Question 1: Mathematical Routines (MR) — 10 points

A (i) For a multistep derivation that includes the equation $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$ — **Point A1**

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$) — **Point A2**

For a correct expression for the enclosed charge (e.g., $\sigma_1\left(2\pi R_1\ell\right)$) — **Point A3**

Example Response

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E\left(2\pi r\ell\right) = \frac{\sigma_1\left(2\pi R_1\ell\right)}{\varepsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\varepsilon_0 r}$$

(ii) For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E}\cdot d\vec{r}$ — **Point A4**

Scoring Notes:

- Vector notation is not required for this point to be earned.
- The sign of ΔV is not considered for this point to be earned.

For an attempt to solve the integral for $\left|\Delta V\right|$ that includes correct limits (e.g., $\left|\Delta V\right| = \dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_1}^{R_2}\frac{1}{r}dr$) — **Point A5**

Scoring Note: This point may be earned regardless of the order of the limits of integration; for example, $\dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_2}^{R_1}\frac{1}{r}dr$.

Example Response

$$\left|\Delta V\right| = \left|-\int_a^b \vec{E}\cdot d\vec{r}\right|$$

$$\left|\Delta V\right| = \int_{R_1}^{R_2}\frac{\sigma_1 R_1}{r\varepsilon_0}dr = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r}dr$$

$$\left|\Delta V\right| = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\left(r\right)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

(iii) For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$ — **Point A6**

For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ — **Point A7**

Scoring Note: The curve does not have to intersect the vertical dashed lines for this point to be earned.

Example Response

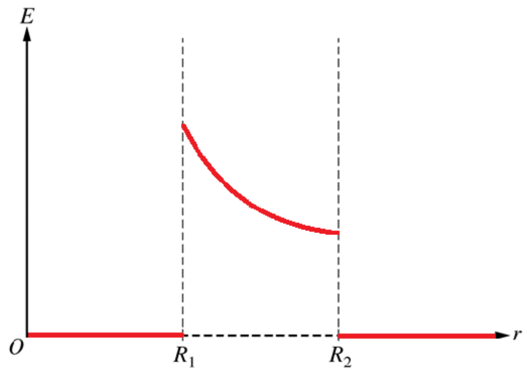


Figure 2

B

For a multistep derivation that includes the equation $C = \dfrac{Q}{\Delta V}$ — **Point B1**

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa \dfrac{Q}{\Delta V}$) — **Point B2**

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A — **Point B3**

Example Response

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1 (2\pi L R_1)}{\left(\dfrac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\dfrac{R_2}{R_1}\right)\right)}$$

$$C = \frac{2\pi L \varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

With the dielectric material

$$C = \kappa \frac{Q}{\Delta V}$$

$$C = \frac{2\pi L \kappa \varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: This point can be earned if a negative sign is included in the final answer or if units are missing and/or the units are incorrect.

Example Response

$$|\Phi_E| = 226 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{-2.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}}$$

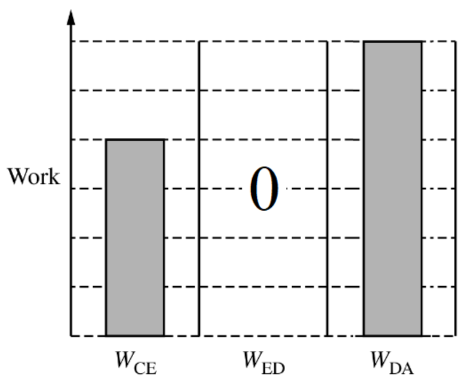
$$|\Phi_E| = 226 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{ED} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of six units — **1 point**

Example Response



(b)(ii)	For using an equation that relates the electric field to potential difference	**1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_x = -\frac{dV}{dx} \quad \textbf{OR} \quad |E_x| = \left|-\frac{dV}{dx}\right| \quad \textbf{OR} \quad |E_x| = \left|-\frac{\Delta V}{\Delta x}\right| \quad \textbf{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B — **1 point**

Example Response

$$|E_x| = \left|-\frac{20.0\text{ V} - 0.0\text{ V}}{0.65\text{ m}}\right|$$

Example Solution

$$E_x = -\frac{dV}{dx}$$

$$|E_x| = \left|-\frac{dV}{dx}\right|$$

$$|E_x| = \left|-\frac{\Delta V}{\Delta x}\right|$$

$$|E_x| = \left|-\frac{20.0\text{ V} - 0.0\text{ V}}{0.65\text{ m}}\right|$$

$$|E_x| = 31\,\frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c)	For selecting only $+y$ with an attempt at a relevant justification	**1 point**
	For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position D	**1 point**
	For indicating one of the following:	**1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the sphere have charges of opposite sign.
- The test charge moves in the direction of the electric field, which is directed upward.

Example Response

$+y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position D, the electric force is upward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) 3 points

(d)(i)	For using an appropriate equation for determining the electric potential from a line of uniform charge	**1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_\text{P} = k\sum\frac{Q}{x_\text{P} - x} \quad \textbf{OR} \quad V_\text{P} = k\int\left(\frac{1}{x_\text{P} - x}\right)dq$$

For a correct integral with λdx substituted for dq **1 point**

Example Response

$$V_\text{P} = k\lambda\int\left(\frac{1}{x_\text{P} - x}\right)dx$$

For the correct limits of integration **1 point**

Example Response

$$V_\text{P} = k\lambda\int_0^{4L}\left(\frac{1}{x_\text{P} - x}\right)dx$$

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{x_P - x}\right)dq$$

$$dq = \lambda\, dx$$

$$V_P = k\lambda\int\left(\frac{1}{x_P - x}\right)dx$$

$$V_P = k\lambda\int_0^{4L}\left(\frac{1}{x_P - x}\right)dx$$

$$V_P = -k\lambda\ln(x_P - x)\Big|_0^{4L} = k\lambda\ln(x_P - x)\Big|_{4L}^{0}$$

$$V_P = k\lambda\ln\left(\frac{x_P}{x_P - 4L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(x) = \int\frac{k}{r^2}\,dq$$

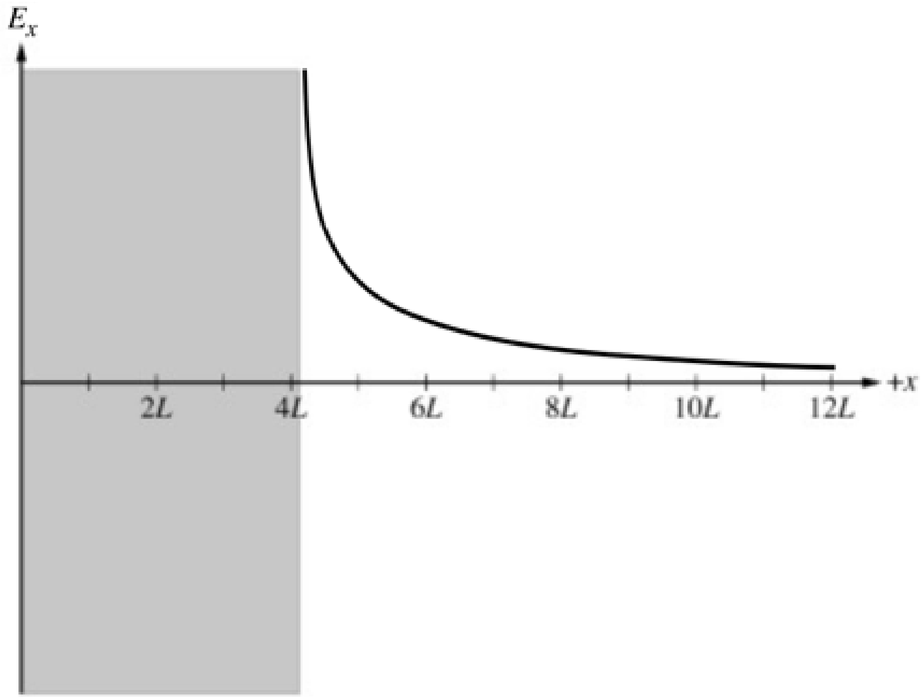
$$E(x) = \int_0^{4L}\frac{k\lambda}{(x - x')^2}\,dx' = k\lambda\left(\frac{1}{x - 4L} - \frac{1}{x}\right)$$

$$V_P = -\int_\infty^{x_P}E(x)\,dx = -k\lambda\int_\infty^{x_P}\left(\frac{1}{x - 4L} - \frac{1}{x}\right)dx$$

$$V_P = k\lambda\ln\left(\frac{x_P}{x_P - 4L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave up curve that is always positive	**1 point**

Example Response



Total for part (d) **6 points**

Total for question 1 **15 points**

Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: Correct units are not required to earn this point.

Example Response

$$\Phi_E = 113 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Example Solution

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{1.0 \times 10^{-9}\ \mathrm{C}}{8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}}$$

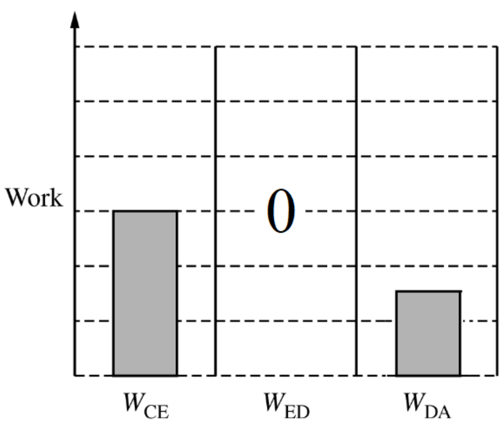
$$\Phi_E = 113 \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{\mathrm{ED}} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units — **1 point**

Example Response



(b)(ii) For using an equation that relates the electric field to potential difference **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$E_y = -\frac{dV}{dy}$ **OR** $|E_y| = \left|-\frac{dV}{dy}\right|$ **OR** $|E_y| = \left|-\frac{\Delta V}{\Delta y}\right|$ **OR** $\Delta V = -\int \vec{E} \cdot d\vec{r}$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$|E_y| = \left|-\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right|$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$|E_y| = \left|-\frac{dV}{dy}\right|$$

$$|E_y| = \left|-\frac{\Delta V}{\Delta y}\right|$$

$$|E_y| = \left|-\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right|$$

$$|E_y| = 7.1\,\frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c) For selecting only $-y$ with an attempt at a relevant justification **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C **1 point**

For indicating one of the following: **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) 3 points

(d)(i) For using an appropriate equation for determining the electric potential from a line of uniform charge **1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_\text{P} = k\sum\frac{Q}{y_\text{P} - y} \quad \textbf{OR} \quad V_\text{P} = k\int\left(\frac{1}{y_\text{P} - y}\right)dq$$

For a correct integral with $\lambda\, dy$ substituted for dq **1 point**

Example Response

$$V_\text{P} = -k\lambda\int\left(\frac{1}{y_\text{P} - y}\right)dy$$

For the correct limits of integration **1 point**

Example Response

$$V_\text{P} = -k\lambda\int_0^{2L}\left(\frac{1}{y_\text{P} - y}\right)dy$$

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

$$dq = -\lambda\, dy$$

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = k\lambda\ln(y_P - y)\,\Big|_0^{2L} = -k\lambda\ln(y_P - y)\,\Big|_{2L}^{0}$$

$$V_P = -k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(y) = \int\frac{k}{r^2}\,dq$$

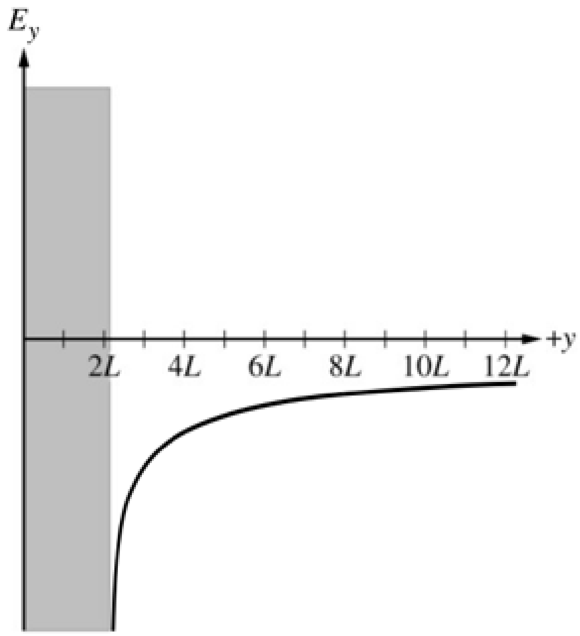
$$E(y) = \int_0^{2L}\frac{k\lambda}{(y - y')^2}\,dy' = k\lambda\left(\frac{1}{y - 2L} - \frac{1}{y}\right)$$

$$V_P = -\int_\infty^{y_P}E(y)\,dy = -k\lambda\int_\infty^{y_P}\left(\frac{1}{y - 2L} - \frac{1}{y}\right)dy$$

$$V_P = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave down curve that is always negative	**1 point**

Example Response



Total for part (d)	**6 points**
Total for question 1	**15 points**

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +3Q - (+Q)$$
$$q_{outer} = +2Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — 1 points

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

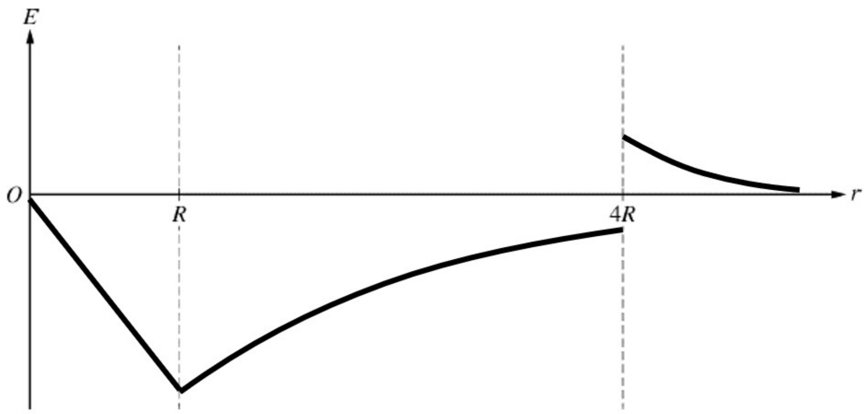
Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

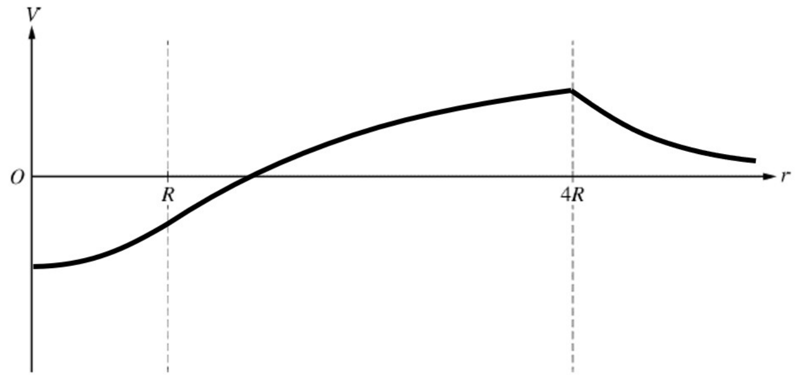
Total for part (b) — 3 points

(c)	For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere **Example Response** $E \propto \frac{1}{r^2}$	**1 point**
	For the correct magnitude, including units, of the electric field at $2R$ **Example Response** $E_{new} = \frac{E_{old}}{4} = \frac{8 \text{ N/C}}{4}$ $E_{new} = 2 \text{ N/C}$	**1 point**
	Total for part (c)	**2 points**
(d)	For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **Example Response** $V_f - V_i = -\int_{s_i}^{s_f} E dr$ $\Delta V = -\int_{r=R}^{r=4R} E dr$	**1 point**
	For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) **Example Response** $\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2} dr$ $\Delta V = \frac{Q}{4\pi\varepsilon_0} \int_{r=R}^{r=4R} \frac{1}{r^2} dr$	**1 point**

	For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration **Example Response** $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 point**
	Alternate Solution	
	For using a difference in potentials treating the inner sphere as a point charge by symmetry $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$	**1 Point**
	For using the correct charge on both of the terms; Q *not* Q *and* $3Q$	**1 Point**
	For using the correct values for r $\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$ $\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$	**1 Point**
	Total for part (d)	**3 points**
(e)(i)	For indicating that E is negative and the magnitude of E increases linearly from 0 to R	**1 point**
	For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$	**1 point**
	For a positive, concave-up, and decreasing curve for $r > 4R$	**1 point**
	Example Response  	

(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e) 6 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +4Q - (-Q)$$
$$q_{outer} = +5Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) — 1 point

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E(2\pi rL)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{Q}{\pi R^2 L}\right)\left(\pi r^2 L\right) = Q\frac{r^2}{R^2}$$

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0(2\pi rL)} = \left(Q\frac{r^2}{R^2}\right)\frac{1}{\varepsilon_0(2\pi rL)}$$
$$E = Q\frac{r}{2\pi\varepsilon_0 R^2 L}$$

Total for part (b) — 3 points

(c) For recognizing that the E-field varies as the reciprocal of the separation distance from the center of the nonconducting cylinder **1 point**

Example Response

$$E \propto \frac{1}{r}$$

For the correct value, including units, of electric field at $2R$ **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{2} = \frac{12 \text{ N/C}}{2}$$
$$E_{new} = 6 \text{ N/C}$$

Total for part (c) 2 points

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E dr$$
$$\Delta V = -\int_{r=R}^{r=3R} E dr$$

For substituting the correct expression for the electric field or an expression consistent with the explicit functional dependence of $E(r)$ from part (c) **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=3R} \left(\frac{Q}{2\pi\varepsilon_0 rL} \right) dr = -\frac{Q}{2\pi\varepsilon_0 L} \int_{r=R}^{r=3R} \frac{1}{r} dr$$

For correctly integrating the electric field expression that was substituted in the previous point with the correct limits of of integration **1 point**

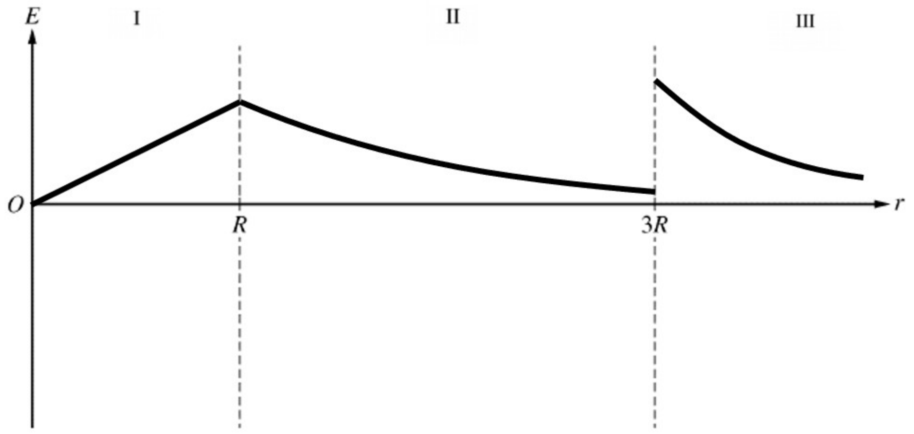
Example Response

$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(r)]_{r=R}^{r=3R} = -\frac{Q}{2\pi\varepsilon_0 L} [\ln(3R) - \ln(R)]$$
$$\Delta V = -\frac{Q}{2\pi\varepsilon_0 L} \ln(3)$$

Total for part (d) 3 points

(e)(i)	For indicating E is positive and the magnitude increases linearly from 0 to R	**1 point**
	For a positive, decreasing concave up graph for $r > R$	**1 point**
	For a discontinuous increase in magnitude at $r = 3R$	**1 point**

Example Response

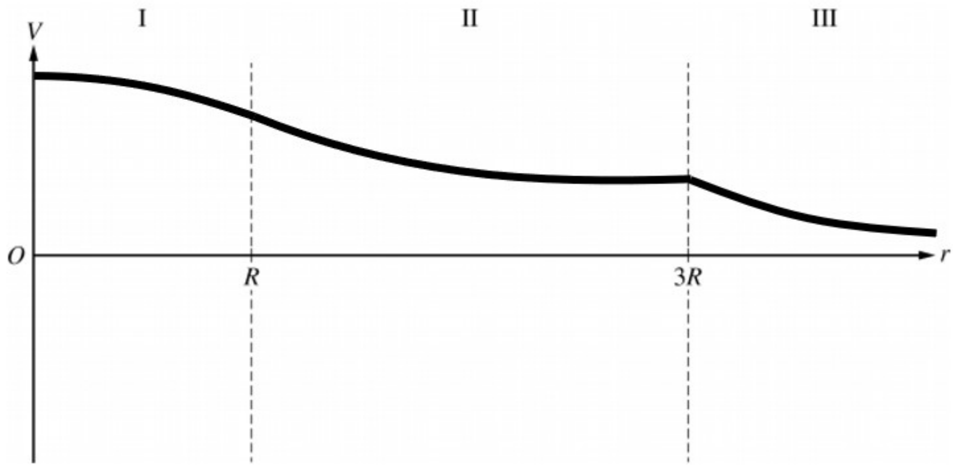


(e)(ii)	For a graph that is concave down and always decreasing as r increases within Region I	**1 point**
	For a graph that is concave up and always decreasing as r increases within each of the Regions II and III	**1 point**
	For a continuous graph across all three regions	**1 point**

Scoring Notes:

- The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.
- The response can earn a maximum of 2 points if all three segments are reflected across the horizontal axis.

Example Response



Total for part (e) 6 points

Total for question 1 15 points

2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

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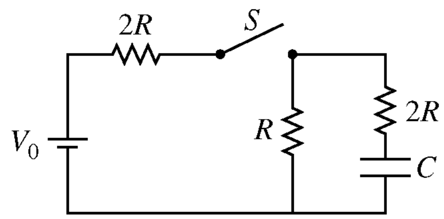
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \dfrac{V_0}{3R}$		

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$		
$q = CV_R = C\left(\frac{1}{3}V_0\right) = \frac{1}{3}CV_0$		

Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \dfrac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\dfrac{q(t)}{C} = I(R + 2R)$		
Note: Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$		

Question 1 (continued)

(e)

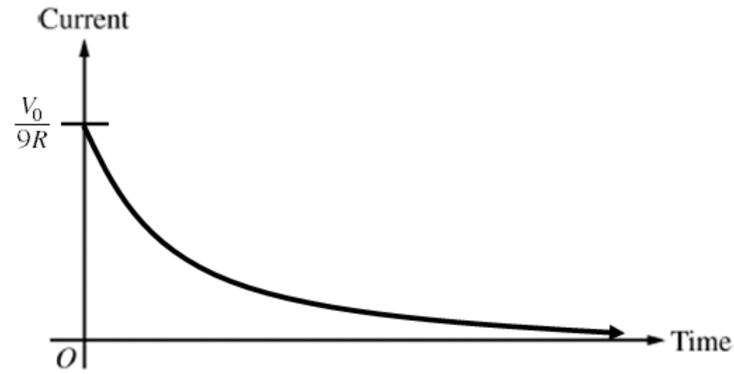
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \dfrac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$		

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

2019 SCORING GUIDELINES

Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
Alternate Solution		*Alternate Points*
For selecting "Less than"		*1 point*
For a correct justification invoking conservation of energy		*1 point*
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

2019 SCORING GUIDELINES

Question 1 (continued)

Learning Objectives

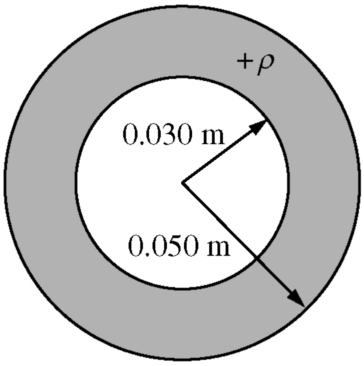
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m: } \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m: } \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m: } \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r}(4\pi r^2)dr$		
For using the proper limits of integration		1 point
$Q = 4\pi(1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi(1.6 \times 10^{-6})\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8}$ C		

Question 2 (continued)

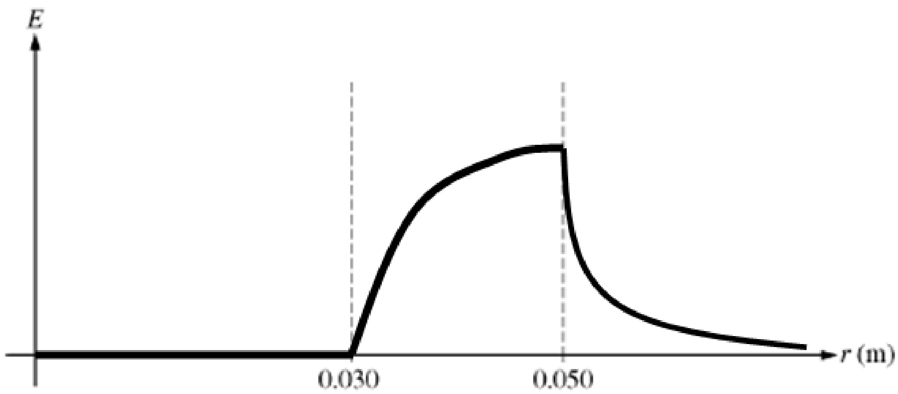
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \bullet dA = E\left(4\pi r^2\right)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\dfrac{Q_{enc}}{\varepsilon_0} = E\left(4\pi r^2\right) \therefore E = \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \dfrac{\left(1.61 \times 10^{-8}\ \text{C}\right)}{(4\pi)\left(8.85 \times 10^{-12}\right)(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4\ \text{N/C}$		

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m		1 point

Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential		1 point
For substituting $r = 0.05$ m into a correct expression for electric potential		1 point
$V_R = \dfrac{Q_{\text{tot}}}{4\pi\varepsilon_0 r} = \dfrac{(9\times 10^9)(1.61\times 10^{-8}\ \text{C})}{(0.05\ \text{m})} = 2900\ \text{V}$		
Alternate Solution		*Alternate Points*
For substituting the total charge from part (a) into an integration for electric potential		*1 point*
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} dr = -\int \dfrac{(9\times 10^9)(1.61\times 10^{-8}\ \text{C})}{r^2} dr$		
For integrating with correct limits of integration		*1 point*
$V_R = -\displaystyle\int_{r=\infty}^{r=0.05\ \text{m}} \dfrac{145}{r^2} dr = -145\left[-\dfrac{1}{r}\right]_{r=\infty}^{r=0.05\ \text{m}} = 145\left(\dfrac{1}{(0.05\ \text{m})} - \dfrac{1}{\infty}\right) = 2900\ \text{V}$		

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)
i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field		1 point
$F = ma \therefore qE = ma \therefore a = \dfrac{qE}{m}$		
For correctly substituting into equation above		1 point
$a = \dfrac{qE}{m} = \dfrac{(1.6\times 10^{-19}\ \text{C})(5.79\times 10^4\ \text{N/C})}{1.67\times 10^{-27}\ \text{kg}} = 5.55\times 10^{12}\ \text{m/s}^2$		

2019 SCORING GUIDELINES

Question 2 (continued)

(e) continued
ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2\left(1.6\times10^{-19}\ \text{C}\right)(0-2900\ \text{V})}{\left(1.67\times10^{-27}\ \text{kg}\right)}} = 7.45\times10^{5}\ \text{m/s}$		

Learning Objectives

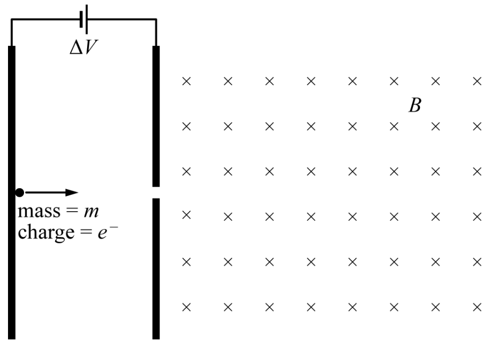
ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)

i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page
____ Toward the top of the page ____ Toward the bottom of the page

For selecting "Toward the bottom of the page"		1 point

ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)		1 point
Example: The sphere will move in a circular path toward the bottom of the page.		

Question 3 (continued)

(b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

(c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\mathrm{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99

Question 3 (continued)

(d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:____________
Horizontal axis:____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_eV}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2}V$$

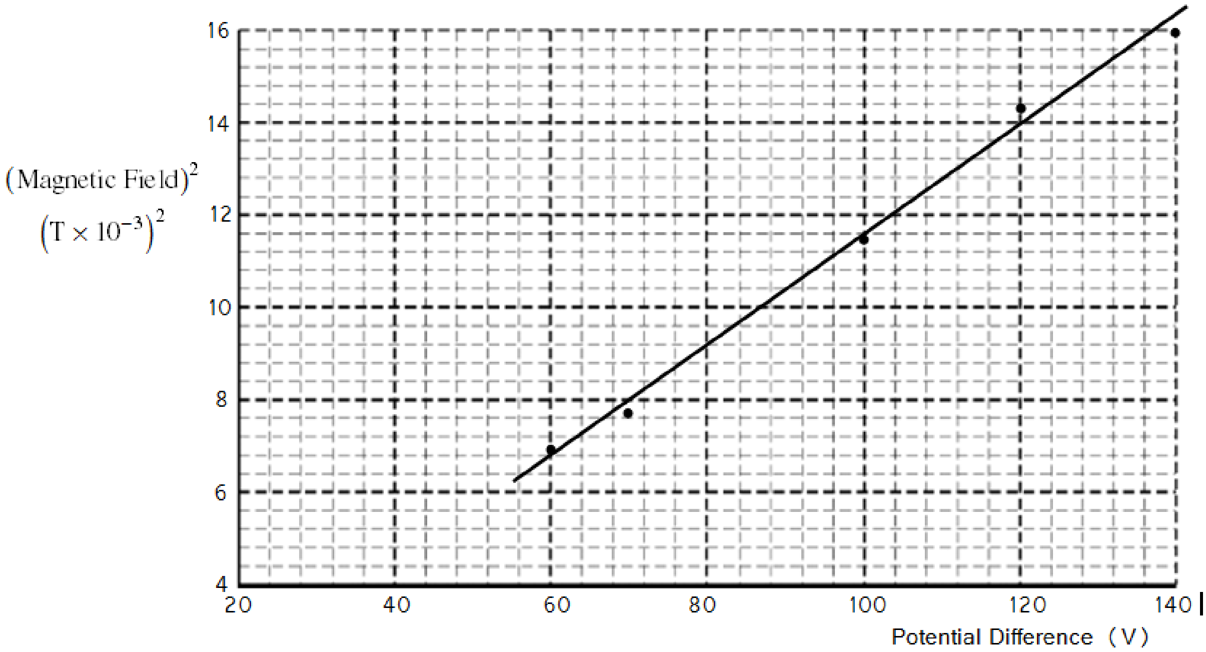
Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\mathrm{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99
$(\text{Magnetic field})^2$ $\left(\mathrm{T} \times 10^{-3}\right)^2$	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

Question 3 (continued)

(e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units		1 point
For correctly scaling and labeling the axes so that the points use at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a straight line that represents the data		1 point

(f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{14-9.2}{120-80} = 0.12\ \dfrac{(\mathrm{T}\times 10^{-3})^2}{V} = 1.2\times 10^{-7}\ \mathrm{T^2/V}$		
For a correct expression relating the slope to the mass to charge ratio of an electron		1 point
$B^2 = \dfrac{2m_e}{er^2}V \ \therefore\ \text{slope} = \dfrac{2m_e}{er^2} \ \therefore\ \dfrac{m_e}{e} = \dfrac{r^2 \times \text{slope}}{2}$		
For substituting correct values into the above equation		1 point
$\dfrac{m_e}{e} = \dfrac{(0.01\ \mathrm{m})^2 \times \left(1.2\times 10^{-7}\ \mathrm{T^2/V}\right)}{2} = 6.0\times 10^{-12}\ \mathrm{kg/C}$		

Question 3 (continued)

Learning Objectives

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.
CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.
CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

2018

AP® CollegeBoard

AP Physics C: Electricity and Magnetism

Scoring Guidelines

2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

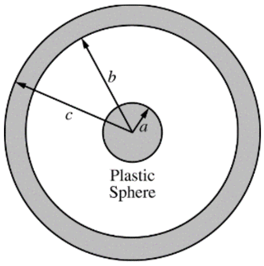
5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m}/\text{s}^2$, but the use of $10\ \text{m}/\text{s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**



Conducting Spherical Shell

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i. 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii. 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\dfrac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A}$		
$\dfrac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\dfrac{\beta\pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \dfrac{\beta r^2}{4\varepsilon_0}$		

ii. 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\dfrac{\beta\pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$		

2018 SCORING GUIDELINES

Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)		1 point
$q_{inner} = -\beta\pi a^4$		
For a correct justification		1 point
$E_{shell} = 0 = \frac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii. 1 point

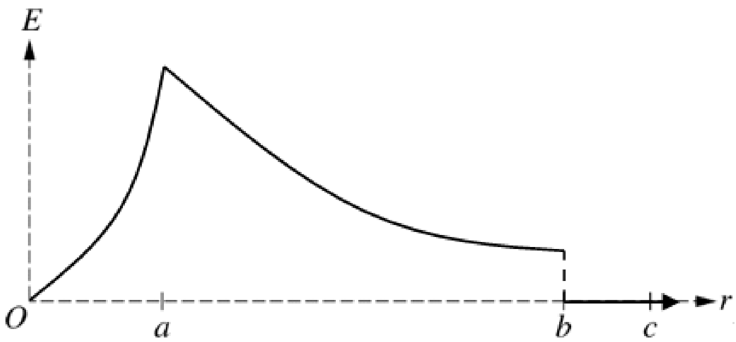
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

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Question 1 (continued)

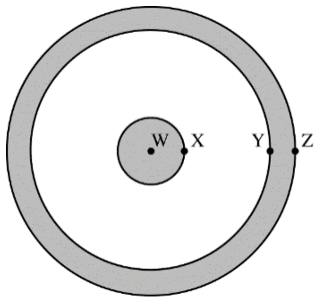
Distribution of points

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W   __2__ X   __3__ Y   __3__ Z



For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (because Y = Z is the first point)		1 point

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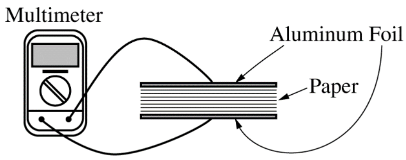
Question 2

15 points total | **Distribution of points**



An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C *Horizontal axis:* $\dfrac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

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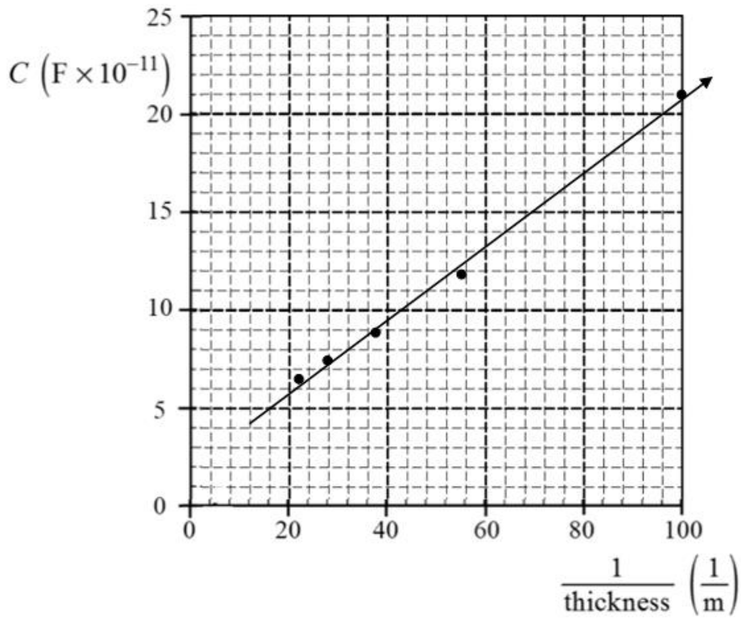
Question 2 (continued)

Distribution of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axes including units		1 point
For correctly plotting the data		1 point
For drawing a straight line consistent with the plotted data		1 point

(c) 3 points

Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)\left(\mathrm{F}\times10^{-11}\right)}{(80-32)(1/\mathrm{m})} = 0.19\ \mathrm{F{\bullet}m}$ (Linear regression $= 0.187\ \mathrm{F{\bullet}m}$)		
For correctly relating the slope to the dielectric constant		1 point
$\text{slope} = \kappa\varepsilon_0 A$		
$\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{(0.19\ \mathrm{F{\bullet}m})}{\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)\left(0.060\ \mathrm{m^2}\right)}$		
For a correct answer		1 point
$\kappa = 3.58$ (Linear regression $= 3.52$)		

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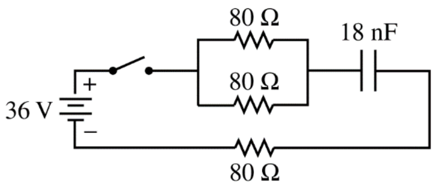
Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm’s law with the potential difference across the capacitor equal to zero		1 point
$I = \frac{V}{R} = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

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Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.		

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Question 3

15 points total **Distribution of points**

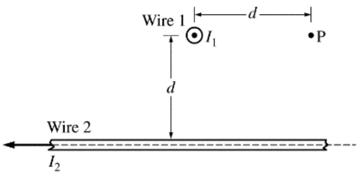

Figure 1. Side view

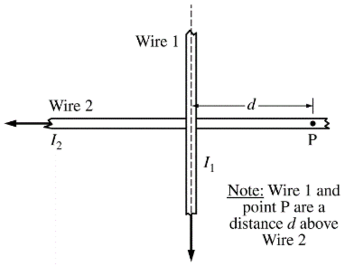

Figure 2. Top view

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point *P*		1 point
$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$		
$B(2\pi d) = \mu_0 I$		
For a correct answer		1 point
$B = \dfrac{\mu_0 I}{2\pi d}$		

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$		1 point
$B_1 = \dfrac{\mu_0 I}{2\pi d}$ & $B_2 = \dfrac{\mu_0(2I)}{2\pi d} = \dfrac{\mu_0 I}{\pi d}$		
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2		1 point
$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\dfrac{\mu_0 I}{2\pi d}\right)^2 + \left(\dfrac{\mu_0(2I)}{2\pi d}\right)^2} = \dfrac{\sqrt{5}\mu_0 I}{2\pi d}$		

AP PHYSICS C: ELECTRICITY AND MAGNETISM
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Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\dfrac{B_1}{B_2}\right) = \tan^{-1}\left(\dfrac{\left(\dfrac{\mu_0 I}{2\pi d}\right)}{\left(\dfrac{\mu_0(2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\dfrac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel		1 point
For stating there is a net torque which causes rotation		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.		

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Question 3 (continued)

Distribution of points

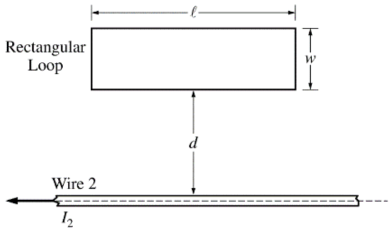

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0 (2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0 (2I_0)(1-kt)\ell w}{2\pi} dr$

__X__ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0 (2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0 (2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0 (2I_0)(1-kt)}{2\pi r} \ell dr$		1 point

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Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \dfrac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\dfrac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\dfrac{d\Phi}{dt} \right\rvert = \dfrac{\mu_0 I_0 \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right) \left\lvert \dfrac{d}{dt}[(1 - kt)] \right\rvert$		
$\varepsilon = \dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \dfrac{\varepsilon}{R} = \dfrac{\dfrac{\mu_0 I_0 k \ell}{\pi} \ln\left(\dfrac{d + w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \dfrac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\dfrac{d + w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

____ Clockwise ____ Counterclockwise

____ Undefined because there is no current induced in the loop

Justify your answer.

For selecting "Clockwise" with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz's law to relate the decrease in the flux to the clockwise current		1 point
Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz's law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.		

2017

AP Physics C: Electricity and Magnetism

Scoring Guidelines

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General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

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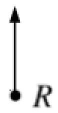
Question

15 points total **Distribution of ints**

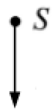
(a)

i. 1 point



For correctly drawing a single vector pointing upward at point *R* — 1 point

ii. 1 point



For correctly drawing a single vector pointing downward at point *S* — 1 point

(b)

i. 2 points

For using Gauss's law to calculate the electric flux — 1 point

$$\Phi = \frac{Q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

For a correct answer — 1 point

$$\Phi = \frac{\rho_0 A h}{\varepsilon_0}$$

ii. 3 points

For correctly applying Gauss's law to the slab — 1 point

$$\frac{Q}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A} = EA_{top} + EA_{bottom}$$

For substituting the flux from part (b)(i) into equation above and for including the contributions of the top and bottom parts of the Gaussian surface — 1 point

$$\frac{\rho_0 A h}{\varepsilon_0} = 2EA$$

For correctly relating *h* to 2*z* — 1 point

$$\frac{\rho_0 A (2z)}{\varepsilon_0} = 2EA$$

$$E = \frac{\rho_0 z}{\varepsilon_0}$$

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Question 1 (continued)

Distribution of points

(c)

i. 1 point

Selecting "$z_0 < 0$"

For a correct justification — 1 point

Example: The electric field due to the plates alone is directed toward the top of the page. Therefore, the net electric field can only be zero where the electric field of the slab is directed toward the bottom of the page. So the net electric field can only be zero when $z_0 < 0$.

ii. 3 points

For setting the net electric field between the plates equal to zero and relating it to the sum of the electric fields of the plates and the slab — 1 point

$$E_{total} = 0 = E_{plate} + E_{slab}$$

For using the correct formula for the electric field between the plates regardless of sign — 1 point

$$\frac{\sigma}{\varepsilon_0} = -\frac{\rho_0 z_0}{\varepsilon_0}$$

$$z_0 = -\frac{\sigma}{\rho_0}$$

Substitute into equation above

$$z_0 = -\frac{\left(2.0 \times 10^{-6}\ \mathrm{C/m^2}\right)}{\left(1.0 \times 10^{-3}\ \mathrm{C/m^3}\right)}$$

For an answer that is consistent with the formula provided by the student above (must include units) — 1 point

$$z_0 = -2.0 \times 10^{-3}\ \mathrm{m}$$

Question 1 (continued)

Distribution of points

(d) 4 points

For using the sum of the electric potential of the slab and the plates as the total potential difference between the center and top of the slab — 1 point

$$\Delta V = \Delta V_{slab} + \Delta V_{plate}$$

$$\Delta V = \int E_{total}\, dr = \int \left(E_{slab} + E_{plate}\right) dr$$

$$\Delta V = \int E_{slab} dr + \int E_{plate}\, dr$$

For integrating with proper limits or constants of integration — 1 point

$$\Delta V = \int_{z=0}^{z=0.005} \frac{\rho_0 z}{\varepsilon_0} dz + \int_{z=0}^{z=0.005} \frac{\sigma}{\varepsilon_0} dz$$

For correctly integrating each term regardless of whether they contain the correct expressions — 1 point

$$\Delta V = \frac{\rho_0}{\varepsilon_0}\left[\frac{z^2}{2}\right]_{z=0}^{z=0.005} + \frac{\sigma}{\varepsilon_0}[z]_{z=0}^{z=0.005} = \frac{\rho_0}{2\varepsilon_0}\left(0.005^2 - 0\right) + \frac{\sigma}{\varepsilon_0}(0.005 - 0)$$

$$\Delta V = \frac{\rho_0 z^2}{2\varepsilon_0} + \frac{\sigma z}{\varepsilon_0}$$

$$\Delta V = \frac{\left(1.0 \times 10^{-3}\ \mathrm{C/m^3}\right)\left(5.0 \times 10^{-3}\ \mathrm{m}\right)^2}{(2)\left(8.85 \times 10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)} + \frac{\left(2.0 \times 10^{-6}\ \mathrm{C/m^2}\right)\left(5.0 \times 10^{-3}\ \mathrm{m}\right)}{\left(8.85 \times 10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}$$

For a correct answer — 1 point

$$\Delta V = 2540\ \mathrm{V}$$

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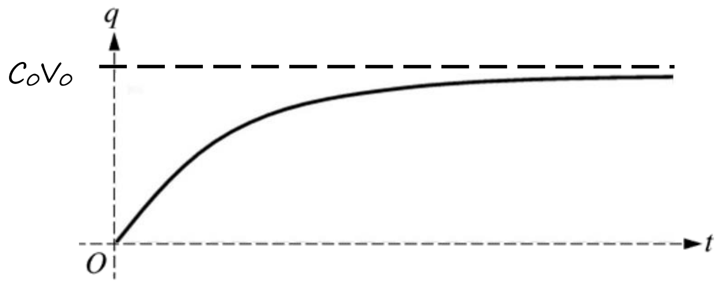
Question

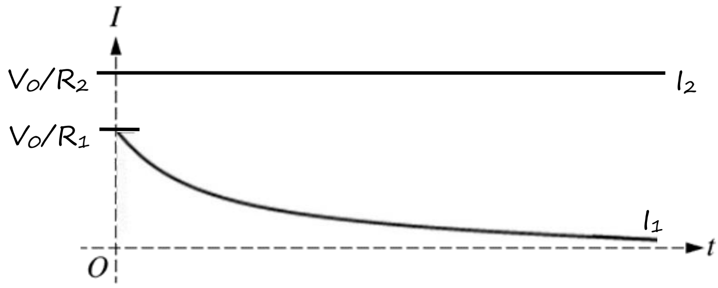
15 points total | **Distribution of points**

(a) 3 points



For a concave down curve starting at the origin — 1 point
For a horizontal asymptote — 1 point
For properly labeling the horizontal asymptote — 1 point

(b) 4 points



For showing that I_2 is horizontal with the correct vertical intercept — 1 point
For showing that I_1 is concave up and asymptotic to I = 0 — 1 point
For correctly labeling the vertical intercept of the I_1 graph — 1 point
For drawing the I_2 graph always above the I_1 graph — 1 point

(c) 2 points

Select "$\Delta t_C < \Delta t_D$"

For indicating that the equivalent resistance during discharging is greater than during charging — 1 point
For a statement relating the greater resistance to the greater time constant, or to a smaller current — 1 point

Example: Because the resistance as the capacitor discharges is greater than when it charges, the time constant is larger for discharging. Therefore, the time to charge to 50% of its maximum charge is less than the time to discharge to 50% of its maximum charge.

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Question 2 (continued)

Distribution of points

(d)

i. 2 points

For using a correct statement of Ohm's law or loop rule with the switch open — 1 point

$V = IR$

$I = \dfrac{V_C}{R_{tot}} = \dfrac{V_0}{R_1 + R_2}$

Substitute correct values into equation above

$I = \dfrac{(1.5\ \text{V})}{(100\ \Omega + 150\ \Omega)}$

For a correct answer — 1 point

$I = 6.0\ \text{mA}$ with units, or 0.006 without units

ii. 1 point

For a correct justification — 1 point

Example: Once the capacitor begins to discharge, the charge stored in the capacitor will decrease and the potential difference across the capacitor decreases. Therefore, the current through resistor R_2 decreases.

(e)

i. 1 point

For correctly substituting into an equation for the energy stored in a capacitor, including units for numerical values — 1 point

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(80\ \mu\text{F})(1.5\ \text{V})^2 = 90\ \mu\text{J}$

ii. 2 points

For setting up a valid equation or argument to calculate the energy dissipated by resistor R_1 — 1 point

$E_{dis} = Pt = I^2Rt \therefore E_{dis} \sim R \therefore E_{dis,1} = E_{dis,tot}\dfrac{R_1}{(R_1 + R_2)}$

For correctly substituting into the equation above — 1 point

$E_{dis,1} = (90\ \mu\text{J})\dfrac{(150\ \Omega)}{(150\ \Omega + 100\ \Omega)} = 54\ \mu\text{J}$

Alternate Solution on next page.

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Question 2 (continued)

Distribution of points

(e)(ii) continued

Alternate Solution — *Alternate Points*

Derive an equation for the current through resistor R_1 as a function of time

$$I(t) = I_{\text{MAX}} e^{-t/\tau} = \frac{V_{\text{MAX}}}{(R_1 + R_2)} e^{-t/(R_1+R_2)C} = \frac{(1.5\text{ V})}{(250\ \Omega)} e^{-t/(250\ \Omega)(80\ \mu\text{F})}$$

$$I(t) = (6.0\text{ mA})e^{-t/(0.02)}$$

For using a correct formula with integral calculus to calculate the energy dissipated by resistor R_1 — 1 point

$$E = \int P dt = \int I^2 R dt = \int I(t)^2 R_1 dt = \int \left((6.0\text{ mA})e^{-t/(0.02)}\right)^2 (150\ \Omega)\, dt$$

For integrating the above equation with correct limits or constants of integration — 1 point

$$E = (0.0054)\int_{t=0}^{t=\infty} \left(e^{-t/(0.02)}\right)^2 dt = (0.0054)\int_{t=0}^{t=\infty} e^{-100t} dt = (0.0054)\left[\frac{e^{-100t}}{(-100)}\right]_{t=0}^{t=\infty}$$

$$E = \left(-5.4 \times 10^{-5}\right)\left(e^{-\infty} - e^{0}\right) = 54\ \mu\text{J}$$

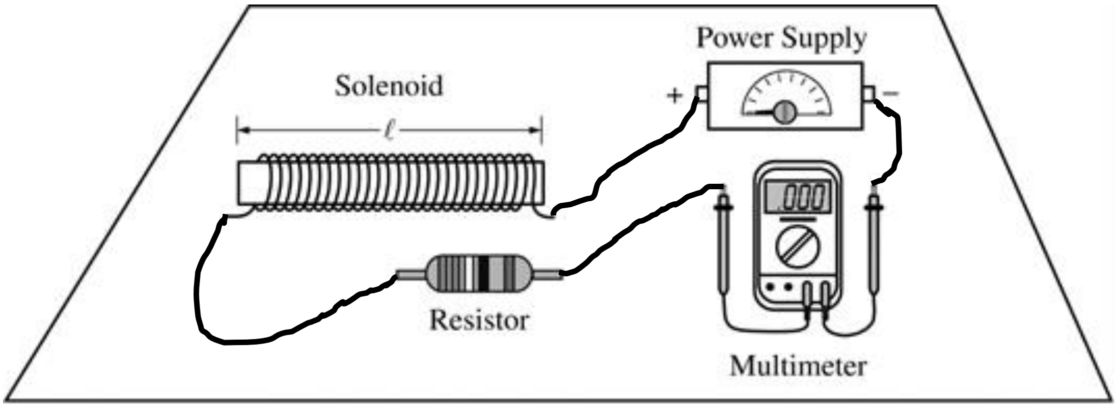
2017 SCORING GUIDELINES

Question

15 points total | **Distribution of points**

(a)

i. 2 points



For connections that create a circuit with the power supply, solenoid, and resistor in series with each other — 1 point

For connections that create a circuit with the multimeter in series with the inductor/resistor combination — 1 point

ii. 1 point

For selecting the answer consistent with the answer in part (a)(i) — 1 point

iii. 3 points

For selecting point *B* — 1 point

For any indication that the magnetic field outside the solenoid is less than inside the solenoid — 1 point

For any indication that the closer you are to the center of the solenoid, the better the approximation of an ideal solenoid due to the lack of edge effects — 1 point

Example: The equation for the magnetic field for a solenoid is for an ideal solenoid. This does not work outside the solenoid (this eliminates *C* and *D*). In addition, as you approach the edge of the solenoid, the ideal approximation is not as accurate (this eliminates *A*). Thus, point *B* gives the best approximation of an ideal solenoid.

2017 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

(b) 2 points

For any indication that Solenoid 1 has more turns per unit length — 1 point

For any indication that Solenoid 1 is longer and therefore more like an ideal solenoid — 1 point

Examples:

The equation for the magnetic field of a solenoid $(B = \mu_0 nI)$ is based on an ideal solenoid in which you can ignore edge effects. Because Solenoid 1 has more turns per unit length, it is a better approximation of an ideal solenoid and thus its slope will give a better value for the permeability of free space.

The slope for Solenoid 1 is closer to the accepted value than the slope for Solenoid 2; therefore, the graph for Solenoid 1 is better.

(c)

i. 3 points

For correctly calculating the slope for the solenoid chosen in part (b) using the best-fit line and not the data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(10-4)\left(\text{T}\times 10^{-5}\right)}{(75-24)(\text{A/m})} = 1.17\times 10^{-6}\ \text{T}\bullet\text{m/A}$$

For correctly relating the magnetic permittivity of free space to the slope — 1 point

$\mu_0 = \text{slope}$

For correct units — 1 point

$\mu_0 = 1.17\times 10^{-6}\ \text{T}\bullet\text{m/A}$

For Solenoid 2

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(8-4)\left(\text{T}\times 10^{-5}\right)}{(82-35)(\text{A/m})} = 8.51\times 10^{-7}\ \text{T}\bullet\text{m/A}$$

ii. 1 point

For using a correct equation for % error including the substitution of the accepted and the measured values — 1 point

$$\%\ \text{error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}}\times 100\% = \frac{\left|\left(1.26\times 10^{-6}\right) - \left(1.18\times 10^{-6}\right)\right|}{\left(1.26\times 10^{-6}\right)}\times 100\% = 6.35\%$$

For Solenoid 2

$$\%\ \text{error} = \frac{|\text{acc} - \text{exp}|}{\text{acc}}\times 100\% = \frac{\left|\left(1.26\times 10^{-6}\right) - \left(8.51\times 10^{-7}\right)\right|}{\left(1.26\times 10^{-6}\right)}\times 100\% = 32.4\%$$

2017 SCORING GUIDELINES

Question 3 (continued)

Distribution of points

(d)

i. 1 point

For a correct explanation involving any extraneous *B* fields — 1 point

Example: A component of Earth's magnetic field is in the direction of the inductor's magnetic field.

ii. 2 points

For stating that either the solenoid or the circuit has near zero resistance — 1 point

For stating that as a result of near zero resistance the current becomes very high — 1 point

Example: The inductor has near zero resistance. Therefore, without the resistor in series with it, the inductor will draw a near infinite current that the multimeter cannot handle.

AP® Physics C: Electricity and Magnetism 2016 Scoring Guidelines

SCORING GUIDELINES

Question

15 points total | **Distribution of points**

3 points

For indicating that the total potential is the sum of the potential from individual point charges — 1 point

ample using point B: $V_B = 0 = V_1 + V_2$

orrectly substituting into the above equation (signs are ignored at this step) — 1 point

$-V_1 = V_2$

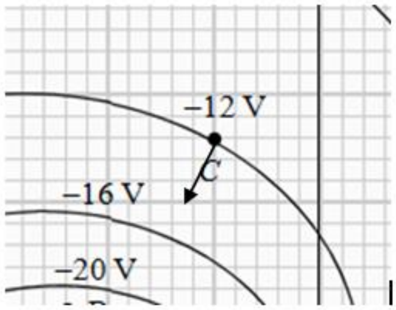
$-\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$

$-\frac{q_1}{(5 \times 0.5m)} = \frac{(2.0 \text{ nC})}{(2 \times 0.5 \text{ m})}$

For a correct answer with correct sign and units — 1 point

$q_1 = -5.0 \text{ nC}$

2 points



For drawing a vector perpendicular to the equipotential line for C — 1 point

For drawing a vector in the direction of the -16 V line — 1 point

2 points

For using the equation relating the electric field to potential difference — 1 point

$E = -\frac{dV}{dx}$

$|E| \simeq \frac{\Delta V}{\Delta x}$

For substituting values from the figure — 1 point

$E = \frac{(-20 \text{ V} - (-24 \text{ V}))}{(2 \times 0.1 \text{ m})}$

$E = 20 \text{ N/C}$

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

2 points

For using a correct equation for the electric flux — 1 point

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi_E = \frac{(2.0 \text{ nC})}{\left(8.85 \times 10^{-12} \ \text{C}^2/\left(\text{N}\bullet\text{m}^2\right)\right)}$$

For a correct answer with units — 1 point

$$\Phi_E = 226 \ \left(\text{N}\bullet\text{m}^2\right)/\text{C}$$

(

2 points

sing an equation that relates the work done to the change in potential energy — 1 point

$$W = -q\Delta V = -(1.6 \times 10^{-19} \text{ C})(-4 \text{ V} - (4 \text{ V}))$$

For a correct answer including sign and units — 1 point

$$W = 1.28 \times 10^{-18} \text{ J} = 8.0 \text{ eV}$$

2 points

ndicating that the kinetic energy of the proton at E will be equal to the work done — 1 point

$$W = \Delta K = \frac{1}{2}mv^2$$

orrectly substituting the answer from part (e) i. into the above equation — 1 point

$$\left(1.28 \times 10^{-18} \text{ J}\right) = \left(\frac{1}{2}\right)\left(1.67 \times 10^{-27} \text{ kg}\right)v^2$$

$$v = 3.92 \times 10^4 \ \text{m/s}$$

) 2 points

orrectly selecting Left — 1 point

For a correct justification — 1 point

Example: Electrons accelerate in the direction perpendicular to equipotential surfaces and toward higher potential. Therefore, at point B, an electron would accelerate toward the left.

No points are earned if the wrong answer is selected.

;CORING GUIDELINES

Question

15 points total | **Distribution of points**

2 points

For selecting "It is the same through both." — 1 point
For a correct justification — 1 point
Example: The sample of wire and the resistor are in series with each other and therefore will have the same current.
Note: Because the voltmeter is ideal, it will have no effect on the result.
No points are earned if the wrong answer is selected.

2 points

For selecting "It will depend on the resistance of the sample." — 1 point
For a correct justification including argument using Ohm's law, voltage drop across resistors, etc. — 1 point
Example: Because the sample of wire has resistance, it cannot be known whether the wire of the resistor has the greatest resistance and which has the higher potential difference across it. Therefore, it will depend on the resistance of the wire sample.
No points are earned if the wrong answer is selected.

(1 point

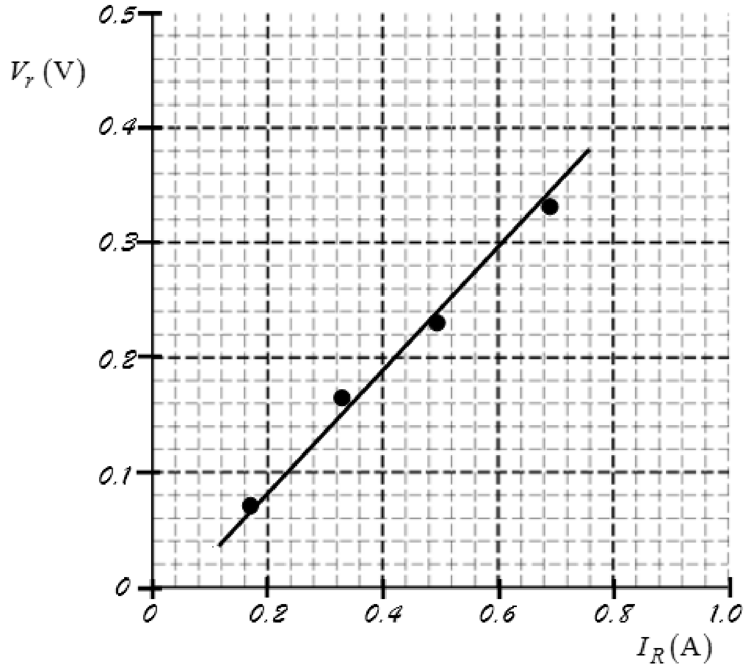
ndicating the correct quantities for each axis — 1 point
Horizontal axis: I_R
Vertical axis: $V_r = \mathcal{E} - V_R$
Note: Credit is received if the axes are reversed.

SCORING GUIDELINES

Question 2 (continued)

Distribution of points

3 points



For a correct scale that uses more than half the grid and for correctly labeling the axes including units — 1 point

For correctly plotting given data — 1 point

For drawing a straight line consistent with the given data — 1 point

(2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(V_2 - V_1)}{(I_2 - I_1)} = \frac{(0.30 - 0.10)\ \text{V}}{(0.60 - 0.22)\ \text{A}} = 0.526\ \text{V/A}$$

Note: Linear regression gives $\text{slope} = 0.485\ \text{V/A}$. (Student must indicate that the value comes from a linear regression from the calculator, and the equation of the line must be present.)

For correctly relating r to the slope — 1 point

$V_r = I_R r$, therefore the slope equals r

$r = 0.526\ \Omega$

Note: Linear regression gives $r = 0.485\ \Omega$.

;CORING GUIDELINES

Question 2 (continued)

Distribution of points

) 1 point

For using the equation relating resistance to resistivity with the correct or consistent substitutions — 1 point

$$R = \frac{\rho L}{A} \text{ so } \rho = \frac{AR}{L}$$

$$\rho = \frac{(\pi)\left(1.00 \times 10^{-3} \text{ m}\right)^2 (0.526\ \Omega)}{(3.00 \text{ m})}$$

$$\rho = 5.51 \times 10^{-7}\ \Omega \bullet \text{m}$$

Note: Linear regression gives $\rho = 5.08 \times 10^{-7}\ \Omega \bullet \text{m}$.

2 points

electing "Less than" with an attempt at a justification — 1 point

For a correct justification — 1 point

Example: The resistance calculated from the graph is the sum of the sample resistance plus the ammeter resistance because it is not ideal. The actual resistance is the calculated resistance minus the ammeter resistance, and therefore less than the calculated resistance.

oints are earned if the wrong answer is selected.

2 points

For selecting "Greater than" with an attempt at a justification — 1 point

For a correct justification — 1 point

Example: If the voltmeter is not ideal, that would add an additional resistor in parallel. A parallel resistor reduces the total resistance of the circuit. This would lead to an increase in current and a higher reading on the ammeter.

No points are earned if the wrong answer is selected.

SCORING GUIDELINES

Question

15 points total **Distribution of points**

1 point

For selecting "Left" — 1 point

2 points

For selecting "Less than" — 1 point
For a correct justification — 1 point
Example: Because as the bar falls the flux at point C is increasing, the emf generated must create a magnetic field to oppose this change. Therefore, it will create a magnetic field to decrease the flux and thus decrease the magnetic field.
No points are earned if the wrong answer is selected.

2 points

For selecting "Greater than" — 1 point
For a correct justification — 1 point
Example: The field at point C, which is above the bar, is less than the original magnetic field, and point D is on the other side of the bar. Therefore, the direction of the magnetic field from the bar at point D is the opposite of the direction at point C, so the net magnetic field at D when the bar is falling must be greater than the original magnetic field.
No points are earned if the wrong answer is selected.

(4 points

For correctly applying Newton's second law to the motion of the bar — 1 point

$F_{net} = Mg - F_M = Mg - BIL$

$I = \mathcal{E}/R$

For attempting to use Faraday's law to obtain an expression for the emf in the bar — 1 point

$\mathcal{E} = d\Phi/dt = BL(dx/dt)$

For correctly using the expression for emf to obtain an expression for the current — 1 point

$$I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$$

$$Ma = Mg - B\frac{BLv}{R}L$$

$$a = g - \frac{B^2L^2v}{MR}$$

vriting the acceleration as dv/dt — 1 point

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

;CORING GUIDELINES

Question 3 (continued)

Distribution of points

2 points

For setting the net force at terminal velocity equal to zero — 1 point

$$0 = Mg - \frac{B^2L^2v_T}{R}$$

$$\frac{B^2L^2v_T}{R} = Mg$$

ın answer consistent with part (c) — 1 point

$$v_T = \frac{MgR}{B^2L^2}$$

(1 point

For a correct or consistent substitution in an appropriate power equation — 1 point

$P = I^2R$ or $V = IR$

$$\frac{BLv_T}{R} = I$$

$$P = \frac{B^2L^2{v_T}^2}{R}$$

Alternate Solution — *Alternate points*

For a correct or consistent substitution in an appropriate power equation — *1 point*

$$P = I^2R$$

$$Mg = BIL$$

$$I = \frac{Mg}{BL}$$

$$P = \frac{M^2g^2R}{B^2L^2}$$

SCORING GUIDELINES

Question 3 (continued)

Distribution of points

) 3 points

Using the equation from part (c)

$$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$$

$$\frac{dv}{dt} = \frac{-B^2L^2}{MR}\left(v - \frac{MRg}{B^2L^2}\right)$$

For attempting separation of variables — 1 point

$$\frac{dv}{v - \frac{MRg}{B^2L^2}} = \frac{-B^2L^2}{MR}dt$$

For attempting to integrate with the correct limits or the correct constant of integration — 1 point

$$\int_{v'=0}^{v'=v(t)} \frac{1}{\left(v' - \frac{MRg}{B^2L^2}\right)}\,dv' = \int_{t'=0}^{t'=t} \frac{-B^2L^2}{MR}dt'$$

$$\ln\left[v' - \frac{MRg}{B^2L^2}\right]_{v'=0}^{v'=v(t)} = \frac{-B^2L^2}{MR}t$$

$$\ln\left(\frac{v(t) - \frac{MRg}{B^2L^2}}{\frac{MRg}{B^2L^2}}\right) = \frac{-B^2L^2}{MR}t$$

For a correct answer — 1 point

$$v(t) = \frac{MRg}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{MR}t}\right)$$

ng a trial solution in the differential equation and verifying its correctness is also acceptable.