AP Physics C: Electricity and Magnetism Worked Solutions · 解题范例

On this exam the marks concentrate in the setup: the symmetry argument that justifies a law, the choice of dq, the limits of the integral. Each solution below writes the justification first and evaluates last. 得分集中在建立积分的过程：对称性论证、微元选取与积分限。

Electric field and Gauss's law

A solid insulating sphere of radius R carries a uniform charge density ρ. Find the electric field inside and outside.

Inside: $E = \rho r/3\varepsilon_0$. Outside: $E = \rho R^3/3\varepsilon_0 r^2$.

The distribution is spherically symmetric, so E is radial and constant in magnitude on any concentric sphere – that statement is what makes Gauss's law usable and it is a scored line. Take a Gaussian sphere of radius r. Inside ($r < R$) the enclosed charge is $\rho(4/3)\pi r^3$, so $E(4\pi r^2) = \rho(4/3)\pi r^3/\varepsilon_0$ and $E = \rho r/3\varepsilon_0$, growing linearly with r. Outside ($r > R$) the whole charge $\rho(4/3)\pi R^3$ is enclosed, giving $E = \rho R^3/3\varepsilon_0 r^2$, which falls as $1/r^2$. Check continuity at $r = R$: both expressions give $\rho R/3\varepsilon_0$, so the field is continuous – always perform that check.

A thin rod of length L carries uniform linear charge density λ. Find the potential at a point on the axis, a distance d from the near end.

$V = (\lambda/4\pi\varepsilon_0) \ln((d + L)/d)$.

Potential is a scalar, so no components are needed – say so, since choosing potential over field is the decision being tested. Take an element dx at distance x from the point, carrying $dq = \lambda dx$. Its contribution is $dV = \lambda dx/(4\pi\varepsilon_0 x)$. Integrate from $x = d$ to $x = d + L$: $V = (\lambda/4\pi\varepsilon_0)[\ln x]$ from d to d+L $= (\lambda/4\pi\varepsilon_0)\ln((d+L)/d)$. Check the far limit: for d much greater than L the logarithm approaches L/d, giving $V \approx \lambda L/4\pi\varepsilon_0 d$ – the potential of a point charge λL, as it must be.

Capacitance and circuits

A parallel-plate capacitor with plate area A and separation d is charged to voltage V and disconnected. A dielectric of constant κ is then inserted. Find the new voltage and the change in stored energy.

V' = V/κ; the energy falls to U/κ.

Disconnected means the CHARGE is fixed – identify that first, because the whole answer depends on it. The capacitance rises to κC, and since $Q = CV$ is fixed, $V' = Q/\kappa C = V/\kappa$. The energy $U = Q^2/2C$ becomes $Q^2/2\kappa C = U/\kappa$, so it falls. The energy went into pulling the dielectric in: the field polarises it and the resulting attraction does work on it. If instead the battery had stayed connected, V would be fixed and the energy would RISE to κU – the exam asks for both cases and the distinction is the point.

In an RC circuit with $\varepsilon = 12$ V, R = 2.0 kΩ and C = 100 μF, initially uncharged, find the current at t = 0, the final charge, and the time to reach half the final charge.

6.0 mA; 1.2 mC; 0.139 s.

At t = 0 an uncharged capacitor has no voltage across it, so it behaves as a plain wire and the full e.m.f. is across the resistor: I = 12/2000 = 6.0 mA. After a long time no current flows, so the full e.m.f. is across the capacitor: Q = CV = 100e−6 × 12 = 1.2 mC. For the half-charge time, q = Q(1 − e^(−t/RC)) with q = Q/2 gives e^(−t/RC) = ½, so t = RC ln2 = 0.20 × 0.693 = 0.139 s. Reasoning from the two limiting behaviours answers the first two parts with no exponential at all.

Magnetism and induction

A long solenoid of n turns per metre carries current I. Use Ampère's law to find the field inside.

B = μ_0nI, uniform and parallel to the axis.

Justify the loop before drawing it: the field outside an ideal solenoid is negligible and inside it is parallel to the axis, so choose a rectangular loop with one side of length ℓ inside along the axis and the opposite side outside. Only the inside side contributes to the line integral, giving $B\ell$. The enclosed current is $n\ell I$. Ampère's law gives $B\ell = \mu_0 n\ell I$, so $B = \mu_0 nI$ – independent of position within the solenoid and of its radius. Naming which sides contribute nothing, and why, is where the credit sits.

A conducting bar of length L slides at constant speed v on frictionless rails in a uniform field B perpendicular to the plane, with a resistor R closing the circuit. Find the current, the force needed to maintain v, and the power dissipated.

I = BLv/R; F = B^2L^2v/R; P = $B^2L^2v^2/R$.

The flux through the circuit changes because the area changes, so $\varepsilon = d\Phi/dt = BLv$, and the current is $I = BLv/R$. That current in the field feels a force $F = BIL = B^2L^2v/R$, directed to OPPOSE the motion by Lenz's law, so an equal applied force is needed to keep v constant. The mechanical power delivered is $Fv = B^2L^2v^2/R$, and the electrical power dissipated is $I^2R = (BLv/R)^2R = B^2L^2v^2/R$. They agree – that energy balance is a scored consistency argument, not just a check.