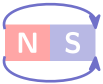
# Magnetic Fields & Electromagnetism

AP Physics C: E&M ▪ Unit 12

AP Physics C: E&M



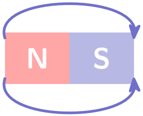
# What we will cover

1 Magnetic fields

2 Force on a moving charge

3 Fields of current-carrying wires

4 Ampère's law



# 1 Magnetic fields

# Magnetic fields

A **magnetic field** 磁场 $\vec{B}$ has field lines in closed loops – there are **no monopoles** 磁单极子. Materials differ by how their **atomic dipoles** 原子磁偶极子 align:

- **ferromagnetic** 铁磁性 – an external field aligns whole **magnetic domains** 磁畴, and the alignment survives the field being removed – a permanent magnet; align strongly, stay (iron)
- **paramagnetic** 顺磁性 – align weakly
- **diamagnetic** 抗磁性 – weakly oppose

*Aurora magnetism*

# The magnetic field, and why there are no monopoles

| | |
|---|---|
| **magnetic field 磁场 $\vec{B}$** | a **vector field 矢量场** surrounding magnets, moving charges and currents; it sets the force on any *moving* charge |
| **field lines 磁感线** | run out of the north pole and into the south *outside* the magnet, and continue through it – they always close on themselves |
| **Gauss's law for $B$** | $\oint \vec{B} \cdot d\vec{A} = 0$ through *any* closed surface – as many lines leave as enter |
| **what that means** | no isolated **magnetic monopole 磁单极子** has ever been found; cut a magnet in half and you get two magnets |
| **a compass 指南针** | a small **magnetic dipole 磁偶极子** that lines up **tangent 相切** to the local field |

Compare with electric flux, which equals $Q_{\text{enc}}/\varepsilon_0$ and is non-zero. The zero on the right is the whole difference between the two fields.

# 2 Moving charges

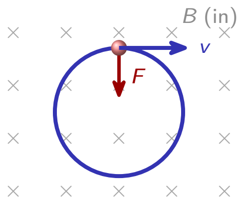
# Force on a moving charge

## A cross product

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Size $qvB\sin\theta$; $\perp$ to $\vec{v}$, so it does no work. A wire feels $\vec{F} = I\vec{L} \times \vec{B}$.

Perpendicular entry $\Rightarrow$ a circle:
$qvB = \dfrac{mv^2}{r}$, so $r = \dfrac{mv}{qB}$.

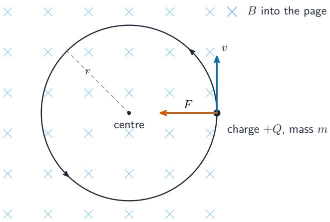


# How a material answers a magnetic field

| | |
|---|---|
| **ferromagnetic 铁磁性** | iron, nickel, cobalt: domains align *with* the field and stay aligned – a strong response that can be made permanent |
| **paramagnetic 顺磁性** | a weak alignment *with* the field, lost as soon as the field is removed |
| **diamagnetic 抗磁性** | present in all materials: the electronic structure produces a usually weak alignment *opposite* to the field |
| **permeability 磁导率** | the measure of that response. Free space has the constant $\mu_0 = 4\pi \times 10^{-7}$ T m/A |

> Diamagnetism is universal but weak, so it only shows when nothing stronger is present. The other two are extra effects layered on top of it.

# A moving charge both makes and feels a field




$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

A moving charge does two things: it *creates* a field, perpendicular to its velocity and circling it, and it *feels* a force in an external one. The force is perpendicular to both $\vec{v}$ and $\vec{B}$ – fingers along $\vec{v}$, curl toward $\vec{B}$, thumb gives the force on a positive charge – so it does no work and the path is a circle.

# The velocity selector

| | |
|---|---|
| **the setup** | an electric and a magnetic field, crossed, in the same region |
| **they act independently** | the two forces add as vectors: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ |
| **balance them** | $qE = qvB$, so only particles with $v = E/B$ pass straight through |
| **note** | the selected speed depends on neither $q$ nor $m$ – so it filters by speed alone |

Put a **velocity selector** 速度选择器 in front of a mass spectrometer and the second stage's radius $r = mv/qB$ then measures $m/q$ cleanly, because $v$ is already known.

# Worked example – a proton circling in a field

**A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) enters a $0.50$ T field at $2.0 \times 10^5$ m/s, perpendicular to it. Find $r$ and $T$.**

- the magnetic force is the **centripetal force** 向心力: $qvB = \frac{mv^2}{r}$
- $r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27})(2.0 \times 10^5)}{(1.6 \times 10^{-19})(0.50)} = 4.2 \times 10^{-3}$ m
- $T = \frac{2\pi m}{qB} = 1.3 \times 10^{-7}$ s

$T$ contains no $v$. Faster particles ride larger circles in exactly the same time – which is the principle the cyclotron is built on.

# 3 Wire fields

# Fields of current-carrying wires

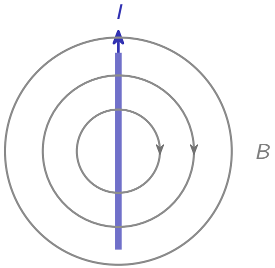
Moving charge makes magnetic field:

## Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

Integrate for standard results:

- long straight wire: $B = \dfrac{\mu_0 I}{2\pi r}$
- loop centre: $B = \dfrac{\mu_0 I}{2R}$



# Biot–Savart, and the arc it makes easy

| | |
|---|---|
| **Biot–Savart law 毕奥-萨伐尔定律** | $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$ – the field from one current element |
| **a straight wire** | gives **concentric circles 同心圆** around it, $B = \frac{\mu_0 I}{2\pi r}$, direction from the right-hand rule |
| **at a loop's centre** | every element $d\vec{l}$ is perpendicular to $\hat{r}$ and equally distant $R$ – say that sentence, it is the marked step – giving $B = \frac{\mu_0 I}{2R}$ |
| **an arc** | a fraction of that circle contributes the same fraction: $B = \frac{\mu_0 I\theta}{4\pi R}$ for angle $\theta$ |

For several wires, **superpose 叠加**: work out each field's magnitude and direction separately, then add as vectors.

# The field of a current, and one measurement

**Biot–Savart law**

The **Biot–Savart law** gives the field of a current element: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$ – integrate it for any geometry.

**Magnetic permeability**

$\mu_0$ is the **magnetic permeability** of free space, the constant that fixes the strength of the magnetic interaction.

**Superposition**

**Superposition** does the rest: the field of several wires is the vector sum of their separate fields.

**Hall effect**

The **Hall effect**: a magnetic field pushes the carriers to one side of a conductor, producing a transverse voltage – which reveals the *sign* of the charge carriers.

**Magnetic field lines** form closed loops – there is no magnetic charge for them to start on.

# 4 Ampere

# Ampère's law

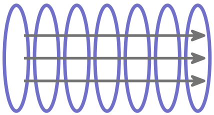
For enough symmetry, Ampère's law beats integrating Biot-Savart:

## The loop integral

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

> Choose an **Amperian loop** 安培环路 where $B$ is constant and parallel, so the integral becomes $B\times$ length.

Inside a **solenoid** 螺线管: $B = \mu_0 n I$, uniform.



$B = \mu_0 n I$

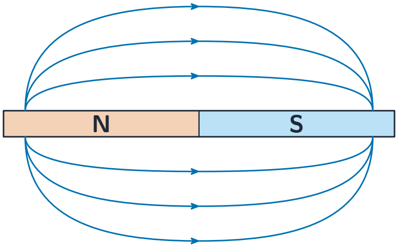
# Worked example – inside a current-carrying cylinder

**A solid cylinder of radius $R$ carries current $I$ spread uniformly over its cross-section. Find $B$ at radius $r < R$.**

- loop: a circle of radius $r$, concentric – $B$ is constant on it and parallel to it by symmetry
- enclosed current scales with area: $I_{\text{enc}} = I\dfrac{\pi r^2}{\pi R^2} = I\dfrac{r^2}{R^2}$
- $B(2\pi r) = \mu_0 I \dfrac{r^2}{R^2}$, so $B = \dfrac{\mu_0 I r}{2\pi R^2}$

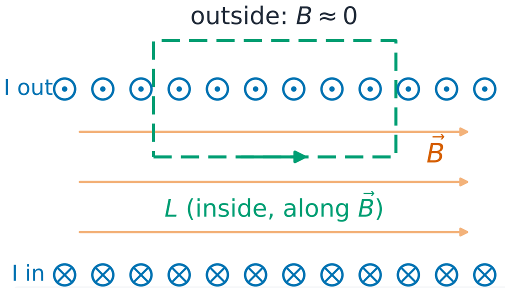
Inside, $B$ grows linearly with $r$; outside it falls as $1/r$, so the field peaks at the surface. Check both limits agree at $r = R$.

# Closed loops, and what that means




Field lines run N to S outside the magnet and close on themselves inside it. That is Gauss's law for magnetism – the second of **Maxwell's equations** 麦克斯韦方程组: the net flux through *any* closed surface is zero, as many lines leaving as entering. It is the same statement as "no isolated **magnetic monopole** 磁单极子 has ever been found".

# Ampère's Law, continued




$$\oint \vec{B} \cdot d\vec{\ell} = BL = \mu_0 (nL) I \quad \Rightarrow \quad B = \mu_0 n I \quad \text{(inside)}$$

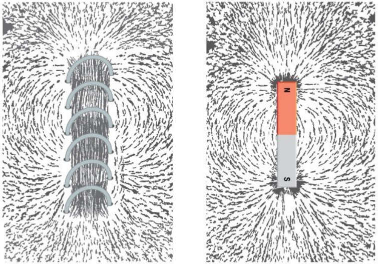
A rectangular Amperian loop derives the uniform field inside a solenoid

# Every Ampère's-law answer earns its marks in the setup

| | |
|---|---|
| **1. name the loop** | say which **Ampèrian loop 安培环路** you chose and where it runs |
| **2. justify it** | why $B$ is constant on it and parallel to it – by symmetry, in words |
| **3. count $I_{enc}$** | only current *through* the loop; for a spread-out current use the **current density 电流密度** and the enclosed area |
| **4. solve** | $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$, giving $B(2\pi r) = \mu_0 I_{enc}$ for a circular loop |

For symmetric distributions this beats Biot–Savart by a long way – but only when the symmetry argument is stated, not assumed.

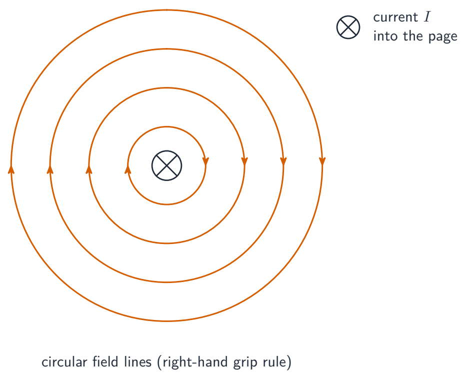
# Magnetic Fields of Current-Carrying Wires




(a) (b)

A current-carrying coil (solenoid) makes a magnetic field shaped just like a bar magnet's

# Concentric circular field lines surround a straight




Concentric circular field lines surround a straight current-carrying wire

# Exam tips

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is **perpendicular** to velocity – use the right-hand rule and note it does **no work**.
- A charge in a uniform field moves in a circle with $r = \frac{mv}{qB}$.
- Find the field of currents with **Biot–Savart** $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$ or **Ampère's law** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ when there is symmetry.
- Force on a wire is $\vec{F} = I\vec{L} \times \vec{B}$; keep the cross-product direction straight.
- Distinguish the force **on** a charge from the field the current **creates**.

# 5 Recap

# Unit 12 – key takeaways

### 1 Fields

lines in closed loops; no magnetic monopoles

### 2 Force

$\vec{F} = q\vec{v} \times \vec{B}$, does no work; circle $r = mv/qB$

### 3 Wire fields

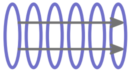
Biot-Savart; long wire $\mu_0 I/2\pi r$

### 4 Ampère

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$; solenoid $\mu_0 nI$

# Check yourself

1. Why does the magnetic force on a moving charge do no work?
2. Which law gives the field of a symmetric current quickly?
3. Double the distance from a straight wire – what happens to $B$?
4. Is the field inside an ideal solenoid uniform?



**Answers** **1.** it is perpendicular to $\vec{v}$ **2.** Ampère's law **3.** it halves **4.** yes