Electric Charges, Fields, & Gauss's Law

AP Physics C: E&M ▪ Unit 8

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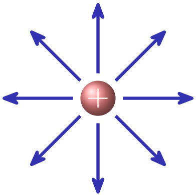
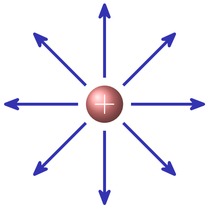


What we will cover

1. Charge and Coulomb's law
2. The process of charging
3. Electric fields and field lines
4. Fields of charge distributions
5. Electric flux
6. Gauss's law



1 Charge & Coulomb

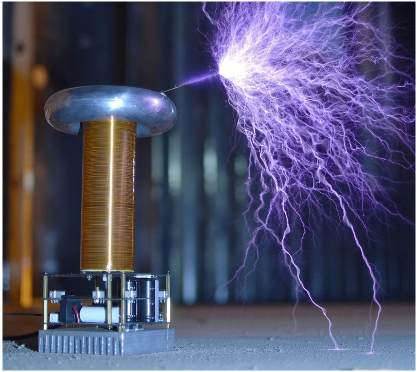
Charge and Coulomb's law

Charge 电荷 is **quantized** 量子化 ($q = ne$) and **conserved** 守恒:

A vector, inverse-square force

$$\vec{F} = k\,\frac{q_1 q_2}{r^2}\,\hat{r}, \qquad k = \frac{1}{4\pi\varepsilon_0}$$

> Forces from many charges add by **superposition** 叠加原理 – the vector sum.



Tesla coil discharge

Charge is quantized, and it is conserved

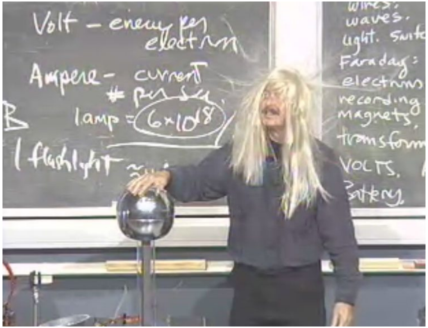
quantized 量子化	every charge is a whole-number multiple of the **elementary charge 基本电荷** $e = 1.6 \times 10^{-19}$ C
conservation of charge 电荷守恒	charge is never created or destroyed, only moved – the total of an isolated system is fixed
point charge 点电荷	a model that ignores the size of the object; Coulomb's law is exact only for these
ε_0	the **permittivity of free space** 真空介电常数; $k = \dfrac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ \mathrm{N\,m^2/C^2}$

Charge is a **fundamental property** 基本属性 of matter, like mass. Like charges repel, opposites attract – and no experiment has ever produced a fraction of e in isolation.

2 Charging

The process of charging

- a **conductor** 导体 lets charge move; an **insulator** 绝缘体 holds it fixed
- **friction** 摩擦起电 – electrons rubbed across
- **conduction** 传导起电 – touch: same sign
- **induction** 感应起电 – near + **ground** 接地: opposite sign



Static charge builds on a Van de Graaff generator

Three ways to charge an object

charging by friction 摩擦起电	rubbing two materials transfers electrons from one to the other – they end up equal and opposite
charging by conduction 接触起电	a charged body *touches* a neutral one; charge flows until both share it, so same sign results
charging by induction 感应起电	a charged body is brought *near*, the neutral body is grounded then ungrounded – the result is the opposite sign, with no contact
polarization 极化	an **insulator** 绝缘体 cannot pass charge, but its molecules stretch or turn so one face goes slightly positive – which is why a charged rod attracts paper

An **electroscope** 验电器 shows all of this: its leaves diverge when like charge spreads onto both of them.

3 Fields

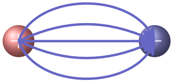
Electric fields and field lines

The **electric field** 电场 is force per unit positive **test charge** 检验电荷:

Field of a point charge

$$\vec{E} = \frac{\vec{F}}{q_0}, \qquad \vec{E} = k\,\frac{q}{r^2}\,\hat{r}$$

- lines leave $+$, enter $-$; denser $=$ stronger
- lines never cross; $\vec{F} = q\vec{E}$ on any charge



dipole field

Worked example – Coulomb's law with numbers

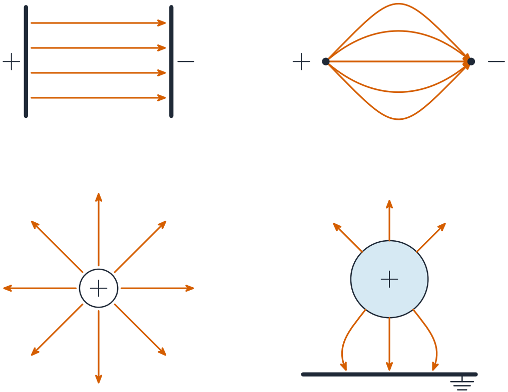
Two $+2.0\ \mu$C point charges sit 0.30 m apart. Find the force on each.

- $F = \dfrac{kq_1q_2}{r^2}$
- $= \dfrac{(9.0 \times 10^9)(2.0 \times 10^{-6})^2}{(0.30)^2}$
- $= 0.40$ N, repulsive

Both charges feel 0.40 N: Newton's third law, whatever the sizes of q_1 and q_2. Get the direction from the signs, never from the algebra.

Electric field-line patterns for parallel plates

a) parallel plates (uniform field) b) dipole (+ and −)



c) point charge (radial) d) sphere above earthed plate

> Electric field-line patterns for parallel plates, a dipole, and a point charge

4 Distributions

Fields of charge distributions

Split a spread-out charge into pieces dq and integrate:

Continuous superposition

$$\vec{E} = \int k \frac{dq}{r^2}\,\hat{r}$$

Densities 电荷密度: $\lambda = \frac{dq}{d\ell}$, $\sigma = \frac{dq}{dA}$, $\rho = \frac{dq}{dV}$.
Use **symmetry** 对称性 to cancel components.

Two standard results

$$\text{ring (axis)}: \quad E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$\text{infinite line}: \quad E = \frac{2k\lambda}{r}$$

Point $\sim 1/r^2$, line $\sim 1/r$, sheet $\sim$ uniform.

Choose the density that matches the geometry

linear 线 λ — $dq = \lambda\, dl$ – a rod, a ring, an arc

surface 面 σ — $dq = \sigma\, dA$ – a disc, a sheet, a shell

volume 体 ρ — $dq = \rho\, dV$ – a solid sphere or cylinder

then integrate — $\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$, and use **symmetry 对称性** to kill the components that cancel

> Picking λ, σ or ρ correctly is the first marked step of every distribution FRQ. Write dq before you write the integral.

Charge densities, and the symmetry to match

Linear charge density	**Linear charge density** λ in C/m – for a rod or a wire.
Surface charge density	**Surface charge density** σ in C/m^2 – for a sheet or a conductor's surface.
Volume charge density	**Volume charge density** ρ in C/m^3 – for a solid insulating ball.
Match the symmetry	Gauss's law is only useful with symmetry: a Gaussian cylinder for **cylindrical symmetry**, a sphere for **spherical symmetry**, a pillbox for **planar symmetry**.

> Choose the surface so that $\vec{E}$ is constant and either parallel or perpendicular to it everywhere. Then the integral becomes multiplication.

5 Flux

Electric flux

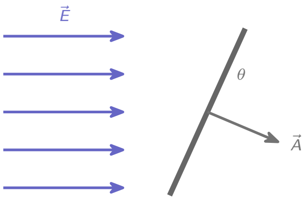
Electric flux 电通量 counts the field lines through a surface:

Uniform, then general

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Largest when $\vec{E} \perp$ the surface ($\theta = 0$); zero when parallel.



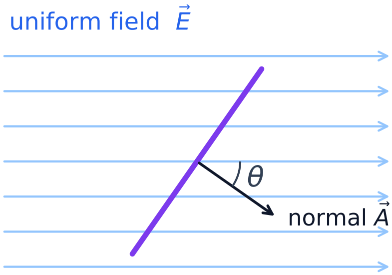
Worked example – the field on a ring's axis

A ring of radius R carries total charge Q. Find E at a point a distance z along its axis.

- every element is the same distance $\sqrt{R^2 + z^2}$ away
- the perpendicular components cancel by symmetry; only the axial part survives
- $\cos\theta = \dfrac{z}{\sqrt{R^2 + z^2}}$, so $E = \dfrac{kQz}{(R^2 + z^2)^{3/2}}$

Check the limits: at $z = 0$, $E = 0$ (the centre); for $z \gg R$ it becomes kQ/z^2, the point charge again. State the cancellation – that sentence is the mark.

Electric flux through a surface depends on



$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta \quad \cdot \quad \text{max when } \vec{E} \parallel \vec{A}$$

> Electric flux through a surface depends on the angle between the field and the surface normal

6 Gauss

Gauss's law

One of Maxwell's equations – flux out of a closed surface reveals the charge inside:

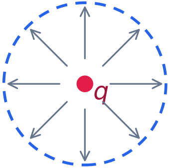
Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Choose a **Gaussian surface** 高斯面 matching the symmetry so E comes out of the integral.

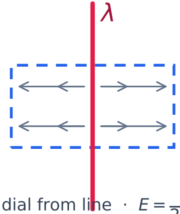
Gauss's law, in detail

Spherical (point / sphere)



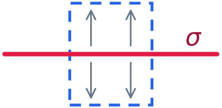
E radial, constant on r · $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Cylindrical (line / cylinder)



E radial from line · $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Planar (sheet / pillbox)



$E \perp$ sheet, both sides · $E = \frac{\sigma}{2\varepsilon_0}$

Gauss: $\oint E \cdot dA = Q_{\text{enc}}/\varepsilon_0$

Choose a Gaussian surface so E is constant (or zero) on each face — then $\Phi_E = Q_{\text{enc}}/\varepsilon_0$

Sphere, cylinder, or pillbox – matched to the symmetry.

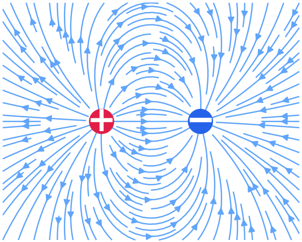
> A Gaussian surface is chosen to match the symmetry of the charge

A full-credit Gauss's-law answer has four parts

1. **name the surface** — say which **Gaussian surface** 高斯面 you chose and where it sits
2. **state the symmetry** — why E is constant on it and perpendicular to it – the **area vector** 面积矢量 $d\vec{A}$ points along the outward normal
3. **compute the flux** — $\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA$, with A the surface's own area
4. **equate and solve** — $EA = \dfrac{Q_{\text{enc}}}{\varepsilon_0}$, then say clearly *which* charge is enclosed

Three symmetries qualify and no others: spherical, **cylindrical** 柱对称 and **planar** 面对称. Anything else needs integration.

Electric Fields

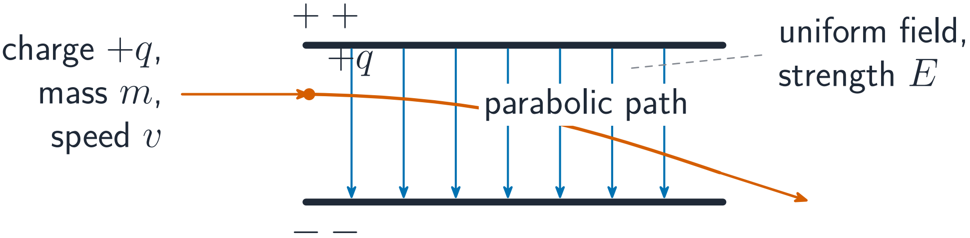


$\vec{p} = q\vec{d}$ · lines leave + enter − ·

denser $\Leftrightarrow$ stronger $|\vec{E}|$

> Electric field lines of a dipole point from the positive to the negative charge

Electric Fields, continued



A charge crossing a uniform field follows a parabolic path

In a uniform field, a charge is a projectile

the force	$\vec{F} = q\vec{E}$ is constant, so the acceleration $a = qE/m$ is constant too
launched along the field	straight-line motion with constant acceleration – the SUVAT equations, unchanged
launched *across* the field	constant velocity one way, constant acceleration the other – a parabola
the analogy	exactly **projectile motion** 抛体运动 under gravity, with qE/m in place of g

> Every capacitor-deflection question is a projectile question wearing different symbols. Split the motion into the two perpendicular directions and reuse the mechanics you already have.

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda\, dl,\ \sigma\, dA,\ \rho\, dV$; use symmetry to cancel components.
- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\varepsilon_0}$ straight and check units.

7 Recap

Unit 8 – key takeaways

1 Coulomb

$\vec{F} = kq_1q_2\hat{r}/r^2$; superpose vectors

2 Field

$\vec{E} = kq\hat{r}/r^2$; integrate $\int k\,dq\,\hat{r}/r^2$ for distributions

3 Flux

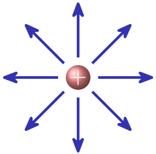
$\Phi_E = \oint \vec{E} \cdot d\vec{A}$; lines through a surface

4 Gauss

$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$; use symmetry

Check yourself

1. Do charges outside a closed surface contribute net flux through it?
2. What integral gives the field of a continuous charge distribution?
3. When is the flux through a flat surface zero?
4. Where does all excess charge sit on a conductor?



Answers **1.** no (zero net) **2.** $\int k\, dq\, \hat{r}/r^2$ **3.** when $\vec{E}$ is parallel to it **4.** on its outer surface