Electromagnetic Induction

AP Physics C: E&M ▪ Unit 13

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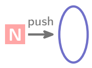
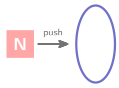


What we will cover



1 Flux & Faraday

Magnetic flux and Faraday's law

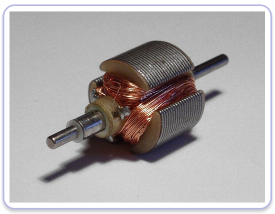
Flux, then Faraday

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The minus sign is **Lenz's law** 楞次定律: the induced current opposes the change – energy conservation.

Motional emf: $\varepsilon = BLv$; a coil of N turns: $\times N$.



Motor rotor

Flux, and the three ways it changes

Magnetic flux 磁通量 measures how much field passes through a surface. For a uniform field through a flat loop it is a dot product, so the **area vector** 面积矢量 – normal to the surface – sets the sign:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

- B changes – a stronger or weaker field
- A changes – the loop grows, shrinks or leaves the field
- θ changes – the loop rotates

Any one of the three induces an e.m.f. A generator uses the third.

The sliding rod on rails

$L = 0.20\,\mathrm{m}$ at $3.0\,\mathrm{m/s}$ across $0.50\,\mathrm{T}$

$\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30\,\mathrm{V}$

The rod sliding on conducting rails, closed by a resistor, is the setup every question is built from: the growing loop area changes the flux.

Once a current flows, the field pushes back on it ($\vec{F} = \int I\,d\vec{l} \times \vec{B}$), and by Lenz's law that force always resists the motion that caused it. Push harder, get more current, feel more drag – the mechanical work is what becomes electrical energy.

Faraday's law is also the third of **Maxwell's equations** 麦克斯韦方程组: a changing magnetic flux creates a circulating electric field.

2 Induced currents

Induced currents and forces

The **induced current** 感应电流 (Lenz's law) always creates a **retarding force** 阻碍力 that resists the motion causing it.

- mechanical work → electrical energy → heat
- **eddy currents** 涡电流 in solid metal damp motion (magnetic braking)

Substation transformer

The induced current fights the change that made it

the force	once an induced current flows, the external field pushes on it: $\vec{F} = \int I\, d\vec{l} \times \vec{B}$
Lenz's law	that force always resists the motion that is changing the flux
why	energy conservation – if it helped the motion you would get free energy
eddy currents 涡流	loops induced in a solid conductor; the same opposing force is what brakes a magnet falling down a copper tube

> The FRQ walks one chain: flux → emf → current → force → Newton's second law. Write each link separately and check the direction at every step.

3 Inductance

Inductance

An **inductor** 电感 resists a change in current, creating a **back emf** 反电动势:

Back emf and stored energy

$$\varepsilon = -L\frac{dI}{dt}, \qquad U = \tfrac{1}{2}LI^2$$

Energy density $u = \dfrac{B^2}{2\mu_0}$ – the magnetic mirror of $\frac{1}{2}\varepsilon_0 E^2$.

Worked example

$2\,\mathrm{H}$ with I rising at $3\,\mathrm{A/s}$:

$$\varepsilon = L\tfrac{dI}{dt} = 2 \times 3 = 6\,\mathrm{V}$$

At a steady $4\,\mathrm{A}$: $U = 16\,\mathrm{J}$, and the back emf vanishes.

The inductor, and the solenoid that makes one

inductance 电感	L relates flux to current: $\varepsilon = -L\dfrac{dI}{dt}$ – it opposes a *change* in current, not current itself
inductor 电感器	a component built to have large L – usually a **solenoid** 螺线管
the geometry	$L = \dfrac{\mu_0 N^2 A}{\ell}$: turns squared, so doubling N quadruples L
stored energy	$U_L = \frac{1}{2}LI^2$ – held in the magnetic field, the mirror of $\frac{1}{2}C(\Delta V)^2$ in a capacitor

> An inductor is to current what mass is to velocity: it resists change. That analogy carries all the way through the RL and LC results.

4 LR & LC

LR and LC circuits

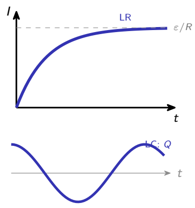
LR: smooth growth

$$\tau = \frac{L}{R}, \quad I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

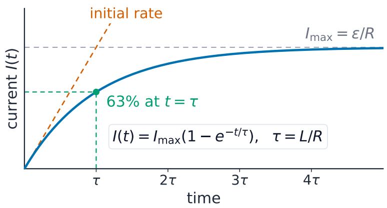
An inductor blocks current at first, passes it freely later – opposite to a capacitor.

LC: oscillation

$$\omega = \frac{1}{\sqrt{LC}}$$



The current in an RL circuit rises exponentially



RL rise: $I = I_0(1 - e^{-t/\tau}) \cdot \tau = L/R$

> The current in an RL circuit rises exponentially, reaching 63% at one time constant

The two moments an RL question always asks about

just after closing ($t = 0$)	current cannot jump, so it is still zero; the inductor self-induces an emf equal and opposite to the applied one, and behaves like an open circuit
long after ($t \to \infty$)	$dI/dt \to 0$, so the inductor's emf vanishes and it behaves like a plain wire: $I = \varepsilon/R$
the capacitor is the mirror	a capacitor is a *wire* at $t = 0$ and an *open circuit* at $t \to \infty$ – exactly the other way round
in between	the exponential, governed by τ

> Almost every circuit FRQ asks for these two limits before it asks for anything else. They need no calculus at all – just the two rules above.

The RL circuit, and its time constant

the loop rule	$\varepsilon - IR - L\frac{dI}{dt} = 0$ – a **differential equation 微分方程**, not an algebraic one
the solution	$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$, with $\tau = \frac{L}{R}$
time constant 时间常数	after one τ a rising current reaches about 63% of its final value; a decaying one falls to 37%
bigger L, smaller R	both make τ longer – the circuit takes longer to settle

> $\tau = L/R$ has units of seconds. Checking that is the fastest way to catch an inverted expression.

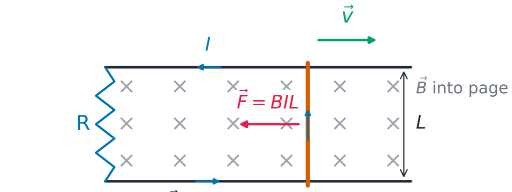
Worked example – the LC oscillator

$L = 2.0$ H and $C = 8.0\ \mu$F. Find the angular frequency and the period.

- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0(8.0 \times 10^{-6})}}$
- $= 250$ rad/s
- $T = \frac{2\pi}{\omega} = 0.025$ s

> This is the same equation as a mass on a spring: **simple harmonic motion** 简谐运动 with charge in place of displacement, L in place of mass and $1/C$ in place of k. **Conservation of energy** 能量守恒 moves it between the capacitor's field and the inductor's.

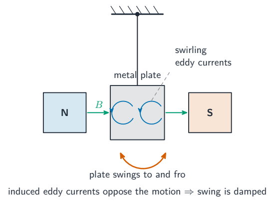
Induced Currents and Magnetic Forces



$\mathcal{E} = BLv$ · induced $\vec{F}$ opposes the motion (Lenz)
motional emf: $\mathcal{E} = B\ell v$ · $F_{\text{mag}} = I\ell B$ opposes

> A rod sliding on rails: the induced current feels a force opposing the motion

Induced eddy currents oppose motion



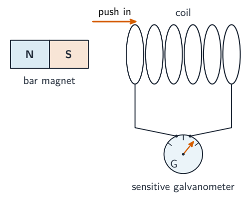
> Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field

Flux, the rules, and what falls out of them

Surface integral	Magnetic flux is a **surface integral**: $\Phi_B = \int \vec{B} \cdot d\vec{A}$ – and only the component through the surface counts.
Right-hand rule	The **right-hand rule** fixes every direction here: the field of a current, the force on a moving charge, and the sense of an induced current.
Permeability	μ_0, the **magnetic permeability** of free space, appears in Ampère's law and again in the speed of light.
Electromagnetic waves	Maxwell's equations give $c = 1/\sqrt{\mu_0 \varepsilon_0}$ – so **electromagnetic waves** travel at a speed built from two electrostatic and magnetostatic constants.

> Lenz's law is conservation of energy in disguise: the induced current opposes the change that made it, or you would get energy for nothing.

Electromagnetic Induction



> Moving a magnet into a coil induces an e.m.f. that drives a current

Exam tips

- Compute **flux** $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and get the induced EMF from **Faraday's law** $\varepsilon = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways – changing B, changing area, or changing angle – identify which and differentiate.
- For a rod of length L moving at speed v, the motional EMF is BLv.
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).

5 Recap

Unit 13 – key takeaways

1 Faraday

$\varepsilon = -d\Phi_B/dt$; Lenz opposes the change

2 Induced force

always retarding; eddy currents damp motion

3 Inductance

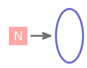
$\varepsilon = -L\,dI/dt$; stores $U = \frac{1}{2}LI^2$

4 Circuits

LR grows with $\tau = L/R$; LC oscillates at $1/\sqrt{LC}$

Check yourself

1. Write Faraday's law for the induced emf.
2. Which law fixes the direction of the induced current?
3. At the first instant, how does an inductor behave?
4. What sets the oscillation frequency of an LC circuit?



Answers **1.** $\varepsilon = -d\Phi_B/dt$ **2.** Lenz's law **3.** it blocks current (open) **4.** $\omega = 1/\sqrt{LC}$