Electric Potential

AP Physics C: E&M ▪ Unit 9

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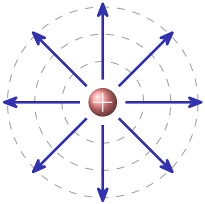
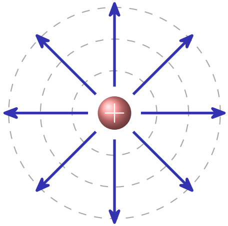


What we will cover

1. Electric potential energy
2. Electric potential (voltage)
3. Potential and field
4. Conservation of electric energy



1 Potential energy

Electric potential energy

Electric potential energy 电势能 is the work to assemble charges from infinity:

A pair, and a system

$$U = k\frac{q_1 q_2}{r}, \qquad U_{tot} = k\sum_{\text{pairs}} \frac{q_i q_j}{r_{ij}}$$

Like charges: $U > 0$ (fly apart). Unlike: $U < 0$ (a well). Note r, not r^2; $W = -\Delta U$.

Power lines

The electric force is conservative

conservative force 保守力	the work it does depends only on the start and end points, never the path taken
so	$W_{\text{elec}} = -\Delta U_E$ – a potential energy exists at all only because of this
a group of charges	add the **superposition 叠加** of every pair, counting each pair once: $U_{\text{tot}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$
signs matter	U is a scalar with a sign – carry the sign of each q into the sum, do not take magnitudes

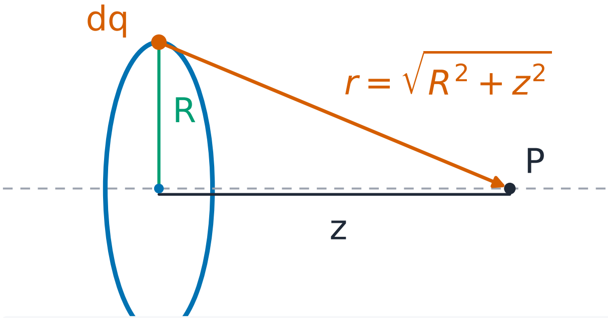
Three charges give three pairs, four give six. Count the pairs before you write the sum and you will not lose a term.

Continuous charge distributions

For a **continuous charge distribution 连续电荷分布**, cut it into small pieces dq, treat each piece as a point charge, and **integrate 积分**:

> AP expects you to handle four shapes with calculus: a thin ring (at a point on its axis), an arc (at its centre), a finite line or wire (collinear, or on its perpendicular bisector).

Continuous charge distributions, in detail



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$ (all dq share the same r)

A ring of charge: every element dq is the same distance from an axis point

Potential is easier than field, and here is why

V is a scalar 标量	no components, no cancellation – just add $\dfrac{k\,dq}{r}$ over the distribution
a ring or arc	every element sits the same distance r from the centre, so $V = \dfrac{kQ}{R}$ whatever the arc's shape
say it	"r is the same for every element" is the marked step on the FRQ – write the sentence, not just the answer
potential difference 电势差	$\Delta V = \Delta U_E/q$: the energy change per unit charge moved. A battery is a device that maintains one

> When a question asks for a *field*, you must handle direction. When it asks for a *potential*, you never do – so reach for V whenever the question lets you.

2 Potential

Electric potential (voltage)

Electric potential 电势 is energy per unit charge – a scalar:

Point charge, sum, integral

$$V = \frac{U}{q} = k\frac{q}{r}$$

$$V = k\sum_i \frac{q_i}{r_i} = k\int \frac{dq}{r}$$

> Adding potentials is easy – just add numbers with signs, no vectors.

Worked example

$+3\,\mathrm{nC}$ and $-3\,\mathrm{nC}$, each $2\,\mathrm{m}$ away:

$$V = k\left(\tfrac{+3}{2} + \tfrac{-3}{2}\right) \times 10^{-9} = 0$$

Yet $\vec{E} \neq 0$ there – V and $\vec{E}$ tell different stories.

Potential and field

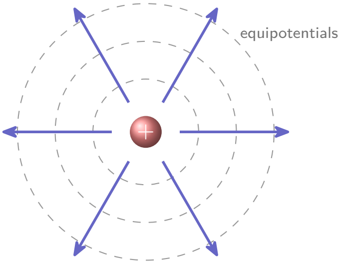
Field and potential are linked by calculus:

Integrate, or take the gradient

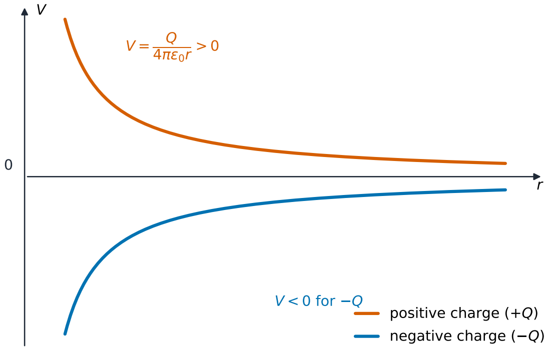
$$V = -\int \vec{E}\cdot d\vec{l}, \qquad \vec{E} = -\nabla V$$

In 1-D, $E_x = -\dfrac{dV}{dx}$: the field points downhill in V.

Equipotentials 等势面 are always perpendicular to field lines; no work moving along one.



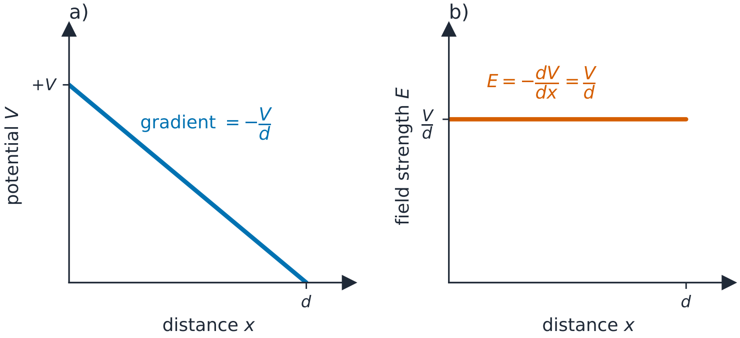
The potential near a point charge varies as – one



The potential near a point charge varies as 1/r

> The potential near a point charge varies as 1/r

The potential near a point charge varies as – two



In a uniform field the potential falls steadily with distance

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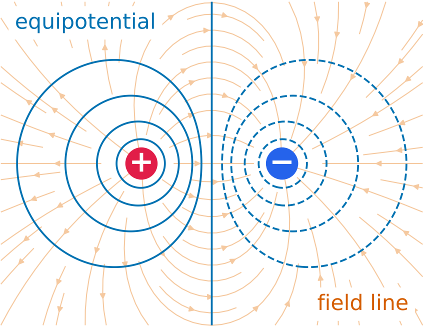
Equipotential maps

- Isolines are **perpendicular 垂直** to field lines 电场线 everywhere they cross.
- $\vec{E}$ points from high V to low V – never along an isoline. Moving a charge along an isoline takes no work.
- Closely spaced isolines mean a strong field: $E \approx -\Delta V/\Delta x$.
- You can sketch the field map from an isoline map, and the isoline map from a field map.

Equipotential maps, in detail



$\vec{E} \perp$ equipotential · $\Delta V = 0$ along an equipotential · $V = 0$ midplane

dipole: V high near $+q$, zero on perpendicular bisector

> Moving a charge along an isoline takes no work.

Reading the field off an equipotential map

the relation	$E = -\dfrac{dV}{dx}$: the field points down the steepest slope of V
estimating	isolines every 10 V, spaced 2 cm: $E \approx \dfrac{10}{0.02} = 500$ V/m
direction	perpendicular to the isolines, pointing from high V to low V
crowding	lines close together mean a strong field; widely spaced means weak

An **equipotential surface** 等势面 costs no work to move along, because $\Delta V = 0$ everywhere on it. That is why $\vec{E}$ must be perpendicular to it.

3

Conservation

Conservation of electric energy

Moving a charge through a **potential difference** 电势差 does work:

Work and kinetic energy

$$W = q\,\Delta V, \qquad \Delta K = q\,\Delta V$$

- a + charge speeds up moving to lower V
- 1 **eV** 电子伏特 $= 1.6 \times 10^{-19}\,\mathrm{J}$ (an energy)

Worked example

An electron through $200\,\mathrm{V}$:

$$\Delta K = 200\,\mathrm{eV} = 3.2 \times 10^{-17}\,\mathrm{J}$$

$$v = \sqrt{\frac{2\,\Delta K}{m}} \approx 8.4 \times 10^{6}\,\mathrm{m/s}$$

Which sign speeds up which charge

positive charge	speeds up moving to lower potential – it falls down the potential hill
negative charge	speeds up moving to higher potential – it falls *up* the hill
why	$\Delta U_E = q\,\Delta V$: flip the sign of q and you flip which direction loses energy
the check	if only the electric force acts, $\Delta K = -\Delta U_E$ – kinetic energy rises exactly as potential energy falls

> Watch both signs together, the charge's and the potential change's. Getting one right and the other wrong gives a plausible number pointing the wrong way.

Worked example – a proton accelerated through a potential drop

A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) starts at rest and falls through 500 V. Find its final speed.

- $\Delta K = q\Delta V = (1.6 \times 10^{-19})(500) = 8.0 \times 10^{-17}$ J
- $\frac{1}{2}mv^2 = 8.0 \times 10^{-17}$
- $v = \sqrt{\dfrac{2(8.0 \times 10^{-17})}{1.67 \times 10^{-27}}} = 3.1 \times 10^5$ m/s

Only the end-point potentials enter. Choose energy over kinematics whenever the field is non-uniform or the path curves – there is no constant a to put in a SUVAT equation.

Exam tips

- Relate field and potential by $V = -\int \vec{E} \cdot d\vec{l}$ and $\vec{E} = -\nabla V$ (in 1-D, $E_x = -\frac{dV}{dx}$).
- Potential is a **scalar** – add contributions with signs, no vector components needed.
- Potential energy of a pair is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; use energy conservation for a charge's speed.
- Know that no work is done moving along an **equipotential**, which is perpendicular to $\vec{E}$.
- Choose the zero of potential (usually infinity) and state it.

4 Recap

Unit 9 – key takeaways

1 Energy

$U = kq_1q_2/r$; sum over distinct pairs; $W = -\Delta U$

2 Potential

$V = kq/r$, a scalar; $V = k\int dq/r$ for distributions

3 Field link

$V = -\int \vec{E}\cdot d\vec{l}$, $\vec{E} = -\nabla V$; equipotentials $\perp$ field

4 Conservation

$\Delta K = q\Delta V$; energy in electron volts

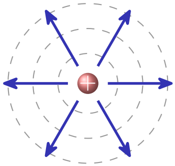
Check yourself

1. Is electric potential a scalar or a vector?

2. How do you get the field from the potential?

3. How much work moves a charge along an equipotential?

4. An electron falls through $50\,\mathrm{V}$ – how much energy in eV?



Answers **1.** a scalar **2.** $\vec{E} = -\nabla V$ **3.** zero **4.** $50\,\mathrm{eV}$