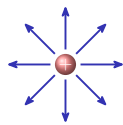


Electric Charges, Fields, & Gauss's Law

AP Physics C: E&M • Unit 8

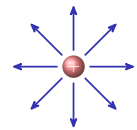
AP Physics C: E&M



Electric Charges, Fields, & Gauss's Law

What we will cover

- 1 Charge and Coulomb's law
- 2 The process of charging
- 3 Electric fields and field lines
- 4 Fields of charge distributions
- 5 Electric flux
- 6 Gauss's law



1 Charge & Coulomb

Electric Charges, Fields, & Gauss's Law

↳ Charge & Coulomb

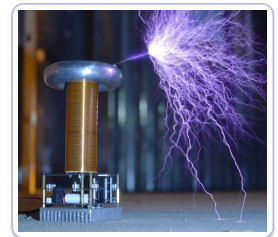
Charge and Coulomb's law

Charge 电荷 is **quantized** 量子化 ($q = ne$) and **conserved** 守恒:

A vector, inverse-square force

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}, \quad k = \frac{1}{4\pi\epsilon_0}$$

Forces from many charges add by **superposition** 叠加原理 – the vector sum.



Tesla coil discharge

Electric Charges, Fields, & Gauss's Law

↳ Charge & Coulomb

Charge is quantized, and it is conserved

quantized 量子化

every charge is a whole-number multiple of the **elementary charge** 基本电荷 $e = 1.6 \times 10^{-19} \text{ C}$

conservation of charge 电荷守恒

charge is never created or destroyed, only moved – the total of an isolated system is fixed

point charge 点电荷

a model that ignores the size of the object; Coulomb's law is exact only for these

ϵ_0

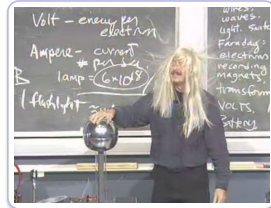
the **permittivity of free space** 真空介电常数: $k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$

Charge is a **fundamental property** 基本属性 of matter, like mass. Like charges repel, opposites attract – and no experiment has ever produced a fraction of e in isolation.

2 Charging

The process of charging

- a **conductor** 导体 lets charge move; an **insulator** 绝缘体 holds it fixed
- **friction** 摩擦起电 – electrons rubbed across
- **conduction** 传导起电 – touch: **same** sign
- **induction** 感应起电 – near + **ground** 接地: **opposite** sign



Static charge builds on a Van de Graaff generator

Three ways to charge an object

- charging by friction** 摩擦起电: rubbing two materials transfers electrons from one to the other – they end up equal and opposite
- charging by conduction** 接触起电: a charged body *touches* a neutral one; charge flows until both share it, so **same sign** results
- charging by induction** 感应起电: a charged body is brought *near*, the neutral body is grounded then ungrounded – the result is the **opposite** sign, with no contact
- polarization** 极化: an **insulator** 绝缘体 cannot pass charge, but its molecules stretch or turn so one face goes slightly positive – which is why a charged rod attracts paper

An **electroscope** 验电器 shows all of this: its leaves diverge when like charge spreads onto both of them.

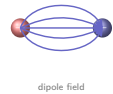
3 Fields

Electric fields and field lines

The **electric field** 电场 is force per unit positive **test charge** 检验电荷:

Field of a point charge

$$\vec{E} = \frac{\vec{F}}{q_0}, \quad \vec{E} = k \frac{q}{r^2} \hat{r}$$



dipole field

- lines leave +, enter –; denser = stronger
- lines never cross; $\vec{F} = q\vec{E}$ on any charge

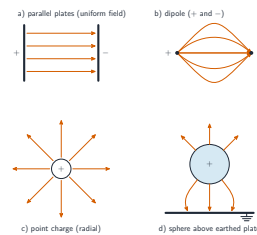
Worked example – Coulomb's law with numbers

Two $+2.0 \mu\text{C}$ point charges sit 0.30 m apart. Find the force on each.

$$\begin{aligned} \blacksquare F &= \frac{kq_1q_2}{r^2} \\ \blacksquare &= \frac{(9.0 \times 10^9)(2.0 \times 10^{-6})^2}{(0.30)^2} \\ \blacksquare &= 0.40 \text{ N, repulsive} \end{aligned}$$

Both charges feel 0.40 N : Newton's third law, whatever the sizes of q_1 and q_2 . Get the **direction** from the signs, never from the algebra.

Electric field-line patterns for parallel plates



Electric field-line patterns for parallel plates, a dipole, and a point charge

4 Distributions

Fields of charge distributions

Split a spread-out charge into pieces dq and **integrate**:

Continuous superposition

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

Densities 电荷密度: $\lambda = \frac{dq}{dl}$, $\sigma = \frac{dq}{dA}$, $\rho = \frac{dq}{dV}$.
Use **symmetry** 对称性 to cancel components.

Two standard results

$$\text{ring (axis)} : E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$\text{infinite line} : E = \frac{2k\lambda}{r}$$

Point $\sim 1/r^2$, line $\sim 1/r$, sheet $\sim \text{uniform}$.

Choose the density that matches the geometry

linear 线 λ $dq = \lambda dl$ – a rod, a ring, an arc

surface 面 σ $dq = \sigma dA$ – a disc, a sheet, a shell

volume 体 ρ $dq = \rho dV$ – a solid sphere or cylinder

then integrate $\vec{E} = \int k \frac{dq}{r^2} \hat{r}$, and use **symmetry** 对称性 to kill the components that cancel

Picking λ , σ or ρ correctly is the **first marked step** of every distribution FRQ. Write dq before you write the integral.

Charge densities, and the symmetry to match

Linear charge density Linear charge density λ in C/m – for a rod or a wire.

Surface charge density Surface charge density σ in C/m² – for a sheet or a conductor's surface.

Volume charge density Volume charge density ρ in C/m³ – for a solid insulating ball.

Match the symmetry Gauss's law is only useful with symmetry: a Gaussian cylinder for cylindrical symmetry, a sphere for spherical symmetry, a pillbox for planar symmetry.

Choose the surface so that $\vec{E}$ is constant and either parallel or perpendicular to it everywhere. Then the integral becomes multiplication.

5 Flux

Electric flux

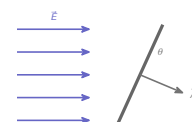
Electric flux 电通量 counts the field lines through a surface:

Uniform, then general

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Largest when $\vec{E} \perp$ the surface ($\theta = 0$); zero when parallel.



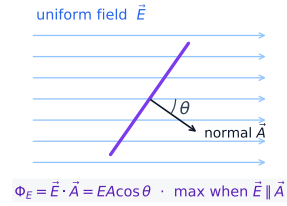
Worked example – the field on a ring's axis

A ring of radius R carries total charge Q . Find E at a point a distance z along its axis.

- every element is the same distance $\sqrt{R^2 + z^2}$ away
- the perpendicular components **cancel** by symmetry; only the axial part survives
- $\cos \theta = \frac{z}{\sqrt{R^2 + z^2}}$, so $E = \frac{kQz}{(R^2 + z^2)^{3/2}}$

Check the limits: at $z = 0$, $E = 0$ (the centre); for $z \gg R$ it becomes kQ/z^2 , the point charge again. State the cancellation – that sentence is the mark.

Electric flux through a surface depends on



Electric flux through a surface depends on the angle between the field and the surface normal

6 Gauss

Gauss's law

One of Maxwell's equations – flux out of a closed surface reveals the charge inside:

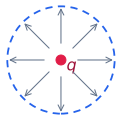
Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

Choose a **Gaussian surface** 高斯面 matching the **symmetry** so E comes out of the integral.

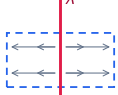
Gauss's law, in detail

Spherical
(point / sphere)



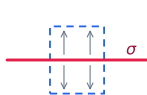
$$E \text{ radial, constant on } r \cdot E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Cylindrical
(line / cylinder)



$$E \text{ radial from line} \cdot E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Planar
(sheet / pillbox)



$$E \perp \text{ sheet, both sides} \cdot E = \frac{\sigma}{2\epsilon_0}$$

Gauss: $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$

Choose a Gaussian surface so E is constant (or zero) on each face — then $\Phi_E = Q_{\text{enc}}/\epsilon_0$

Sphere, cylinder, or pillbox – matched to the symmetry.

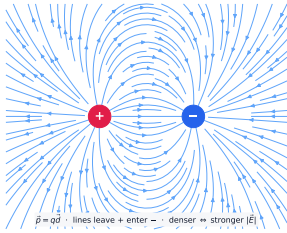
A Gaussian surface is chosen to match the symmetry of the charge

A full-credit Gauss's-law answer has four parts

- name the surface** say which **Gaussian surface** 高斯面 you chose and where it sits
- state the symmetry** why E is **constant** on it and **perpendicular** to it – the **area vector** 面积矢量 $d\vec{A}$ points along the outward normal
- compute the flux** $\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA$, with A the surface's own area
- equate and solve** $EA = \frac{Q_{\text{enc}}}{\epsilon_0}$, then say clearly *which* charge is enclosed

Three symmetries qualify and no others: **spherical**, **cylindrical** 柱对称 and **planar** 面对称. Anything else needs integration.

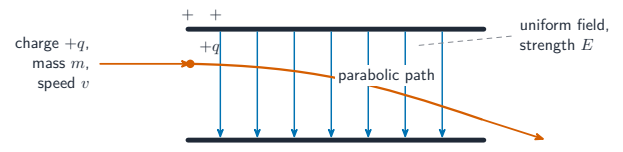
Electric Fields



Electric field lines of a dipole point from the positive to the negative charge

Electric Charges, Fields, & Gauss's Law

Electric Fields, continued



A charge crossing a uniform field follows a parabolic path

Electric Charges, Fields, & Gauss's Law

In a uniform field, a charge is a projectile

the force	$\vec{F} = q\vec{E}$ is constant, so the acceleration $a = qE/m$ is constant too
launched along the field	straight-line motion with constant acceleration – the SUVAT equations, unchanged
launched across the field	constant velocity one way, constant acceleration the other – a parabola
the analogy	exactly projectile motion 抛体运动 under gravity, with qE/m in place of g

Every capacitor-deflection question is a projectile question wearing different symbols. Split the motion into the two perpendicular directions and reuse the mechanics you already have.

Electric Charges, Fields, & Gauss's Law

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda dl, \sigma dA, \rho dV$; use symmetry to cancel components.
- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\epsilon_0}$ straight and check units.

7

Recap

Electric Charges, Fields, & Gauss's Law

Unit 8 – key takeaways

1 Coulomb

$$\vec{F} = kq_1 q_2 \hat{r}/r^2; \text{ superpose vectors}$$

2 Field

$$\vec{E} = kq\hat{r}/r^2; \text{ integrate } \int k dq \hat{r}/r^2 \text{ for distributions}$$

3 Flux

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}; \text{ lines through a surface}$$

4 Gauss

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\epsilon_0; \text{ use symmetry}$$

Check yourself

- 1 Do charges outside a closed surface contribute net flux through it?
- 2 What integral gives the field of a continuous charge distribution?
- 3 When is the flux through a flat surface zero?
- 4 Where does all excess charge sit on a conductor?



Answers 1. no (zero net) 2. $\int k dq/r^2$ 3. when $\vec{E}$ is parallel to it 4. on its outer surface