Electric Circuits

AP Physics C: E&M ▪ Unit 11

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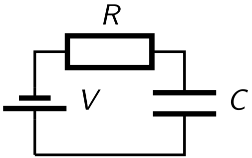
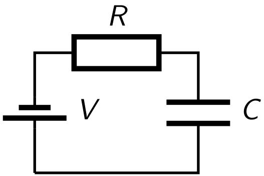


What we will cover



1 Current

Electric current and Ohm's law

Rate of charge flow

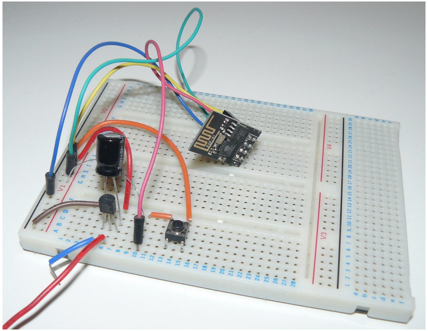
$$I = \frac{dQ}{dt}$$

Conventional current 常规电流 is opposite to the electron **drift** 漂移速度.

Ohm's law

$$V = IR, \qquad R = \frac{\rho L}{A}$$

Resistivity 电阻率 ρ is a material property; R is the object's.



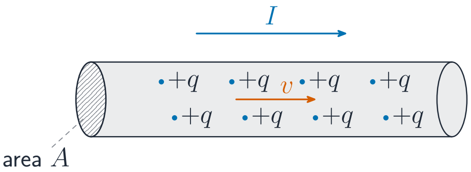
Breadboard

What a circuit is, and what Ohm's law claims

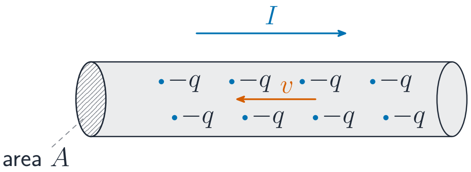
a circuit 电路	closed loops of wires, batteries, resistors, bulbs, capacitors, inductors, switches and meters – charge flows only around a closed loop
steady state 稳态	current the same everywhere along one branch; nothing accumulates
Ohm's law 欧姆定律	$I = \frac{\Delta V}{R}$ for an **ohmic** 欧姆性 element – one whose R does not change with I
resistance	$R = \frac{\rho \ell}{A}$: material through ρ, and geometry through ℓ and A
ideal battery 理想电池	maintains a fixed emf with zero internal resistance; a real one has r in series

Current is a **scalar** 标量 with a chosen direction, not a vector. Conventional current runs the way positive charge would move.

Electric Current



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

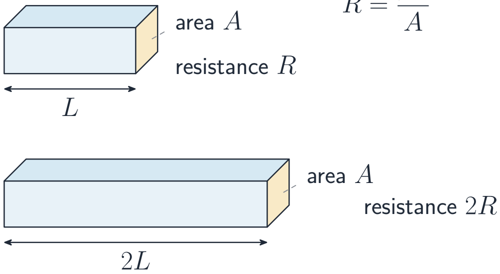
Worked example – how slowly electrons actually drift

A copper wire of area 1.0×10^{-6} m^2 with $n = 8.5 \times 10^{28}$ m^{-3} carries 1.7 A. Find the drift velocity 漂移速度.

- $v_d = \dfrac{I}{nqA}$
- $= \dfrac{1.7}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-6})}$
- $= 1.3 \times 10^{-4}$ m/s – about 0.5 m per hour

The *signal* travels near light speed; the electrons crawl. **Current density** 电流密度 $\vec{J} = nq\vec{v}_d$ is the same idea per unit **cross-sectional area** 横截面积, tied to the field by $\vec{E} = \rho\vec{J}$, with $I = \int \vec{J} \cdot d\vec{A}$ in general.

Resistance, Resistivity, and Ohm's Law



$$R = \frac{\rho L}{A}$$

twice the length $\Rightarrow$ twice the resistance

> A longer conductor has more resistance; a wider one has less

Worked example – stretch a wire to double its length

A wire is stretched to twice its length. What happens to its resistance?

- the volume is fixed, so doubling ℓ halves A
- $R' = \dfrac{\rho(2\ell)}{A/2} = 4\,\dfrac{\rho\ell}{A}$
- $= 4R$

Both geometric factors move, and they move the *same* way. Changing only ℓ would give $2R$ – the missing half of the answer is the area.

2

Internal resistance

Compound circuits and internal resistance

Reduce series/parallel resistors to one **equivalent resistance** 等效电阻. A real battery is an ideal **emf** 电动势 plus a small **internal resistance** 内阻 r:

Terminal voltage

$$V_{term} = \varepsilon - Ir$$

Worked example

$\varepsilon = 12\,\mathrm{V}$, $r = 1\,\Omega$, load $5\,\Omega$:

$$I = \frac{12}{6} = 2\,\mathrm{A}$$

$$V_{term} = 12 - 2(1) = 10\,\mathrm{V}$$

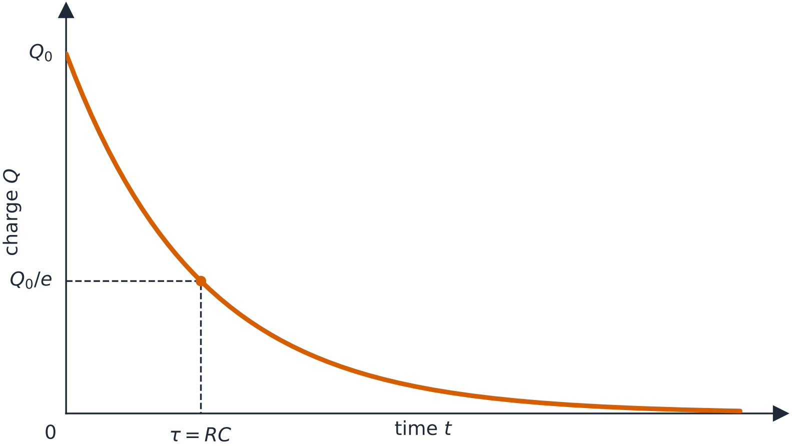
$2\,\mathrm{V}$ lost inside the battery.

Power, and how to rank bulb brightness

where it comes from	a charge q falling through ΔV gives up $q\Delta V$ of energy, so the rate is $P = I\Delta V$
three forms	$P = I\Delta V = I^2 R = \dfrac{(\Delta V)^2}{R}$ – use the one whose two quantities you actually know
in a resistor	all of it becomes heat
brightness	rank bulbs by power, never by current or by voltage alone – a bulb can carry more current and still be dimmer

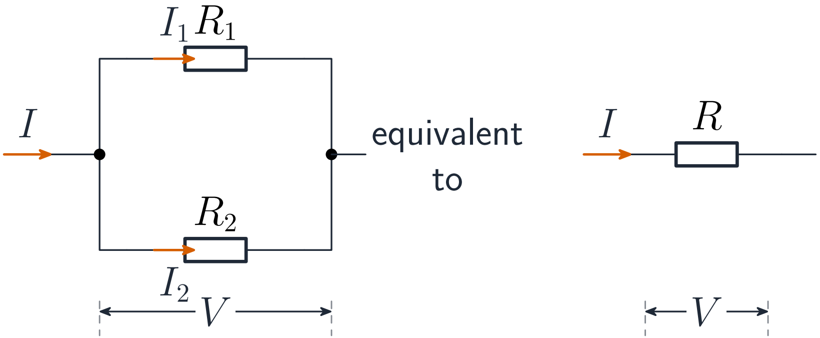
In series the current is shared, so $P = I^2 R$ ranks them: bigger R is brighter. In parallel the voltage is shared, so $P = (\Delta V)^2/R$ ranks them: smaller R is brighter. The ranking reverses.

Resistor-Capacitor (RC) Circuits



The charge on a capacitor decays exponentially as it discharges

Compound Direct Current Circuits



Resistors in parallel combine to a smaller equivalent resistance

Where each meter goes, and why

ammeter 电流表	in series at the point whose current you want – ideal $R = 0$, so it changes nothing
voltmeter 电压表	in parallel across the element – ideal $R = \infty$, so it draws no current
get it backwards	an ammeter in parallel is a short circuit; a voltmeter in series stops the current
circuit diagram 电路图	redraw before you calculate – series and parallel are far easier to see in a tidy diagram than in the given one

The ideal resistances are chosen so the meter does not disturb what it measures. Every real meter fails that a little, which is what internal-resistance questions probe.

3 Kirchhoff

Kirchhoff's rules

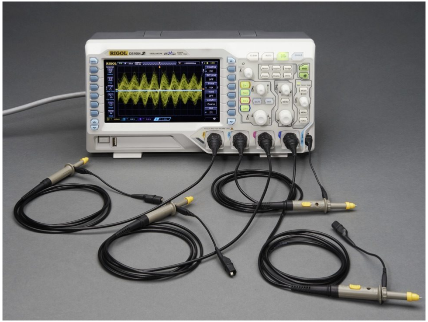
Loop rule (energy)

$$\sum \Delta V = 0 \quad \text{around a loop}$$

A rise across an emf, a drop $-IR$ across a resistor.

Junction rule (charge)

$$\sum I_{in} = \sum I_{out}$$



Oscilloscope

> An oscilloscope: voltage against time reveals how current and charge evolve in a circuit

Both Kirchhoff rules are conservation laws

junction rule	charge is conserved: current in equals current out at any node
loop rule	energy is conserved: charges exchange $\Delta U_E = q\,\Delta V$, so $\sum \Delta V = 0$ around any closed path
signs	walk the loop one way; a drop across a resistor with the current is $-IR$, a rise through a battery from $-$ to $+$ is $+\varepsilon$
how many equations	one per independent loop plus one per independent junction – enough for the unknowns, never more

> If your loop equation does not close, you have a sign error, not an arithmetic one. Redo the walk and check each element's polarity.

4 RC

RC circuits

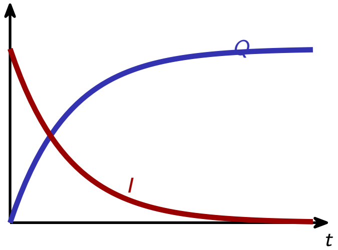
A capacitor charges through a resistor with **time constant** 时间常数 $\tau = RC$:

Charging

$$Q(t) = C\varepsilon\left(1 - e^{-t/\tau}\right)$$

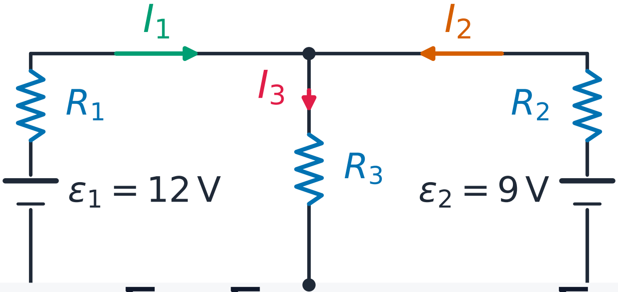
$$I(t) = \frac{\varepsilon}{R} e^{-t/\tau}$$

Empty capacitor = a wire at first; full = blocks steady current.



$R = 2$, $C = 3$: $\tau = 6\,\mathrm{s}$.

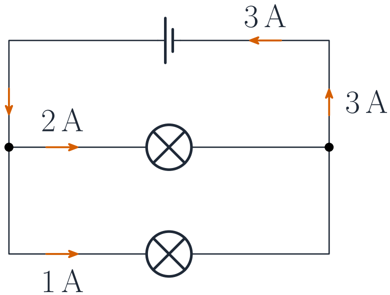
Kirchhoff's Junction Rule



KCL (node): $\sum I_{\mathrm{in}} = \sum I_{\mathrm{out}}$ · KVL (loop): $\sum \Delta V = 0$

> Two loop equations and one junction equation solve this two-battery circuit

Current divides at a junction



at each junction: $3\,\mathrm{A} = 2\,\mathrm{A} + 1\,\mathrm{A}$

> Current divides at a junction: what flows in equals what flows out

Exam tips

- Relate current to charge flow $I = \frac{dQ}{dt}$ and use $J = \sigma E$, $R = \frac{\rho L}{A}$ for resistance.
- Apply **Kirchhoff's laws** (junction: charge; loop: energy) with consistent sign conventions.
- Analyse **RC circuits** with calculus: charging/discharging give exponentials with time constant $\tau = RC$.
- Combine resistors (series add, parallel reciprocal) and track power $P = IV = I^2R$.
- At $t = 0$ a capacitor acts like a wire; after a long time ($t \to \infty$) it acts like an open branch.

5 Recap

Unit 11 – key takeaways

1 Current & Ohm

$I = dQ/dt$; $V = IR$; $P = IV = I^2R = V^2/R$

2 Real battery

$V_{term} = \varepsilon - Ir$, small internal resistance

3 Kirchhoff

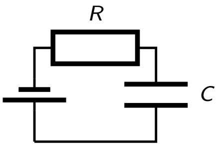
loop $\sum \Delta V = 0$; junction $\sum I_{in} = \sum I_{out}$

4 RC

$\tau = RC$; charge and current decay exponentially

Check yourself

1. Write current as a derivative of charge.
2. What conservation law is Kirchhoff's loop rule?
3. At $t = 0$, how does an empty capacitor behave?
4. After a long time, how much current flows through a charged capacitor?



Answers **1.** $I = dQ/dt$ **2.** conservation of energy **3.** like a plain wire **4.** zero