Conductors and Capacitors

AP Physics C: E&M ▪ Unit 10

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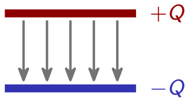
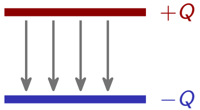


What we will cover

1. Conductors in electrostatic equilibrium
2. Redistribution of charge
3. Capacitors
4. Dielectrics



1 Conductors

Conductors in electrostatic equilibrium

With charges at rest in a **conductor** 导体:

- the field is zero everywhere inside
- all excess charge sits on the **outer surface** 外表面
- outside, the field is perpendicular to the surface
- the whole conductor is one **equipotential** 等势面

> A metal shell keeps its inside field zero – a **Faraday cage** 法拉第笼.

Capacitors

What a conductor in equilibrium must look like

inside	$\vec{E} = 0$ everywhere in the bulk – if it were not, the free electrons would still be moving
the surface	all excess charge sits on it. Negative net charge means extra surface electrons; positive means a shortage
the field just outside	perpendicular to the surface; a parallel component would push charge along it
sharp points	charge density is largest where the surface curves sharply, so the outside field is strongest at points and edges
electrostatic shielding 静电屏蔽	in an external field the conductor **polarizes** 极化 until its own charges cancel the field inside – a cavity is shielded

The whole conductor is one **equipotential surface** 等势面, surface included. Every conductor result on this page follows from $\vec{E} = 0$ inside.

2 Redistribution

Redistribution of charge

Touch two conductors and charge flows until they reach a common potential:

- **conservation of charge** 电荷守恒 fixes the total
- a bigger conductor holds more at the same V
- an external field induces opposite charges on the two sides

Worked example – identical spheres

$12\,\mathrm{nC}$ touched to a neutral twin:

each ends with $6\,\mathrm{nC}$

Touch a much larger sphere instead and most charge flows onto it.

Worked example – two spheres touch and separate

A sphere of radius R carrying $+6.0\ \mu$C touches a distant sphere of radius $2R$, then they separate. Find each charge.

- in contact they are one conductor, so their potentials are equal: $\dfrac{kq_1}{R} = \dfrac{kq_2}{2R}$
- so $q_2 = 2q_1$, and $q_1 + q_2 = 6.0\ \mu$C
- $q_1 = 2.0\ \mu$C (small), $q_2 = 4.0\ \mu$C (large)

Charge splits in proportion to radius, not equally. **Ground** 接地 is the limiting case: an idealised reference at $V = 0$ that can absorb or supply any amount of charge, so grounding near an external charge leaves an **induced charge** 感应电荷 behind.

3 Capacitors

Capacitors

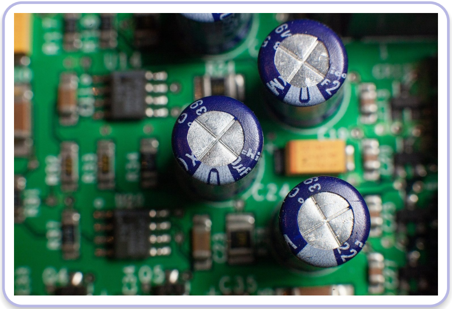
A **capacitor** 电容器 stores charge $\pm Q$ on two plates:

Capacitance and stored energy

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

$$U = \tfrac{1}{2} C V^2 = \frac{1}{2}\frac{Q^2}{C}$$

Energy lives in the field: $u = \frac{1}{2}\varepsilon_0 E^2$. Combine opposite to resistors – parallel adds.



Pcb capacitors

The parallel-plate derivation, in three steps

1. field	Gauss's law plus **superposition** 叠加 give $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$ between the plates, and zero outside
2. potential	the field is uniform, so $\Delta V = Ed = \dfrac{Qd}{\varepsilon_0 A}$
3. capacitance	$C = \dfrac{Q}{\Delta V} = \dfrac{\varepsilon_0 A}{d}$
the energy	$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \dfrac{Q^2}{2C}$

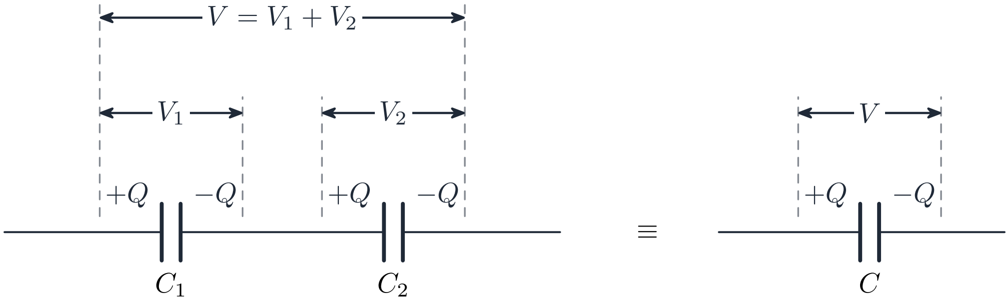
> That $\frac{1}{2}$ is the triangle under the Q–ΔV line: the first charge moved across costs almost nothing, the last costs the full ΔV. The same $E \to \Delta V \to C$ chain does the cylinder and the sphere.

Capacitance is geometry, and nothing else

the definition	$C = \dfrac{Q}{\Delta V}$ – but C depends on neither
what sets it	the plate area, the gap, the shape, and the material filling the gap
double Q	ΔV doubles with it, and C is unchanged
the units	$1\ \mathrm{F} = 1\ \mathrm{C/V}$; real capacitors are pF to μF because a farad is enormous

> If your answer makes C depend on Q or on ΔV, you have made an algebra error. Geometry only.

Capacitors in series carry the same charge – one



Capacitors in series carry the same charge, and their p.d.s add

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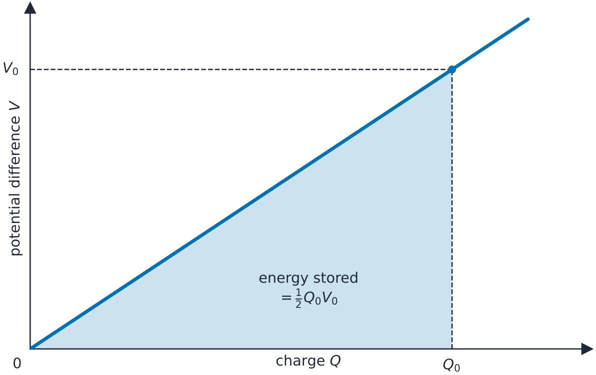
Inside the plates, a charge is a projectile

the field	uniform between the plates, so the force qE and the acceleration are constant
across the gap	velocity along the plates is unchanged; the transverse motion accelerates – exactly **projectile motion** 抛体运动
time in the field	$t = L/v_0$, set entirely by the horizontal motion
deflection	$y = \frac{1}{2}at^2 = \dfrac{qEL^2}{2mv_0^2}$

An electron entering midway at $v_0 = 2.0 \times 10^7$ m/s through $E = 1.0 \times 10^3$ N/C is deflected by exactly this formula. Split the motion into two perpendicular directions and the mechanics is already done.

Capacitors in series carry the same charge – two



The energy stored in a capacitor is the area under its charge-voltage line

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4 Dielectrics

Dielectrics

A **dielectric** 电介质 is an insulator that **polarizes** 极化, building an internal field that opposes the applied one.

Sliding it in

$$C = \kappa C_0$$

the **dielectric constant** 介电常数 κ – and the field between the plates drops.

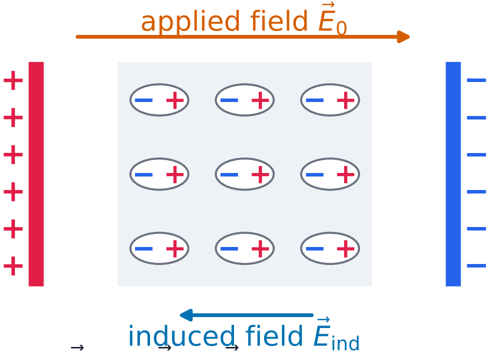
Constant charge: V and U *drop*

Constant voltage: Q and U *rise*

$\kappa = 3$:

$C_0 = 5 \to 15\,\mu\mathrm{F}$.

A polarized dielectric creates an internal field



$\vec{E}_{\text{net}} = \vec{E}_0 - \vec{E}_{\text{ind}}$ · $C = \kappa C_0$

dielectric: $C = \kappa C_0$ · polarization reduces E

A polarized dielectric creates an internal field opposing the applied field

Worked example – insert a dielectric with the battery removed

A 100 pF capacitor is charged to 12 V, , then filled with a $\kappa = 3$ dielectric. Find the new ΔV and U.

- disconnected $\Rightarrow$ Q is trapped: $Q = C_0 \Delta V_0 = 1.2$ nC
- $C = \kappa C_0 = 300$ pF, so $\Delta V = Q/C = 4.0$ V
- $U = \dfrac{Q^2}{2C}$ falls to a third: $7.2 \to 2.4$ nJ

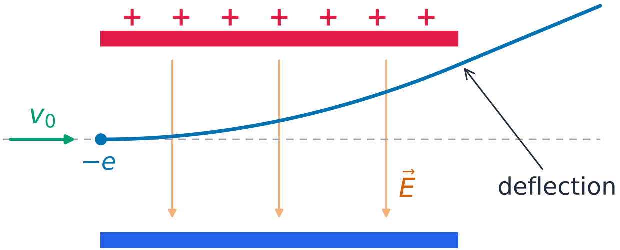
Write down which quantity is held fixed first – Q if the battery is removed, ΔV if it stays connected – then let $C = \kappa C_0$ drive everything else. Get that wrong and every later line is wrong.

What a dielectric is, and what it does

dielectric 电介质	an insulating material: its charges cannot travel, but each molecule stretches or turns in a field
the effect	the material polarizes and sets up its own opposing field, weakening the net field in the gap
so	$C = \kappa C_0$, where κ is the **dielectric constant 介电常数** – the ratio of the material's permittivity to the **permittivity of free space 真空介电常数** ε_0
κ is always > 1	a dielectric can only *raise* capacitance, never lower it

> It also lets the plates sit closer without arcing, which raises C a second time through the smaller d.

Capacitors, continued



$a_y = qE/m$ · $a_x = 0$ (same kinematics as a projectile under gravity)

between plates: $a = qE/m$ constant · parabolic path

> A charged particle between the plates follows a parabola, like a projectile

Battery connected or removed: the two columns

held fixed	battery connected: ΔV fixed	battery removed: Q fixed
charge Q	rises by κ	unchanged
voltage ΔV	unchanged	falls by κ
field E	unchanged (still $\Delta V/d$)	falls by κ
energy U	rises by κ (battery does work)	falls by κ

Both columns raise C by κ. Everything else in the two columns moves the opposite way – which is why naming the fixed quantity is step one.

Exam tips

- In electrostatic equilibrium a conductor has $\vec{E} = 0$ **inside and all excess charge on the surface**.
- Apply **Gauss's law** $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ with a symmetry-matched Gaussian surface (sphere, cylinder, pillbox).
- The whole conductor is one **equipotential**, and the surface field is perpendicular to it.
- For a capacitor use $C = \frac{Q}{V}$, energy $U = \frac{1}{2}CV^2$, and how a **dielectric** raises C.
- Pick the Gaussian surface so $\vec{E}$ is constant and parallel (or zero) on each part.

5 Recap

Unit 10 – key takeaways

1 Conductors

field zero inside; charge on the surface; one equipotential

2 Redistribution

touching conductors reach a common potential

3 Capacitors

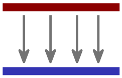
$C = Q/V = \varepsilon_0 A/d$; $U = \frac{1}{2}CV^2$

4 Dielectrics

$C = \kappa C_0$; effect depends on what is held fixed

Check yourself

1. What is the electric field inside a conductor in equilibrium?
2. $12\,\mathrm{nC}$ touched to an identical neutral sphere – charge on each?
3. Double a capacitor's voltage – what happens to its stored energy?
4. Insert a dielectric ($\kappa = 3$) – what happens to the capacitance?



Answers **1.** zero **2.** $6\,\mathrm{nC}$ each **3.** it quadruples **4.** it triples