, decreasing from v_0 toward the horizontal (time) axis as an asymptote; acceleration $a(t)$ – concave down (magnitude decreasing, stays negative), also approaching the horizontal axis as an asymptote. (Full credit is also earned if all three curves are reflected vertically; if a plausible but different nonlinear $v(t)$ shape is drawn, $x(t)$ and $a(t)$ consistent with that shape still earn credit.) 1 point per correct curve (x, v, a).

Question 1

2015 ▪ Q1

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0.

The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

[Diagram: An inclined ramp making angle θ with the horizontal. A Motion Sensor is mounted at the top of the incline, aimed down the ramp, with a y-axis pointing up-left from the sensor and an x-axis pointing along the ramp surface down and to the right. The point $x = 0$ is at the sensor (top of ramp); the point $x = D$ is at the bottom of the ramp where the block starts. A dashed line labeled D spans from the sensor down

Question 1

to a small dashed box partway down the ramp. A block near the bottom of the ramp (at $x = D$) has an arrow labeled $\vec{v}_0$ pointing up the ramp (toward the sensor).]

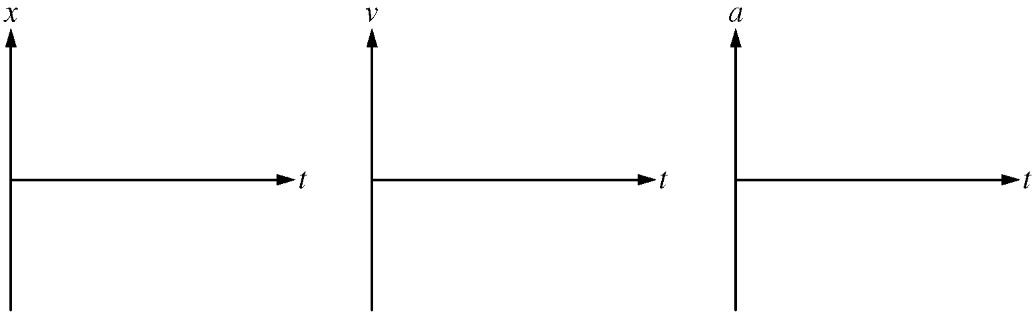
(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

(a)(i) Determine the acceleration a of the block.

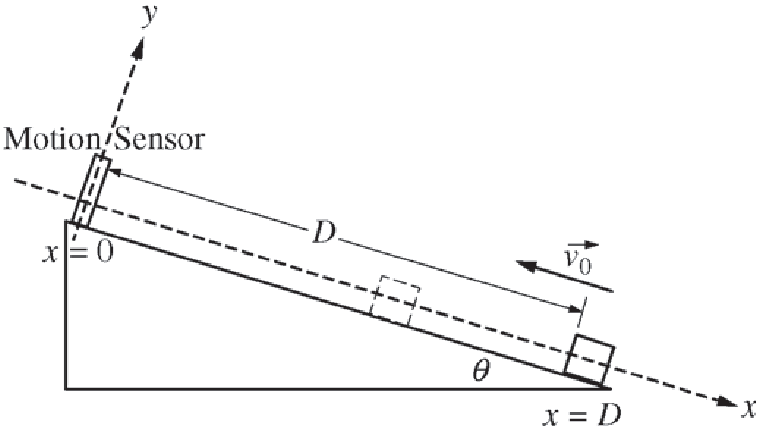
(a)(ii) Determine an expression for the velocity v of the block.

(a)(iii) Determine an expression for the position x of the block.

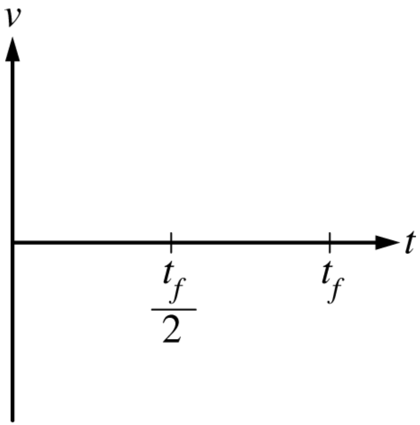
Question 1



Question 1



Question 1



Question 1

2015 ▪ Q1

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[Diagram: An inclined ramp making angle θ with the horizontal. A Motion Sensor is mounted at the top of the incline, aimed down the ramp, with a y-axis pointing up-left from the sensor and an x-axis pointing along the ramp surface down and to the right. The point $x = 0$ is at the sensor (top of ramp); the point $x = D$ is at the bottom of the ramp where the block starts. A dashed line labeled D spans from the sensor down to a small dashed box partway down the

Question 1

ramp. A block near the bottom of the ramp (at $x = D$) has an arrow labeled $\vec{v}_0$ pointing up the ramp (toward the sensor).]

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

Question 1

2015 ▪ Q1

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Question 1

ramp. A block near the bottom of the ramp (at $x = D$) has an arrow labeled $\vec{v}_0$ pointing up the ramp (toward the sensor).]

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

[Three blank axes side by side, each with a horizontal t-axis and a vertical axis. The first is labeled x, the second labeled v, the third labeled a. All axes are unlabeled/blank grids for the student to sketch on — no data is plotted.]

Question 1

2015 ▪ Q1

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Question 1

ramp. A block near the bottom of the ramp (at $x = D$) has an arrow labeled $\vec{v}_0$ pointing up the ramp (toward the sensor).]

(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

Question 1

2015 ▪ Q1

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The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

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Question 1

ramp. A block near the bottom of the ramp (at $x = D$) has an arrow labeled $\vec{v}_0$ pointing up the ramp (toward the sensor).]

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

[Blank axes with vertical axis v and horizontal axis t. Two tick marks are pre-labeled on the t-axis: $\dfrac{t_f}{2}$ and t_f. The graph itself is blank for the student to sketch.]

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[Figure: A horizontal track marked in centimeters from 0 to 120 cm. A cart with a spring attached to the left end of the track sits near the 0 cm mark, with two wheels shown. Five vertical photogates labelled 1, 2, 3, 4, 5 are mounted on the track at positions 20 cm, 40 cm, 60 cm, 80 cm, and 100 cm respectively, labelled "Photogates" above.]

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

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(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

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the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

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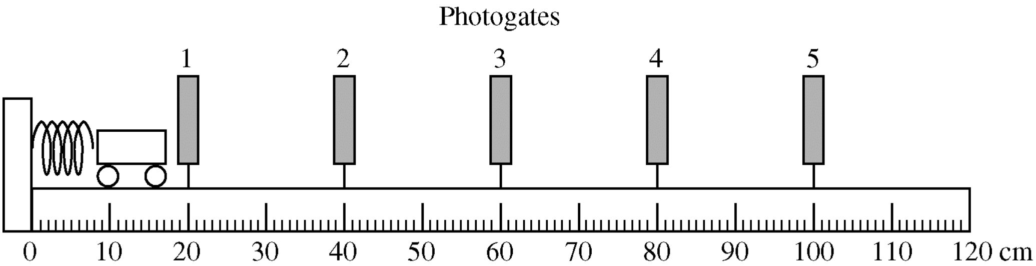

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(b)(i) In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \mathrm{N/m^2}$ and

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$B = 150$ N/m. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart. Calculate the following: The speed of the cart immediately after it loses contact with the spring

(b)(ii) Calculate the following: The impulse given to the cart by the spring

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[Figure: A horizontal track marked in centimeters from 0 to 120 cm. A cart with a spring attached to the left end of the track sits near the 0 cm mark, with two wheels shown. Five vertical photogates labelled 1, 2, 3, 4, 5 are mounted on the track at positions 20 cm, 40 cm, 60 cm, 80 cm, and 100 cm respectively, labelled "Photogates" above.]

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(c) In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after

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the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of ± 0.002 m/s. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

On the axes below, plot the data points for the speed *v* of the cart as a function of position *x*. Clearly scale and label all axes, as appropriate.

[Blank grid of axes for plotting speed *v* vs. position *x*, ruled in a 4x4 arrangement of major squares each subdivided into a 5x5 fine grid, no pre-printed scale numbers or labels.]

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[Figure: A horizontal track marked in centimeters from 0 to 120 cm. A cart with a spring attached to the left end of the track sits near the 0 cm mark, with two wheels shown. Five vertical photogates labelled 1, 2, 3, 4, 5 are mounted on the track at positions 20 cm, 40 cm, 60 cm, 80 cm, and 100 cm respectively, labelled "Photogates" above.]

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(d)(i) Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

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(d)(ii) From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.