

Learn these by heart before the exam.

Kinematics

$\vec{v} = \frac{d\vec{r}}{dt}$, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ differentiate to go down,
integrate (with initial conditions) to go back up

$$\vec{v} = \vec{v}_0 + \int \vec{a} dt, \quad \vec{r} = \vec{r}_0 + \int \vec{v} dt$$

$v dv = a dx$ the trick when $a = a(x)$ rather than $a(t)$ a x
Constant-a formulas apply ONLY when a is constant.
If a depends on t, v or x, you must integrate.

Force and Translational Dynamics

$$\sum \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$F_d = -bv$ or $-cv^2$ terminal velocity is where $\sum F = 0$,
so $mg = bv_t$

$m \frac{dv}{dt} = mg - bv \Rightarrow v(t) = \frac{mg}{b} (1 - e^{-bt/m})$ separate
variables, integrate, apply $v(0) = 0$

$$F_G = \frac{Gm_1m_2}{r^2}, \quad a_c = \frac{v^2}{r}$$

Free-body diagram first, choose axes along the motion,
then $\sum F = ma$ per axis.

Work, Energy, and Power

$$W = \int \vec{F} \cdot d\vec{r}, \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$K = \frac{1}{2}mv^2, \quad W_{\text{net}} = \Delta K$$

$F_x = -\frac{dU}{dx}$, $\Delta U = -\int \vec{F} \cdot d\vec{r}$ force is MINUS the
slope of the potential-energy curve

$U_g = -\frac{GMm}{r}$, $U_s = \frac{1}{2}kx^2$ the $-GMm/r$ form (zero
at infinity) is the Physics C one, not mgh

On a $U(x)$ graph: minima are stable equilibria,
maxima unstable; turning points are where $E = U$.

Linear Momentum

$$\vec{p} = m\vec{v}, \quad \vec{J} = \int \vec{F} dt = \Delta \vec{p}$$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}, \quad \vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm$$

With no external force the centre of mass keeps
moving at constant velocity, whatever happens inside.

Elastic collision conserves p AND K; perfectly
inelastic conserves only p and the bodies stick.

Torque and Rotational Dynamics

$$\vec{\tau} = \vec{r} \times \vec{F}, \quad \tau = rF \sin \theta, \quad \sum \tau = I\alpha$$

$$I = \int r^2 dm$$

$$I_{\text{disc}} = \frac{1}{2}MR^2, \quad I_{\text{rod,cm}} = \frac{1}{12}ML^2, \quad I_{\text{sphere}} = \frac{2}{5}MR^2$$

$$I = I_{cm} + Md^2$$

Rolling without slipping: $v_{cm} = R\omega$ and friction does
no work (the contact point is instantaneously at rest).

Energy and Momentum of Rotating Systems

$$K_{\text{rot}} = \frac{1}{2}I\omega^2, \quad K_{\text{total}} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}, \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Angular momentum is conserved when the net external
torque is zero, even in a collision that loses energy.

Kepler 2 is just conservation of L: equal areas in equal
times.

Oscillations

$\frac{d^2x}{dt^2} = -\omega^2x \Rightarrow x(t) = A \cos(\omega t + \varphi)$ recognise this
differential equation and you have the period immediately

$$\omega_s = \sqrt{\frac{k}{m}}, \quad \omega_p = \sqrt{\frac{g}{L}}, \quad \omega_{\text{phys}} = \sqrt{\frac{mgd}{I}}$$

$$E = \frac{1}{2}kA^2, \quad v_{\text{max}} = A\omega, \quad a_{\text{max}} = A\omega^2$$

To prove SHM: show the restoring force (or torque) is
proportional to $-x$ (or $-\theta$), then read ω^2 off the
coefficient.

Exam technique

Set up the integral or differential equation explicitly.

Physics C awards the setup, so write $\int$ with its limits
before evaluating.

Check limits: does your answer behave sensibly as
 $t \rightarrow \infty$, $m \rightarrow 0$, or $r \rightarrow \infty$? Say so; examiners credit
it.

Symbols before numbers. Derive an algebraic result,
then substitute once at the end.