

Question 2: Translation Between Representations (TBR)

12 points

A	For drawing a bar at $\frac{1}{4}T$	Point A1
	For drawing a bar at $\frac{1}{4}T$ that has a height of 3 units	Point A2
	For indicating that $ \mathcal{E} $ is zero at times $t = 0$ and $\frac{1}{2}T$	Point A3

Example Response

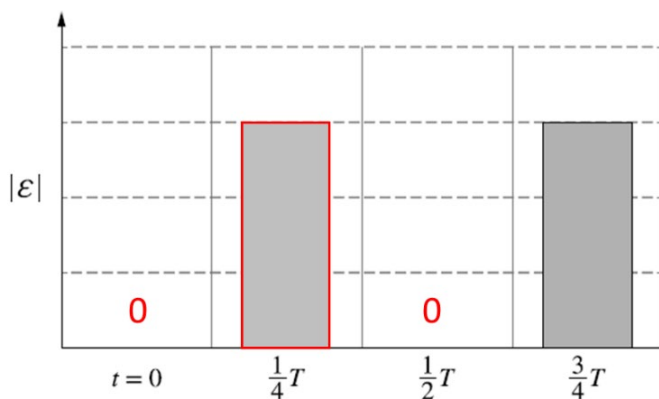


Figure 2

B	For a multistep derivation that includes the equation $\mathcal{E} = -\frac{d\Phi_B}{dt}$	Point B1
	Scoring Note: The negative sign does not have to be present in the expression for this point to be earned.	
	For a correct expression for the induced emf (e.g., $\mathcal{E} = BA\omega \sin(\omega t)$)	Point B2
	For using Ohm's law to relate current and emf (e.g., $I = \frac{\mathcal{E}}{R}$)	Point B3
	For a correct expression for the absolute value of the maximum induced current (e.g., $I = \frac{BA\omega}{R}$)	Point B4

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}[BA \cos(\omega t)] = BA\omega \sin(\omega t)$$

$$\mathcal{E}_{\max} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I_{\max} = \frac{\mathcal{E}_{\max}}{R} = \frac{BA\omega}{R}$$

C	For a curve that is approximately sinusoidal	Point C1
	For a curve that shows exactly two cycles	Point C2
	For a curve that starts at the origin and has equal maximum values and equal minimum values	Point C3

Scoring Note: A curve that starts and ends at $P = 0$ and has a single maximum can earn this point.

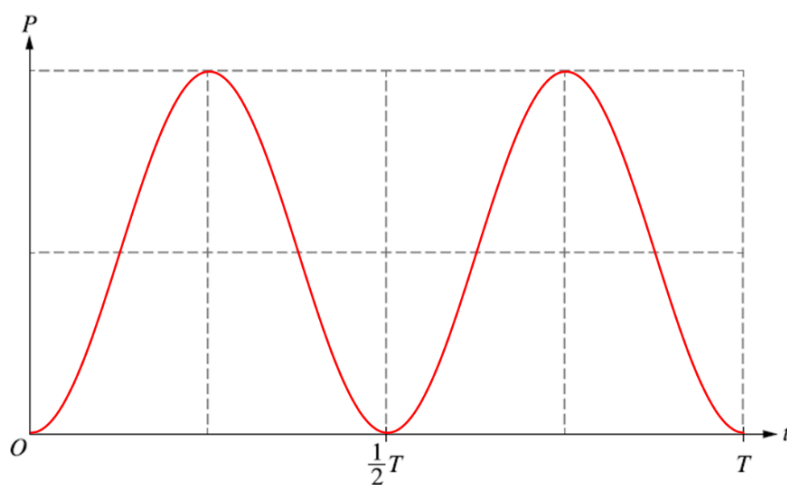
Example Response

Figure 3

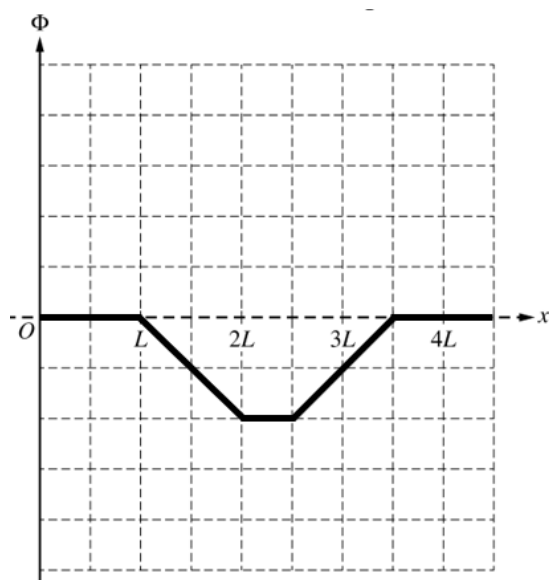
D	For correctly indicating whether the representations are consistent in parts A and C	Point D1
	For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to $\mathcal{E}^2$	Point D2

Example Response

Yes, part C is consistent with part A. P is proportional to $\mathcal{E}^2$. When $|\mathcal{E}|$ is maximum, P is maximum.

Question 3: Free-Response Question**15 points**

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|------------|---|----------------|
| (a) | For a sketch that indicates that the magnetic flux is zero from $x = 0$ to $x = L$ and for the region $x > 3.5L$ | 1 point |
| | For a sketch that indicates that the absolute value of the magnetic flux increases with a constant slope in the region $L < x < 2L$ | 1 point |
| | For a sketch that indicates that the magnetic flux is constant and nonzero in the region $2L < x < 2.5L$ | 1 point |
| | For a sketch that indicates that the absolute value of the magnetic flux decreases from a nonzero value at $x = 2.5L$ to zero at $x = 3.5L$ | 1 point |

Example Response

Scoring Note: A sketch that is reflected across the horizontal axis can earn the four points.

Scoring Note: The absolute values of the slopes in regions $L < x < 2L$ and $2.5L < x < 3.5L$ are not considered for earning points in the response in part (a).

- | | | | |
|---------------|--|---------------------------|-----------------|
| | | Total for part (a) | 4 points |
| (b)(i) | For selecting Counterclockwise with an attempt at a relevant justification | 1 point | |
| | For indicating the magnetic flux through the loop is increasing in the $-z$ -direction | 1 point | |

OR

For indicating the magnetic field due to the induced current will be directed in the $+z$ -direction

Example Response

Counterclockwise. The magnetic flux through the loop is increasing in the $-z$ -direction. Therefore, a magnetic field is induced by the current in the loop to oppose the increasing magnetic flux. To establish this field, the current must be counterclockwise.

(b)(ii) For a multistep derivation that includes Faraday's law 1 point

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$$

For indicating $\frac{dA}{dt}$ is $2Lv$ 1 point

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$\mathcal{E} = B\frac{dA}{dt} = 2BLv$$

For using Ohm's law, resulting in an expression for I_R that is consistent with the expression determined for the emf $\mathcal{E}$ 1 point

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$I_R = \frac{2BLv}{R}$$

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)$$

$$\mathcal{E} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(-B(2L)x)$$

$$\mathcal{E} = 2BL\frac{dx}{dt}$$

$$\mathcal{E} = 2BLv$$

$$I = \frac{\Delta V}{R}$$

$$I_R = \frac{\mathcal{E}}{R}$$

$$I_R = \frac{2BLv}{R}$$

(b)(iii) For using a correct general expression for P **1 point**

Example Responses

$$P = I^2 R \quad \text{OR} \quad P = \frac{(\Delta V)^2}{R} \quad \text{OR} \quad P = I \Delta V$$

For an expression for P that is consistent with the response in part (b)(ii) that is in terms of the provided quantities only **1 point**

Example Response

$$P = \frac{4B^2 L^2 v^2}{R}$$

Example Solution

$$P = I^2 R$$

$$P = \frac{(2BLv)^2}{R}$$

$$P = \frac{4B^2 L^2 v^2}{R}$$

Total for part (b) 7 points

(c) For selecting $E_{\text{new}} < E_{\text{original}}$ with an attempt at a relevant justification **1 point**

For indicating one of the following: **1 point**

- The change in magnetic flux is greater for the original scenario.
- The induced current occurs for a greater amount of time in the original scenario.
- The induced emf occurs for a greater amount of time in the original scenario.

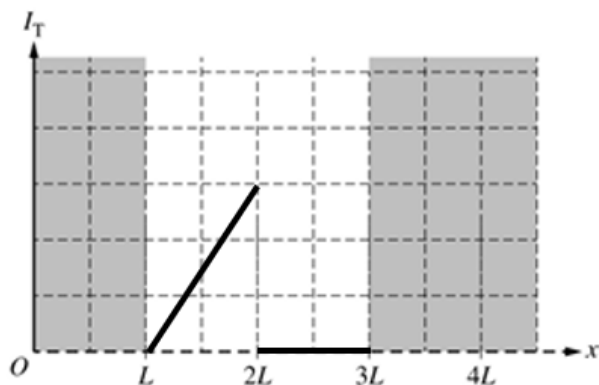
Example Response

The change in magnetic flux is greater in the original scenario, which produces an emf and current for a longer time. Therefore, $E_{\text{new}} < E_{\text{original}}$.

Total for part (c) 2 points

- (d) For a sketch that has an absolute value that only increases from $x = L$ to $x = 2L$ **1 point**
 For a sketch that is zero from $2L$ to $3L$ **1 point**

Example Response



Scoring Note: A response that is reflected across the horizontal axis earns both points.

Scoring Note: Any portion of the graph before $x = L$ and after $x = 3L$ are not scored.

Total for part (d) 2 points

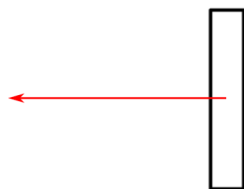
Total for question 3 15 points

Question 2: Free-Response Question

15 points

- (a) For drawing an arrow pointing to the left with no extraneous arrows **1 point**

Example Response



Total for part (a) 1 point

(b)(i) For using Faraday's law to calculate the value of the induced emf **1 point**

Scoring Note: This point may be earned without the negative sign or a numerical answer.

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(BLx)}{dt}$$

For a correct substitution of v for $\frac{dx}{dt}$ **1 point**

Scoring Note: A student can earn points 1 and 2 of part (b)(i) by starting with the expression

$$\mathcal{E} = BLv.$$

Example Response

$$\mathcal{E} = -BL\left(\frac{dx}{dt}\right)$$

$$BL\frac{dx}{dt} = BLv$$

For substituting the correct resistance into an equation for Ohm's law to solve for the current **1 point**

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(BLx)}{dt}$$

$$\mathcal{E} = -BL\frac{dx}{dt}$$

$$\mathcal{E} = -BLv$$

$$\mathcal{E} = -(0.50 \text{ T})(0.40 \text{ m})(2.5 \text{ m/s})$$

$$\mathcal{E} = -0.50 \text{ V}$$

$$I = \frac{\Delta V}{R}$$

$$I = \frac{|\mathcal{E}|}{R}$$

$$I = \frac{|-0.50 \text{ V}|}{0.30 \Omega} = 1.7 \text{ A}$$

(b)(ii)	For substituting the current or an expression for the current obtained from part (b)(i) into an appropriate equation that is related to the magnetic force exerted on the bar	1 point
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Example Responses

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$F = ILB$$

$$F = (1.7 \text{ A})(0.4 \text{ m})(0.5 \text{ T})$$

$$F = 0.33 \text{ N}$$

OR

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$F = ILB$$

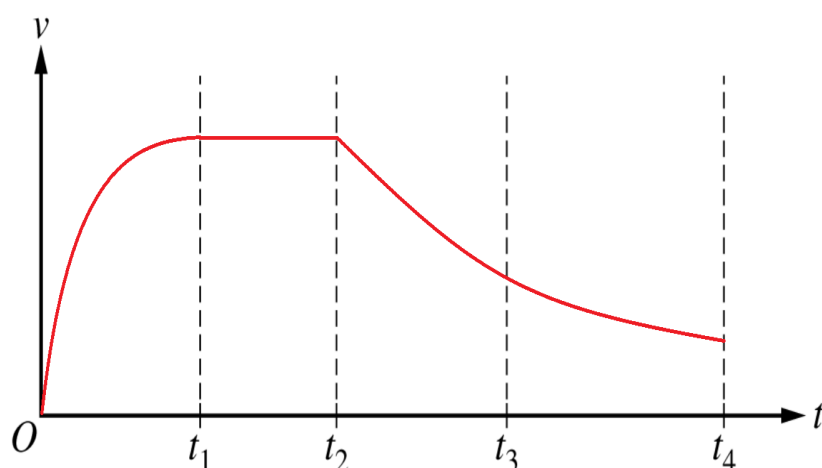
$$F = \left(\frac{BLv}{R} \right) LB$$

$$F = \frac{B^2 L^2 v}{R}$$

$$F = \frac{(0.5 \text{ T})^2 (0.4 \text{ m})^2 (2.5 \text{ m/s})}{0.3 \Omega}$$

$$F = 0.33 \text{ N}$$

Total for part (b)		4 points
(c)	For drawing a curve that starts at the origin, is increasing, and is concave down from $t = 0$ to t_1	1 point
	For drawing a horizontal line from t_1 to t_2	1 point
	For drawing a curve that is decreasing and concave up from t_2 to t_4	1 point
	For drawing a curve that is differentiable at t_3 with a nonzero slope	1 point

Example Response**Total Points for part (c) 4 points**

(d)(i)	For a correct answer with units ($0.15\ \Omega$)	1 point
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Scoring Note: This point can be earned without supporting calculations.

Example Response

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_p} = \frac{1}{0.3\ \Omega} + \frac{1}{0.3\ \Omega}$$

$$\frac{1}{R_p} = \frac{2}{0.3\ \Omega}$$

$$R_p = 0.15\ \Omega$$

(d)(ii)	For a statement that correctly describes the inverse relationship between resistance and current (e.g., as resistance decreases current increases)	1 point
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For a statement that describes the direct relation between current and force (e.g., as current increases force increases)	1 point
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For a statement that describes the direct relation between force and acceleration (e.g., as force increases acceleration increases)	1 point
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Scoring Note: Full credit can be earned with a justification that is consistent with the resistance calculated in part (d)(i).

Example Response

Since there is less resistance in the new circuit, there will be more current in the new circuit, so a larger force on the bar. Thus, since the force on the bar is larger, the new acceleration is greater than the original acceleration.

Total for part (d)	4 points
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- (e) For correctly indicating **one** of the following, with an attempt at a relevant justification: **1 point**
- Decreasing B
 - Decreasing L
 - Increasing m

For correctly justifying the identified modification that will result in a smaller induced potential difference across the original resistor **1 point**

Example Responses

The potential difference due to the induced emf across the original resistor is described by the equation $\mathcal{E} = -BLv$. Induced potential difference $\mathcal{E}$ is proportional to B . Therefore, if the magnitude of the magnetic field is smaller than $B = 0.5 \text{ T}$ in the new scenario compared to the original scenario, $\mathcal{E}$ would be smaller.

OR

The potential difference due to the induced emf across the original resistor is described by the equation $\mathcal{E} = -BLv$. The induced potential difference $\mathcal{E}$ is proportional to L , which represents the distance the conducting rails are separated. Therefore, if L is smaller than $L = 0.4 \text{ m}$, $\mathcal{E}$ would be smaller.

OR

The potential difference due to the induced emf across the original resistor is described by the equation $\mathcal{E} = -BLv$. If the mass of the bar is greater, the velocity entering the magnetic field is less. The induced potential difference $\mathcal{E}$ is proportional to v . Therefore, a smaller v due to a greater mass will induce a smaller $\mathcal{E}$.

Total for part (e) 2 points

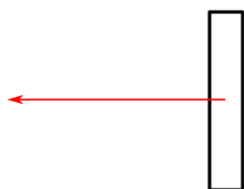
Total for question 2 15 points

Question 2: Free-Response Question

15 points

- (a) For drawing an arrow pointing to the left with no extraneous arrows **1 point**

Example Response



Total for part (a) 1 point

(b)(i) For using Faraday's law to calculate the value of the induced emf**1 point**

Scoring Note: This point may be earned without the negative sign or a numerical answer.**Example Response**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$

For a correct substitution of v for $\frac{dx}{dt}$ **1 point**

Scoring Note: A student can earn points 1 and 2 of part (b)(i) by starting with the expression $\mathcal{E} = B_1 dv$.**Example Response**

$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$

$$\mathcal{E} = -B_1 dv$$

For substituting the correct resistance into an equation for Ohm's law to solve for the current

1 point

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d(B_1 dx)}{dt}$$

$$\mathcal{E} = -B_1 d\left(\frac{dx}{dt}\right)$$

$$\mathcal{E} = -B_1 dv$$

$$\mathcal{E} = -(0.40 \text{ T})(0.30 \text{ m})(2.5 \text{ m/s})$$

$$\mathcal{E} = -0.30 \text{ V}$$

$$\Delta V = IR$$

$$I = \frac{\Delta V}{R}$$

$$I = \frac{|\mathcal{E}|}{R}$$

$$I = \frac{|-0.30 \text{ V}|}{0.20 \Omega} = 1.5 \text{ A}$$

(b)(ii)	For substituting the current or an expression for the current obtained from part (b)(i) into an appropriate equation for calculating the magnetic force	1 point
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Example Responses

$$\vec{F} = \int Id\vec{\ell} \times \vec{B}$$

$$F = IdB_1$$

$$F = (1.5 \text{ A})(0.3 \text{ m})(0.4 \text{ T})$$

$$F = 0.18 \text{ N}$$

OR

$$\vec{F} = \int Id\vec{\ell} \times \vec{B}$$

$$F = IdB_1$$

$$F = \left(\frac{B_1 dv}{R} \right) dB$$

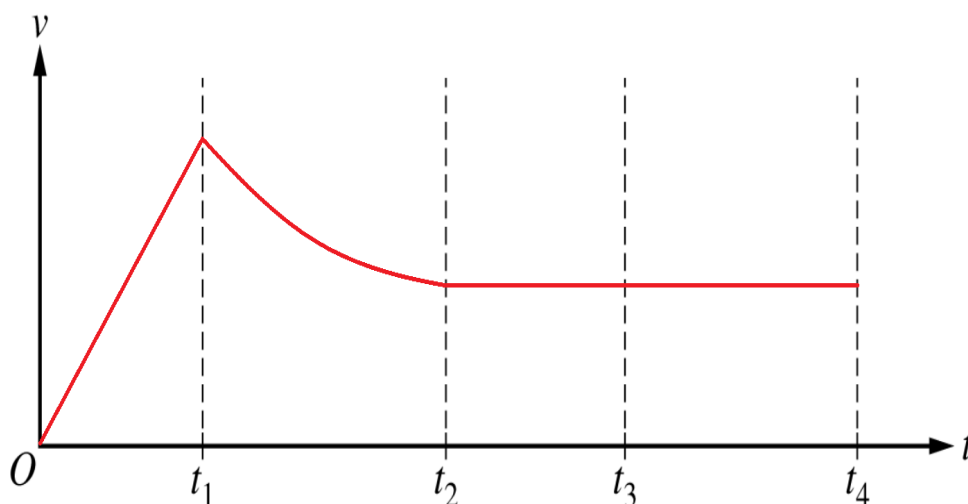
$$F = \frac{B_1^2 d^2 v}{R}$$

$$F = \frac{(0.4 \text{ T})^2 (0.3 \text{ m})^2 (2.5 \text{ m/s})}{0.2 \Omega}$$

$$F = 0.18 \text{ N}$$

Total for part (b) 4 points

(c)	For drawing a straight line with a positive slope that starts at the origin from $t = 0$ to t_1	1 point
	For drawing a curve that is decreasing and concave up from t_1 to t_2	1 point
	For drawing a nonzero horizontal line from t_2 to t_3	1 point
	For drawing a nonzero horizontal line from t_3 to t_4	1 point

Example Response**Total Points for part (c) 4 points**

(d)(i)	For a correct answer with units ($0.4 \, \Omega$)	1 point
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Scoring Note: This point can be earned without supporting calculations.

Example Response

$$R_s = \sum_i R_i$$

$$R_s = (0.2 \, \Omega) + (0.2 \, \Omega)$$

$$R_s = 0.4 \, \Omega$$

(d)(ii)	For a statement that correctly describes the inverse relation between resistance and current (e.g., as resistance increases, current decreases)	1 point
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	For a statement that correctly describes the direct relation between current and force (e.g., as current decreases, magnetic force decreases)	1 point
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	For a statement that correctly describes the direct relation between force and acceleration (e.g., as force decreases acceleration decreases)	1 point
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Scoring Note: Full credit can be earned with a justification that is consistent with the resistance calculated in part (d)(i).

Example Response

The new acceleration a_{new} is less than a_{original} . Greater resistance of the bar causes the current to be less than in the original scenario. Less current causes the magnetic force $F = IdB$ on the bar to also be less than the original. By Newton's second law $F = ma$ so less force on a bar of the same mass results in less acceleration.

	Total for part (d) 4 points
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- (e) For correctly indicating **one** of the following, with an attempt at a relevant justification: **1 point**
- Increasing H
 - Increasing B_1
 - Increasing d

For correctly justifying the identified modification that will result in a larger induced current in the new bar at position x_B **1 point**

Example Responses

Increasing H will increase the induced current. If the ramp is higher, then the potential energy is greater and this results in greater kinetic energy and greater velocity at the bottom of the ramp. A greater velocity causes a greater rate of change in flux as the bar moves through the field. By Faraday's law the emf is greater and therefore also the current.

OR

Increasing B_1 will increase the induced current. If the magnetic field is stronger this increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf is then greater and therefore also the current.

OR

Increasing d will increase the induced current. A larger width results in a greater area encompassed by the circuit. Greater area increases the flux through the circuit and therefore also the rate of change in the flux. By Faraday's law the emf is then greater and therefore also the current.

Total for part (e) 2 points

Total for question 2 15 points

Question 3: Free-Response Question**15 points**

- (a) For a loop rule expression that includes terms for the equivalent resistance $2R$ and the potential difference across the battery **1 point**

For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors **1 point**

Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

$$\mathcal{E} - \frac{Q}{C} - 2R \frac{dQ}{dt} = 0$$

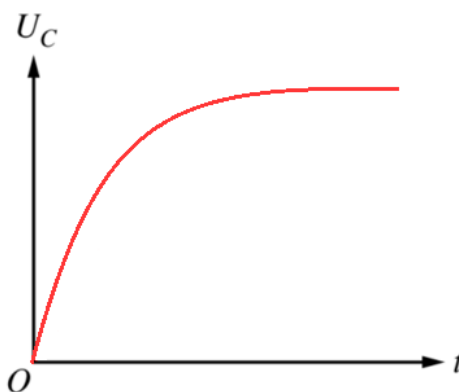
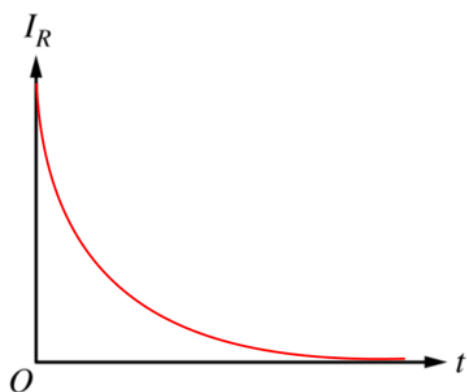
Total for part (a) 2 points

- (b) For sketching a concave up and decreasing curve on the graph of I_R as a function of t **1 point**

For sketching a concave down and increasing curve on the graph of U_C as a function of t **1 point**

For sketching both curves that approach a slope of zero as time increases **1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.

Example Response**Total for part (b) 3 points**

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- (c)(i)** For a correct justification that could include one of the following: **1 point**
- An indication that the current is upwards that includes a statement that indicates that positive charge had accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
 - An indication that the current is upwards that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A
-

Example Responses

When the switch is closed at time t_1 , positive charge has built up on the top plate of the capacitor. This positive charge on the top plate pushes charge up through Resistor 1 and down through Capacitor 2 to charge Capacitor 2.

OR

After a long-time the top plate of Capacitor 1 is at a high potential due to its being charged by the battery. When the switch is closed at time t_1 , the resulting current is up through Resistor 1 as the current goes from high potential on the top plate to low potential clockwise around the circuit through Capacitor 2.

- (c)(ii)** For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ **1 point**
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Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has the same capacitance of Capacitor 1. Therefore, Capacitor 2 stores the same charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores half of the original charge equal to $\frac{Q_0}{2}$.

- (c)(iii) For indicating that the total energy dissipated by Resistor 1 is the difference between the initial electric potential energy of the system at time $t = t_1$ and the final electric potential energy of the system after the new steady state conditions have been reached **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$

- For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store electric potential energy after the new steady-state conditions have been reached, or an alternate response that is consistent with part (c)(ii) **1 point**

Example Response

$$U_{0C} = U_{01} \text{ AND } U_C = U_1 + U_2$$

- For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached based on part (c)(ii) **1 point**

Example Solution

$$\Delta E_R = U_C - U_{0C}$$

$$\Delta E_R = \left(\frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 \right) - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = \frac{Q_0^2}{4C} - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = -\frac{Q_0^2}{4C}$$

Total for part (c) 5 points

- (d) For the correct expression for charge on the top plate of Capacitor 2 $\left(Q_2 = \frac{2Q_0}{3} \right)$ **1 point**

Example Response

$$C_1 = C \text{ and } C_2 = \kappa C = 2C$$

$$\Delta V_{C1} = \Delta V_{C2}$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = \frac{1}{2} Q_2$$

$$Q_1 + Q_2 = Q_0$$

$$\frac{1}{2} Q_2 + Q_2 = Q_0$$

$$Q_2 = \frac{2}{3} Q_0$$

Scoring Note: This point can be earned without supporting calculations.

Total for part (d) 1 point

- (e)(i) For a loop rule equation that includes terms for the potential difference across Resistor 2 and the potential difference across Capacitor 2 with the dielectric **1 point**

Example Response

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

For correct substitution of IR for the potential difference across Resistor 2 **1 point**

Example Response

$$\Delta V_{R,2} = IR$$

For correct substitutions of $2C$ for the new capacitance of Capacitor 2 with the dielectric inserted and of the charge consistent with part (d) into the expression for the potential difference across the capacitor **1 point**

Example Response

$$\Delta V_{C,2} = \frac{Q_2}{C_2}$$

$$\Delta V_{C,2} = \left(\frac{2Q_0}{3} \right) \left(\frac{1}{2C} \right)$$

Example Solution

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

$$IR = \left(\frac{2Q_0}{3} \right) \left(\frac{1}{2C} \right)$$

$$IR = \frac{Q_0}{3C}$$

$$I = \frac{Q_0}{3RC}$$

- (e)(ii) For indicating that the current is zero **1 point**

Total for part (e) 4 points

Total for question 3 15 points

Question 3: Free-Response Question**15 points**

- (a) For selecting “Out of the page” and an attempt at a relevant justification **1 point**

For a justification that correctly relates how the changing current in the long wire changes the flux through the loop with respect to time **1 point**

For indicating the induced current will oppose the change in magnetic flux **1 point**

Example Response

Because the current in the straight wire is decreasing, the magnetic field, which is originally pointing out of the page, is decreasing. Hence, the induced current produces a field that is directed out of the page to compensate for the decreasing flux.

Total for part (a) 3 points

- (b) For using an appropriate integral equation, with a substitution of an expression for magnetic field, to calculate magnetic flux **1 point**

Example Response

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

For writing a correct equation for the magnetic field as a function of distance from the wire **1 point**

Example Response

$$B = \frac{\mu_0 I}{2\pi r}$$

For substituting the value of $t = 3$ s to find the electric current **1 point**

Example Response

$$I(t) = C - Dt$$

$$I(3 \text{ s}) = 10 \text{ A} - (2 \text{ A/s})(3 \text{ s})$$

$$I(3 \text{ s}) = 4 \text{ A}$$

For integrating B with correct limits and a correct substitution for dA , to determine the total flux through the loop **1 point**

Example Response

$$\Phi_B = \int_d^{d+W} \frac{\mu_0 IL}{2\pi r} dr$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln[r]_d^{d+W}$$

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \ln\left[\frac{d+W}{d}\right]$$

$$\Phi_B = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(4 \text{ A})(0.04 \text{ m})}{2\pi} \ln\left[\frac{0.03 \text{ m}}{0.01 \text{ m}}\right] = 3.52 \times 10^{-8} \text{ T}\cdot\text{m}^2$$

Total for part (b) 4 points

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- | | | |
|------------|--|----------------|
| (c) | For using Faraday's law to determine the emf across the light bulb | 1 point |
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Example Response

$$\varepsilon = \left| \frac{d\Phi}{dt} \right|$$

$$\varepsilon = \frac{d}{dt} \left(\frac{\mu_0 (C - Dt)L}{2\pi} \ln \left[\frac{d + W}{d} \right] \right)$$

$$\varepsilon = \left(\frac{\mu_0 DL}{2\pi} \ln \left[\frac{d + W}{d} \right] \right)$$

-
- | | | |
|--|--|----------------|
| | For using Ohm's law to find the current in the light bulb consistent with the emf determined from the previous point | 1 point |
|--|--|----------------|
-

Example Response

$$I = \frac{\varepsilon}{R}$$

$$I = \left(\frac{\mu_0 DL}{2\pi R} \ln \left[\frac{d + W}{d} \right] \right)$$

-
- | | | |
|--|--|----------------|
| | For correct substitutions of the values of $\frac{dI}{dt}$ and R | 1 point |
|--|--|----------------|
-

Example Response

$$I = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(2.0 \text{ A/s})(0.04 \text{ m})}{(2\pi \cdot 10\Omega)} \ln \left[\frac{0.03 \text{ m}}{0.01 \text{ m}} \right]$$

$$I = 1.8 \times 10^{-9} \text{ A}$$

Total for part (c) 3 points

- | | | |
|------------|---|----------------|
| (d) | For selecting: "The current in the long wire changes at a faster rate than expected." | 1 point |
|------------|---|----------------|
-
- | | | |
|--|--|----------------|
| | For correctly justifying the selection | 1 point |
|--|--|----------------|
-

Scoring Note: A response cannot earn this point if the incorrect selection is chosen.

Example Response

If the current in the wire changes at a faster rate, there will be a greater rate of change of magnetic flux. So the induced emf and current will be higher.

Total for part (d) 2 points

(e)	For selecting “ $I_2 < I_1$ ” with an attempt at a relevant justification	1 point
	For indicating the total flux in the loop is less in the new orientation	1 point
	For correctly relating the rate of change of the flux to the total flux inside the loop	1 point

Example Response

With the new orientation, some parts of the rectangle are further away from the straight wire, which means that the magnetic flux through the rectangle will be less. The rate of change of the flux has the same dependence on distance and will also decrease, resulting in a smaller current.

Total for part (e) 3 points**Total for question 3 15 points**

Question 3: Free-Response Question**15 points**

(a) For selecting “Counterclockwise” and an attempt at a relevant justification **1 point**

For a justification that correctly relates how the changing current in the solenoid changes the flux through the loop with respect to time **1 point**

For a justification that has a correct relationship between the change in magnetic flux through the loop of wire and how the current in the loop changes to oppose the change in magnetic flux **1 point**

Example Response

In the end view, the magnetic field due to the solenoid is directed into the page. The current in the solenoid is increasing; thus, the magnetic flux is increasing. According to Lenz’s law, because the magnetic flux is increasing and into the page, the current in the loop must create a magnetic field directed out of the page; thus, the current in the loop must be counterclockwise to counter the change in magnetic flux.

Total for part (a) 3 points

(b) For using an appropriate equation to calculate emf in the loop **1 point**

Example Response

$$\varepsilon = \left| \frac{d\Phi}{dt} \right| = \frac{d(BA)}{dt} = A \frac{dB}{dt}$$

For correctly substituting the magnetic field for a solenoid into above equation **1 point**

Example Response

$$\varepsilon = A \frac{d(\mu_0 n I)}{dt} = \mu_0 n A \frac{dI}{dt}$$

For using Ohm’s law to calculate the current in the loop **1 point**

Example Response

$$I_{\text{loop}} = \frac{\varepsilon}{R} = \frac{\mu_0 n A}{R} \frac{dI}{dt}$$

For correctly substituting into the previous equation for the induced current in the loop **1 point**

Example Response

$$I_{\text{loop}} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}) \left(\frac{500 \text{ turns}}{0.25 \text{ m}} \right) ((\pi)(0.10 \text{ m})^2)}{(3 \Omega)} (5.0 \text{ A/s}) = 1.3 \times 10^{-4} \text{ A}$$

Total for part (b) 4 points

- (c) For using an appropriate expression for the power for a constant current through a resistor **1 point**

Example Response

$$P = I_{\text{loop}}^2 R$$

- For a correct substitution consistent with (b) into a correct expression for electrical power **1 point**

Example Response

$$P = I_{\text{loop}}^2 R = \left(\frac{\mu_0 n A}{R} \frac{dI}{dt} \right)^2 R = (1.3 \times 10^{-4} \text{ A})^2 (3.0 \, \Omega) = 5.2 \times 10^{-8} \text{ W}$$

- For correctly substituting the power dissipated by the loop of wire into a correct energy equation **1 point**

Example Response

$$P = \frac{dE}{dt} = \frac{\Delta E}{\Delta t} \therefore \Delta E = P \Delta t$$

$$\Delta E = (5.2 \times 10^{-8} \text{ W})(2.0 \text{ s}) = 1.0 \times 10^{-7} \text{ J}$$

Total for part (c) 3 points

- (d) For selecting “The plane of the loop is not perpendicular to the axis of the solenoid.” **1 point**

- For correctly justifying the selection **1 point**

Scoring Note: A response cannot earn this point if the incorrect selection is chosen.

Example Response

If the plane of the loop is not perpendicular to the axis of the solenoid it is not perpendicular to the magnetic field. Therefore, the magnetic flux and emf will be less, so the current will be less.

Total for part (d) 2 points

- (e) For selecting “ $\frac{I_2}{I_1} = 2$ ” with an attempt at a relevant justification **1 point**

Scoring Note: The response can earn this point if selecting “ $\frac{I_2}{I_1} > 2$ ” and only referencing the increase in flux through the loop in the justification.

- For a statement that indicates that the resistance of the second loop is double the resistance of the original loop **1 point**

- For a statement that indicates that the new emf is four times greater than the original emf due to the increased flux through the loop resulting from the quadrupled area **1 point**

Example Response

The magnetic flux quadruples. This is because of the increase in area of the loop, which quadruples the emf. The resistance of the loop doubles because the length of the wire doubles. Therefore, $\frac{I_2}{I_1} = 2$.

$$\varepsilon = \mu_0 n (A_{\text{loop}}) \frac{dI}{dt} = \mu_0 n (\pi r^2) \frac{dI}{dt}$$

The radius of the loop doubles. Therefore, the area of the loop quadruples.

$$R = \rho \frac{2\pi r}{A_{\text{wire}}}$$

The radius of the loop doubles. Therefore, the resistance of the loop doubles.

$$\frac{I_2 = \frac{\varepsilon}{R} = \frac{\mu_0 n (4) A}{(2) R} \frac{dI}{dt}}{I_1 = \frac{\varepsilon}{R} = \frac{\mu_0 n A}{R} \frac{dI}{dt}} = 2$$

Total for part (e) 3 points

Total for question 3 15 points

Question 3: Free-Response Question

15 points

- (a) For using Faraday's law to relate emf to change in flux 1 point

$$\mathcal{E} = \left| \frac{d\Phi}{dt} \right| = \left| \frac{d(B \cdot A)}{dt} \right|$$

For a correct expression for the emf 1 point

$$\mathcal{E} = \left| A \frac{dB}{dt} \right| = \left| A \frac{d}{dt} \left(\frac{\beta}{t} \right) \right| = \left| -\frac{A\beta}{t^2} \right| = \frac{A\beta}{t^2}$$

For correctly substituting into Ohm's law 1 point

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R} = \frac{A\beta}{Rt^2}$$

Scoring note: Full credit is earned if the negative sign is included.

Total for part (a) 3 points

- (b) For using a correct equation to calculate the energy dissipated in the ring 1 point

$$E = \int P dt = \int I^2 R dt$$

For correctly substituting into the equation above 1 point

$$E = \int \left(\frac{A\beta}{Rt^2} \right)^2 R dt = \int \frac{A^2 \beta^2}{R^2 t^4} R dt$$

For indicating the correct limits or constant of integration 1 point

$$E = \frac{A^2 \beta^2}{R} \int_1^2 \frac{1}{t^4} dt$$

For a correct answer with units 1 point

$$E = \frac{A^2 \beta^2}{R} \left[-\frac{1}{3t^3} \right]_1^2 = \frac{(0.50 \text{ m}^2)^2 (0.50 \text{ T}\cdot\text{s})^2}{(3)(2.00 \Omega)} \left(-\frac{1}{2^3} - \left(-\frac{1}{1^3} \right) \right) = 0.0091 \text{ J}$$

Total for part (b) 4 points

- (c) For selecting "Less than" and attempting a relevant justification 1 point

For a correct justification 1 point

Example response for part (c)

After the ring rotates, fewer magnetic field lines pass through the ring, and the flux is less than before. Thus, the emf, current, and rate at which energy is dissipated is less.

Total for part (c) 2 points

- (d) For using a correct equation to calculate angular speed 1 point

$$\omega = \frac{2\pi}{T}$$

For substituting appropriate values from the graph in the correct equation

1 point

$$\omega = \frac{(2\pi)}{(2 \text{ s})} = 3.14 \frac{\text{rad}}{\text{s}}$$

Total for part (d) 2 points**(e)** For using a correct equation for solving for the emf in terms of the angular speed**1 point**

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d(B \cdot A)}{dt} = BA \frac{d}{dt}(\cos(\omega t)) = -BA\omega \sin(\omega t)$$

For correctly substituting into the correct equation

1 point

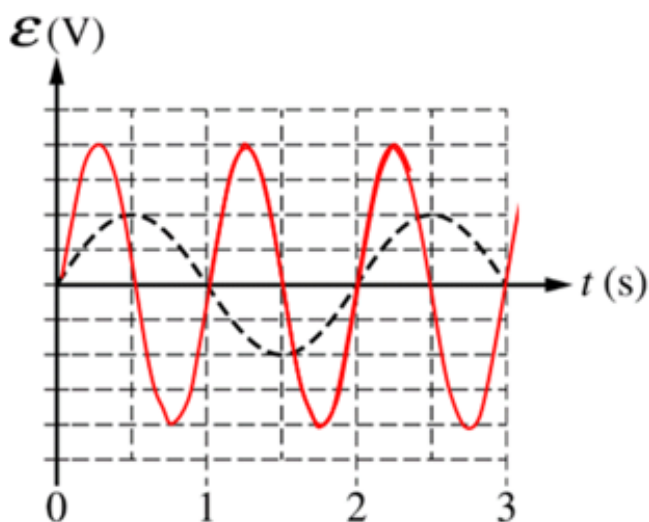
$$\mathcal{E}_{\text{MAX}} = BA\omega = (0.50 \text{ T})(0.50 \text{ m}^2)\left(3.14 \frac{\text{rad}}{\text{s}}\right) = 0.79 \text{ V}$$

Total for part (e) 2 points**(f)** For a sine curve with half the period of the dashed curve and a correct justification**1 point**

For a sine curve with twice the amplitude of the dashed curve and a correct justification

1 point**OR**

a sine curve with an amplitude consistent with work shown in part (e)

Example response for part (f)

Because the rotational speed is twice the original speed, then, according to the equation

$|\mathcal{E}| = B_0 A \omega \sin(\omega t)$ this will double the amplitude of the curve and cut the period in half.

Total for part (f) 2 points**Total for question 3 15 points**

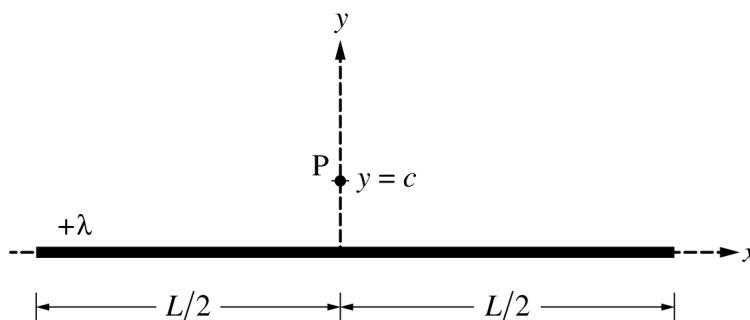
2019**AP[®]** **CollegeBoard**

AP[®] Physics C: Electricity and Magnetism

Scoring Guidelines Set 1

2019 SCORING GUIDELINES**General Notes About 2019 AP Physics Scoring Guidelines**

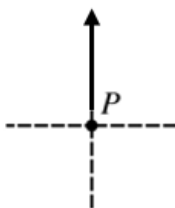
1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2019 SCORING GUIDELINES**Question 1****15 points**Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y -axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y -axis at $y = c$, where $L \gg c$.

- (a)
- LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

For drawing an arrow at point P that points upward

1 point

- LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P

1 point

Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P .

Note: Credit is earned if the student draws the correct surface on the figure.

2019 SCORING GUIDELINES

Question 1 (continued)

(a) continued

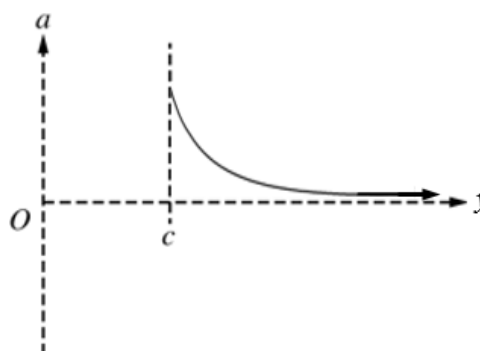
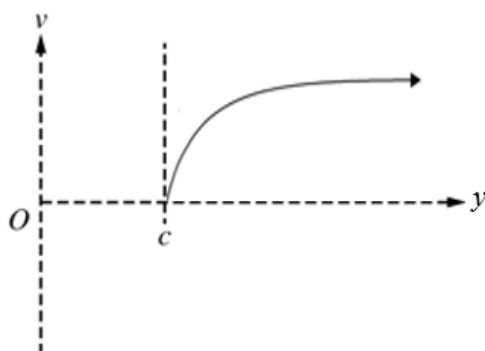
- iii. LO CNV-2.C, SP 5.A, 5.E
3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ , c , L , and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point P		1 point
$\frac{q_{enc}}{\epsilon_0} = \int E \cdot dA \therefore \frac{Q}{\epsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\epsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\epsilon_0 c} = \frac{2k\lambda}{c}$		

- (b) LO ACT-1.D, SP 3.C
2 points

A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

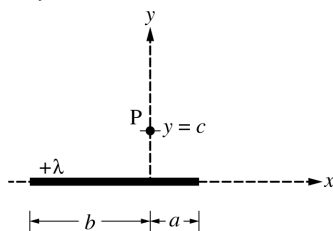


For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

2019 SCORING GUIDELINES

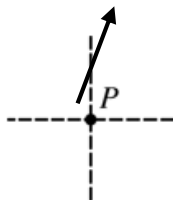
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y -axis is a , and the length of the cylinder to the left of the y -axis is b , where $a < b$.



- (c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points to the right	1 point
For drawing an arrow at point P that points up and to the right	1 point

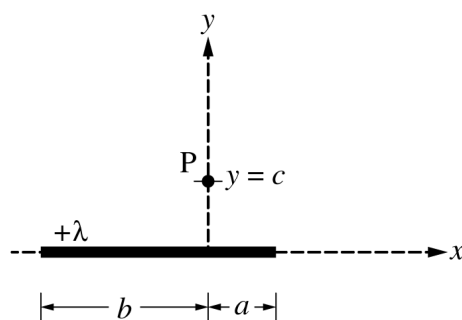
- (d) i. and ii. LO CNV-2.F, SP 7.A
1 point

Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P ?

____ Yes ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P . If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation	1 point
Claim: Select "No."	
Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric.	
Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.	

2019 SCORING GUIDELINES**Question 1 (continued)**

Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

$$\text{Horizontal component: } |E_x| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{x}{(c^2 + x^2)^{3/2}} dx$$

$$\text{Vertical component: } |E_y| = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^a \frac{y}{(c^2 + x^2)} dy$$

- (e)
- i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

_____ Horizontal component _____ Vertical component

For correctly selecting “Vertical component”	1 point
--	---------

2019 SCORING GUIDELINES**Question 1 (continued)**

(e) continued

- ii. LO CNV-3.A, SP 7.D
4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder	1 point
For an appropriate correction	1 point
Claim: Change dy to dx . Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.	
For indicating the power on the denominator term for the vertical component is incorrect	1 point
For an appropriate correction	1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.	

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.

CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.

CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

7.D: Provide reasoning to justify a claim using physical principles or laws.

2019 SCORING GUIDELINES

Question 2

15 points

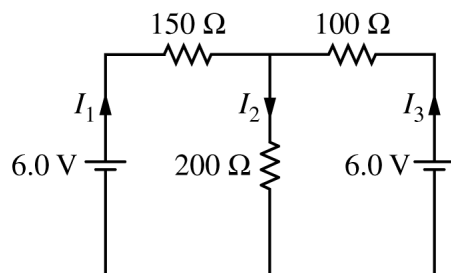


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1 , I_2 , and I_3 in each branch of the circuit are indicated.

- (a)
- LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
<u>Note:</u> Full credit is earned for two correct loop equations using loop currents.		

- LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the $200\ \Omega$ resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -.10$ $-I_3 - 2I_2 = -.06$		
<u>Note:</u> Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -.10 \therefore I_2 = 0.023\text{ A}$		

2019 SCORING GUIDELINES**Question 2 (continued)**

(a) continued

iii. LO CNV-5.A.a, SP 6.C

1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2 R = (0.023\ \text{A})^2 (200\ \Omega) \therefore P = 0.107\ \text{W}$		

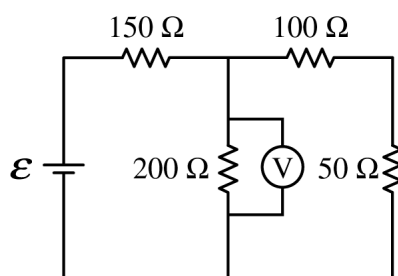


Figure 2

The two $6.0\ \text{V}$ batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is $4.4\ \text{V}$.

(b) LO CNV-6.C.a, SP 6.B, 6.C

2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \frac{V}{R} = \frac{(4.4\ \text{V})}{(100\ \Omega + 50\ \Omega)} \therefore I = 0.029\ \text{A}$		

2019 SCORING GUIDELINES**Question 2 (continued)**

- (c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

Calculate the voltage $\mathcal{E}$ of the battery.

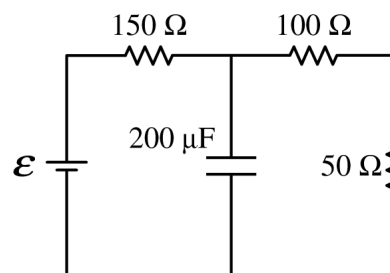
For using a correct equation to determine the current through the $150\ \Omega$ resistor	1 point
For correctly substituting the current from part (b) as I_3	1 point
$I_1 = I_2 + I_3 = \frac{(4.4\ \text{V})}{(200\ \Omega)} + 0.029\ \text{A} = 0.051\ \text{A}$	
For using a correct equation to determine the emf of the battery	1 point
$\mathcal{E} = I_1 R_1 + V_2 = (0.051\ \text{A})(150\ \Omega) + 4.4\ \text{V} = 12.1\ \text{V}$	
<i>Alternate Third Point</i>	
<i>For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery</i>	<i>1 point</i>
$R_T = 150\ \Omega + \frac{1}{\frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}} = 236\ \Omega$	
$\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$	
<i>Alternate Solution</i>	<i>Alternate Points</i>
<i>For using a correct equation to determine the equivalent resistance of the parallel resistors</i>	<i>1 point</i>
$\frac{1}{R_P} = \frac{1}{200\ \Omega} + \frac{1}{150\ \Omega} \therefore R_P = 86\ \Omega$	
<i>For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit</i>	<i>1 point</i>
$I_T = \frac{(4.4\ \text{V})}{(86\ \Omega)} = 0.051\ \text{A}$	
<i>For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery</i>	<i>1 point</i>
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$	
$\mathcal{E} = I_1 R_T = (0.051\ \text{A})(236\ \Omega) = 12.0\ \text{V}$	

2019 SCORING GUIDELINES

Question 2 (continued)

- (d)
i. LO CNV-7.B.a, SP 6.A, 6.C
2 points

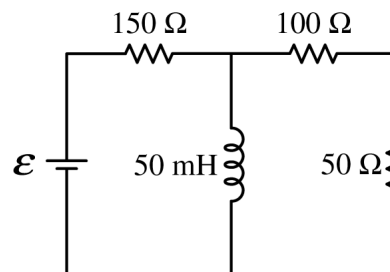
The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu\text{F}$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



For substituting the voltage consistent with part (c) into Ohm's law	1 point
For correctly calculating the equivalent resistance of the circuit	1 point
$I = \frac{\mathcal{E}}{R_{\text{tot}}} = \frac{(12.1\ \text{V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\ \text{mA}$	

- ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal $50\ \text{mH}$ inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?



____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification	1 point
For a correct justification	1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the $50\ \Omega$ resistor.	

2019 SCORING GUIDELINES**Question 2 (continued)****Learning Objectives**

CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.

CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.

CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.

CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

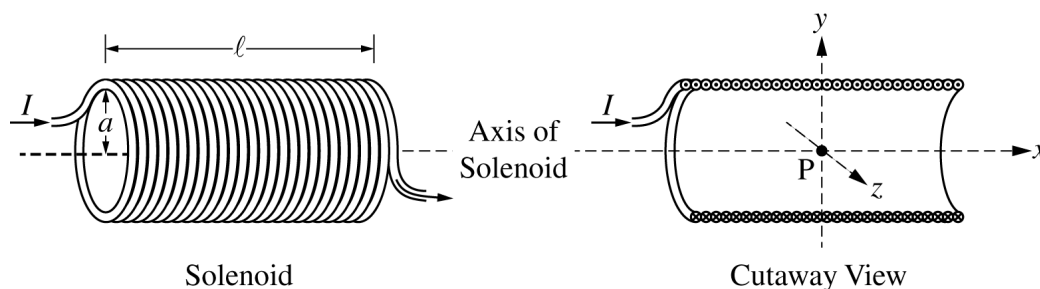
7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2019 SCORING GUIDELINES

Question 3

15 points



Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a , length ℓ , and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I , as shown in the figure on the left above. The x -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz -coordinate system, as shown in the cutaway view on the right above. Assume $\ell \gg a$.

- (a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

☐ + x -direction ☐ + y -direction ☐ + z -direction
☐ - x -direction ☐ - y -direction ☐ - z -direction

Justify your selection.

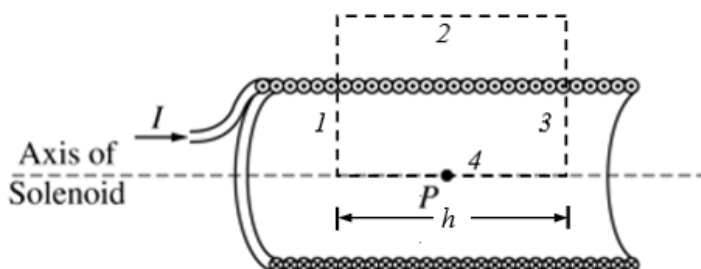
For choosing the “+ x -direction” and providing a justification		1 point
For a correct justification		1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the + x -direction.		
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the + x -direction.		

2019 SCORING GUIDELINES

Question 3 (continued)

- (b)
- i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid	1 point
--	---------

- ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I , ℓ , N , a , and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid	1 point
$\int \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{enc} \therefore \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_2 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_3 + \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_4 = \mu_0 I_{enc}$	
$\left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 + 0 + \left(- \left(\int \mathbf{B} \cdot d\boldsymbol{\ell} \right)_1 \right) + Bh = \mu_0 \frac{N}{\ell} hI$	
$Bh = \mu_0 \frac{N}{\ell} hI$	
For a correct answer	1 point
$B = \frac{\mu_0 NI}{\ell}$	

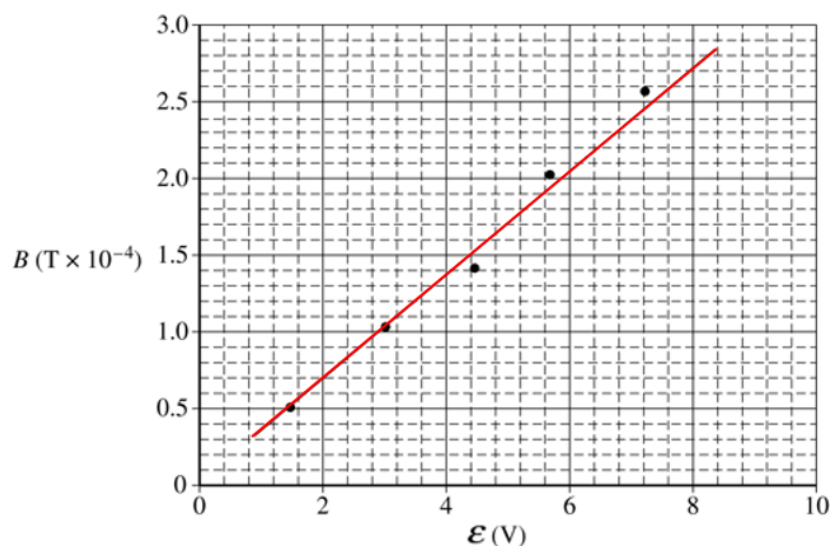
Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

2019 SCORING GUIDELINES**Question 3 (continued)**

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line

1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C

2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line

1 point

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(2.5 - 0.9)(\times 10^{-4} \text{ T})}{(6.4 - 2.0) \text{ V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5} \text{ T/V}$$

For correctly giving the expression that relates the slope to the resistance of the solenoid

1 point

$$B = \frac{\mu_0 N I}{\ell} = \frac{\mu_0 N \epsilon}{\ell R} \therefore \text{slope} = \frac{\mu_0 N}{\ell R_S} \therefore R_S = \frac{\mu_0 N}{\ell \times \text{slope}}$$

$$R_S = \frac{(4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A})(100 \text{ turns})}{(0.40 \text{ m})(3.6 \times 10^{-5} \text{ T/V})} = 8.7 \Omega$$

One of the students notes that the horizontal component of the magnetic field of Earth is $2.5 \times 10^{-5} \text{ T}$.

2019 SCORING GUIDELINES**Question 3 (continued)**

- (d)
- i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select “Yes”		
For a correct justification		1 point
Example: The horizontal component of Earth’s magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
<i>Alternate Solution</i>	<i>Alternate Points</i>	
If the line on the graph does pass through the origin, select “No”		
For a correct justification		1 point
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth’s magnetic field is not affecting the values of B .		

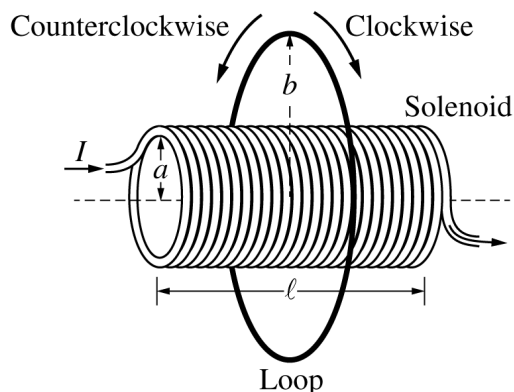
- ii. LO CNV-8.E.a, SP 2.E
1 point

Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.

Select “No”		
For a correct justification		1 point
Example: The horizontal component of Earth’s magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

2019 SCORING GUIDELINES**Question 3 (continued)**

A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt .

(e)

- i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

2019 SCORING GUIDELINES

Question 3 (continued)

(e) continued

- ii. LO FIE-6.A.d, SP 5.A, 5.E
3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I , ℓ , N , a , b , R_L , R_S , Δt , and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop	1 point
$\mathcal{E} = \left \frac{d\Phi}{dt} \right = \frac{d(BA)}{dt} = A \frac{\Delta B}{\Delta t}$	
For using Ohm's law to calculate the current in the loop	1 point
$I_{\text{IND}} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L\Delta t}$	
For using the correct radius for the area in the equation above	1 point
$I_{\text{IND}} = \frac{\pi a^2 \mu_0 N \Delta I / \ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$	

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.

CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.

FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.

FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.

2.E: Identify or describe potential sources of experimental error.

3.D: Create appropriate diagrams to represent physical situations.

4.C: Linearize data and/or determine a best-fit line or curve.

5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.

6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

7.A: Make a scientific claim.

7.C: Support a claim with evidence from physical representations.

2018**AP[®]** **CollegeBoard**

AP Physics C: Electricity and Magnetism

Scoring Guidelines

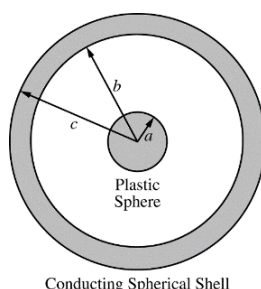
2018 SCORING GUIDELINES**General Notes About 2018 AP Physics Scoring Guidelines**

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at <https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf>.
3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or “Terms Defined” in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

2018 SCORING GUIDELINES

Question 1

15 points total

Distribution
of points

A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r , a , and physical constants, as appropriate.

- (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

- i. 3 points

$$r < a$$

For using the integral to determine the charge enclosed	1 point
$q_{enc} = \int \rho dV = \int \beta r dV$	
For correctly substituting for dV into the integration to determine the charge enclosed	1 point
$q_{enc} = \int (4\pi r'^2)(\beta r') dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4} \right]_{r'=0}^{r'=r}$	
For a correct answer	1 point
$q_{enc} = \beta\pi r^4$	

- ii. 1 points

$$a < r < b$$

For an answer consistent with (a)(i)	1 point
$q_{enc} = \beta\pi a^4$	

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. 2 points

$$r < a$$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\frac{q_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$		
$\frac{q(r)}{\epsilon_0} = EA_{sphere} = E(4\pi r^2)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\frac{\beta\pi r^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta r^2}{4\epsilon_0}$		

ii. 1 point

$$a < r < b$$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\frac{\beta\pi a^4}{\epsilon_0} = E(4\pi r^2)$		
$E = \frac{\beta a^4}{4\epsilon_0 r^2}$		

2018 SCORING GUIDELINES

Question 1 (continued)

**Distribution
of points**

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign)	1 point
$q_{\text{inner}} = -\beta\pi a^4$	
For a correct justification	1 point
$E_{\text{shell}} = 0 = \frac{q_{\text{enc}}}{\epsilon_0} \therefore q_{\text{enc}} = 0 = q_{\text{sphere}} + q_{\text{inner}} \therefore q_{\text{inner}} = -q_{\text{sphere}}$	

ii. 1 point

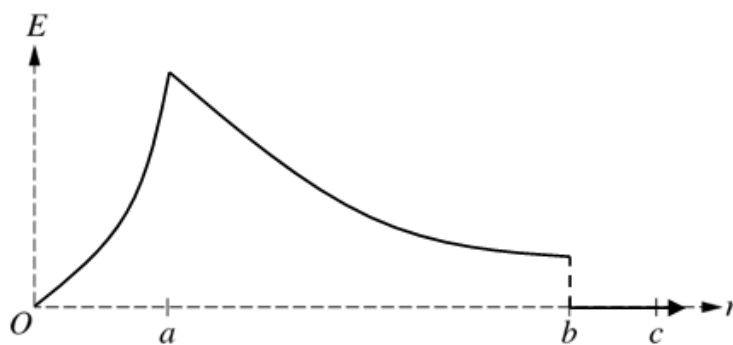
Determine the charge on the outer surface of the conducting shell.

A correct answer	1 point
$q_{\text{outer}} = 0$	

(d)

i. 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$	1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to nonzero value at $r = b$	1 point
For a graph with a value of $E = 0$ for $r > b$	1 point
Note: Due to scale, discontinuity at point b is not required.	

2018 SCORING GUIDELINES

Question 1 (continued)

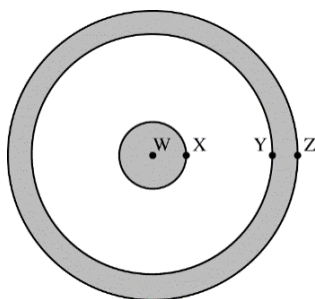
**Distribution
of points**

(d) (continued)

ii. 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W __2__ X __3__ Y __3__ Z

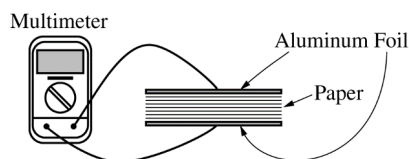


For any ranking that has Y and Z at the same potential	1 point
For ranking the electric potentials: $W > X > Y \text{ \& } Z$ (because $Y = Z$ is the first point)	1 point

2018 SCORING GUIDELINES

Question 2

15 points total

Distribution
of points

An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C Horizontal axis: $\frac{1}{d}$		
Note: Student earns full credit if the axes are reversed or if they use another acceptable combination.		

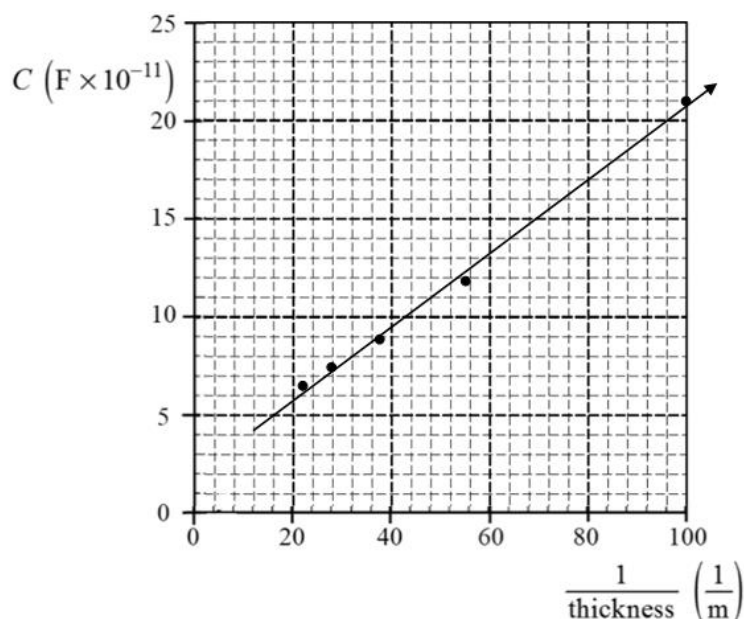
2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid	1 point
For correctly labeling the axes including units	1 point
For correctly plotting the data	1 point
For drawing a straight line consistent with the plotted data	1 point

(c) 3 points

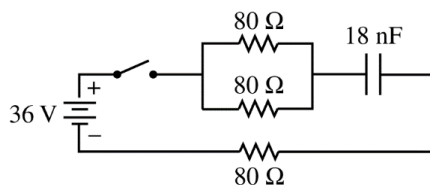
Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line	1 point
$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(17 - 8)(\text{F} \times 10^{-11})}{(80 - 32)(1/\text{m})} = 0.19 \text{ F} \cdot \text{m} \quad (\text{Linear regression} = 0.187 \text{ F} \cdot \text{m})$	
For correctly relating the slope to the dielectric constant	1 point
$\text{slope} = \kappa \epsilon_0 A$	
$\kappa = \frac{\text{slope}}{\epsilon_0 A} = \frac{(0.19 \text{ F} \cdot \text{m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.060 \text{ m}^2)}$	
For a correct answer	1 point
$\kappa = 3.58 \quad (\text{Linear regression} = 3.52)$	

2018 SCORING GUIDELINES

Question 2 (continued)

Distribution
of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors	1 point
$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$	
$R_p = 40\ \Omega$	
For using Ohm's law with the potential difference across the capacitor equal to zero	1 point
$I = \frac{V}{R} = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{R_p + R_3}$	
For substitution of values for resistance including the value for combined resistance above	1 point
$I = \frac{V}{R_p + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$	

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above	1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$	
For an answer with units consistent with part (d)	1 point
$t = 2.16\ \mu\text{s}$	

AP PHYSICS C: ELECTRICITY AND MAGNETISM

2018 SCORING GUIDELINES

Question 2 (continued)

**Distribution
of points**

- (f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.</i>		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

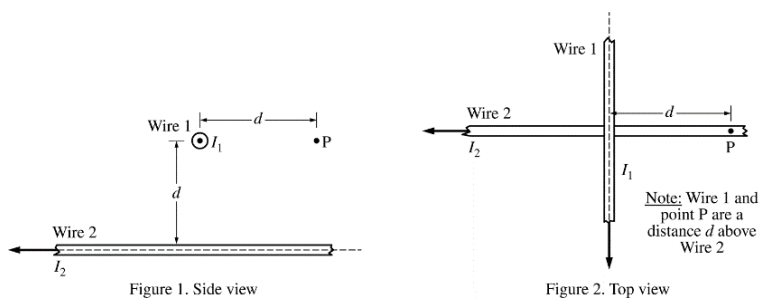
Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
<i>Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.</i>		

2018 SCORING GUIDELINES

Question 3

15 points total

Distribution
of points

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point P	1 point
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$	
$B(2\pi d) = \mu_0 I$	
For a correct answer	1 point
$B = \frac{\mu_0 I}{2\pi d}$	

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$	1 point
$B_1 = \frac{\mu_0 I}{2\pi d}$ & $B_2 = \frac{\mu_0 (2I)}{2\pi d} = \frac{\mu_0 I}{\pi d}$	
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2	1 point
$B_{\text{net}} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\frac{\mu_0 I}{2\pi d}\right)^2 + \left(\frac{\mu_0 (2I)}{2\pi d}\right)^2} = \frac{\sqrt{5}\mu_0 I}{2\pi d}$	

2018 SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right)$	
For correctly substituting B_1 and B_2 into the equation	1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{\left(\frac{\mu_0 I}{2\pi d}\right)}{\left(\frac{\mu_0 (2I)}{2\pi d}\right)}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$	

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel	1 point
For stating there is a net torque which causes rotation	1 point
<i>Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and, thus, will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and, thus, will experience a force to the left. So the net force will be zero, but there will be a net clockwise torque, so the wire will rotate clockwise.</i>	

2018 SCORING GUIDELINES

Question 3 (continued)

**Distribution
of points**

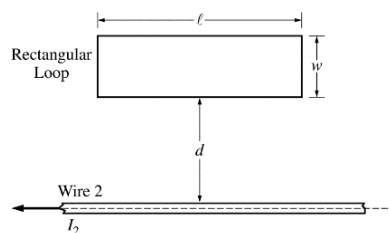


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

___X___ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$	1 point
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2018 SCORING GUIDELINES

Question 3 (continued)

**Distribution
of points**

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf	1 point
$\mathcal{E} = \left -\frac{d\Phi}{dt} \right = \frac{\mu_0 I_0 \ell}{\pi} \ln\left(\frac{d + w}{d}\right) \left \frac{d}{dt} [(1 - kt)] \right $	
$\mathcal{E} = \frac{\mu_0 I_0 k \ell}{\pi} \ln\left(\frac{d + w}{d}\right)$	
For dividing the emf by the resistance to calculate the current	1 point
$I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	
For a correct answer	1 point
$I = \frac{\mu_0 I_0 k \ell}{\pi R} \ln\left(\frac{d + w}{d}\right)$	

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3?

___ Clockwise ___ Counterclockwise

___ Undefined because there is no current induced in the loop

Justify your answer.

For selecting “Clockwise” with an attempt at a relevant justification	1 point
For indicating that the flux inside the loop will decrease	1 point
For using Lenz’s law to relate the decrease in the flux to the clockwise current	1 point
<i>Example: Because the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz’s law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page, and the current in the loop must be clockwise.</i>	

AP[®] Physics C: Electricity and Magnetism 2016 Scoring Guidelines

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SCORING GUIDELINES

Question

15 points total

Distribution
of points

3 points

For indicating that the total potential is the sum of the potential from individual point charges

1 point

ample using point B: $V_B = 0 = V_1 + V_2$

orrectly substituting into the above equation (signs are ignored at this step)

1 point

$$-V_1 = V_2$$

$$-\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

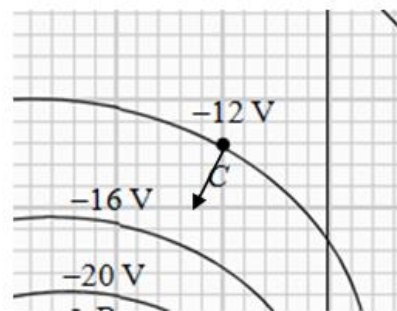
$$-\frac{q_1}{(5 \times 0.5\text{ m})} = \frac{(2.0 \text{ nC})}{(2 \times 0.5 \text{ m})}$$

For a correct answer with correct sign and units

1 point

$$q_1 = -5.0 \text{ nC}$$

2 points

For drawing a vector perpendicular to the equipotential line for C

1 point

For drawing a vector in the direction of the -16 V line

1 point

2 points

For using the equation relating the electric field to potential difference

1 point

$$E = -\frac{dV}{dx}$$

$$|E| \approx \frac{\Delta V}{\Delta x}$$

For substituting values from the figure

1 point

$$E = \frac{(-20 \text{ V} - (-24 \text{ V}))}{(2 \times 0.1 \text{ m})}$$

$$E = 20 \text{ N/C}$$

SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

2 points

For using a correct equation for the electric flux

1 point

$$\Phi_E = \frac{q_{enc}}{\epsilon_0}$$

$$\Phi_E = \frac{(2.0 \text{ nC})}{(8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2))}$$

For a correct answer with units

1 point

$$\Phi_E = 226 \text{ (N}\cdot\text{m}^2)/\text{C}$$

2 points

sing an equation that relates the work done to the change in potential energy

1 point

$$W = -q\Delta V = -(1.6 \times 10^{-19} \text{ C})(-4 \text{ V} - (4 \text{ V}))$$

For a correct answer including sign and units

1 point

$$W = 1.28 \times 10^{-18} \text{ J} = 8.0 \text{ eV}$$

2 points

ndicating that the kinetic energy of the proton at E will be equal to the work done

1 point

$$W = \Delta K = \frac{1}{2}mv^2$$

orrectly substituting the answer from part (e) i. into the above equation

1 point

$$(1.28 \times 10^{-18} \text{ J}) = \left(\frac{1}{2}\right)(1.67 \times 10^{-27} \text{ kg})v^2$$

$$v = 3.92 \times 10^4 \text{ m/s}$$

2 points

orrectly selecting Left

1 point

For a correct justification

1 point

Example: Electrons accelerate in the direction perpendicular to equipotential surfaces and toward higher potential. Therefore, at point B, an electron would accelerate toward the left.

No points are earned if the wrong answer is selected.

SCORING GUIDELINES

Question

15 points total

Distribution
of points

2 points

For selecting “It is the same through both.”

1 point

For a correct justification

1 point

Example: The sample of wire and the resistor are in series with each other and therefore will have the same current.

Note: Because the voltmeter is ideal, it will have no effect on the result.

No points are earned if the wrong answer is selected.

2 points

For selecting “It will depend on the resistance of the sample.”

1 point

For a correct justification including argument using Ohm’s law, voltage drop across resistors, etc.

1 point

Example: Because the sample of wire has resistance, it cannot be known whether the wire of the resistor has the greatest resistance and which has the higher potential difference across it. Therefore, it will depend on the resistance of the wire sample.

No points are earned if the wrong answer is selected.

(1 point

indicating the correct quantities for each axis

1 point

Horizontal axis: I_R Vertical axis: $V_r = \mathcal{E} - V_R$

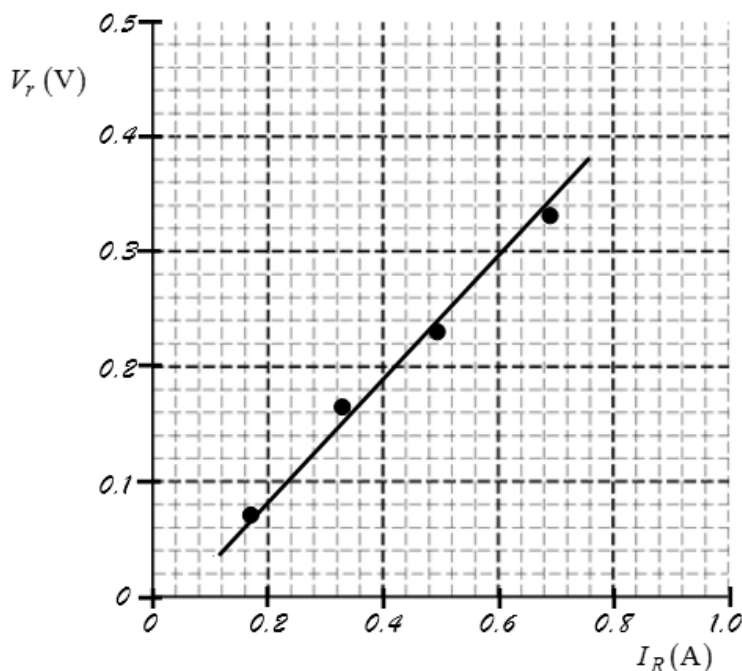
Note: Credit is received if the axes are reversed.

SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

3 points



For a correct scale that uses more than half the grid and for correctly labeling the axes including units

1 point

For correctly plotting given data

1 point

For drawing a straight line consistent with the given data

1 point

(2 points

For correctly calculating slope using the best-fit straight line and not data points

1 point

$$\text{slope} = \frac{(V_2 - V_1)}{(I_2 - I_1)} = \frac{(0.30 - 0.10) \text{ V}}{(0.60 - 0.22) \text{ A}} = 0.526 \text{ V/A}$$

Note: Linear regression gives $\text{slope} = 0.485 \text{ V/A}$. (Student must indicate that the value comes from a linear regression from the calculator, and the equation of the line must be present.)

For correctly relating r to the slope

1 point

$$V_r = I_R r, \text{ therefore the slope equals } r$$

$$r = 0.526 \, \Omega$$

Note: Linear regression gives $r = 0.485 \, \Omega$.

SCORING GUIDELINES

Question 2 (continued)

Distribution
of points

) 1 point

For using the equation relating resistance to resistivity with the correct or consistent substitutions

1 point

$$R = \frac{\rho L}{A} \quad \text{so} \quad \rho = \frac{AR}{L}$$

$$\rho = \frac{(\pi)(1.00 \times 10^{-3} \text{ m})^2 (0.526 \, \Omega)}{(3.00 \text{ m})}$$

$$\rho = 5.51 \times 10^{-7} \, \Omega \cdot \text{m}$$

Note: Linear regression gives $\rho = 5.08 \times 10^{-7} \, \Omega \cdot \text{m}$.

2 points

electing “Less than” with an attempt at a justification

1 point

For a correct justification

1 point

Example: The resistance calculated from the graph is the sum of the sample resistance plus the ammeter resistance because it is not ideal. The actual resistance is the calculated resistance minus the ammeter resistance, and therefore less than the calculated resistance.

oints are earned if the wrong answer is selected.

2 points

For selecting “Greater than” with an attempt at a justification

1 point

For a correct justification

1 point

Example: If the voltmeter is not ideal, that would add an additional resistor in parallel. A parallel resistor reduces the total resistance of the circuit. This would lead to an increase in current and a higher reading on the ammeter.

No points are earned if the wrong answer is selected.

SCORING GUIDELINES

15 points total	Question	Distribution of points
1 point		
For selecting “Left”		1 point
2 points		
For selecting “Less than”		1 point
For a correct justification		1 point
Example: Because as the bar falls the flux at point C is increasing, the emf generated must create a magnetic field to oppose this change. Therefore, it will create a magnetic field to decrease the flux and thus decrease the magnetic field.		
No points are earned if the wrong answer is selected.		
2 points		
For selecting “Greater than”		1 point
For a correct justification		1 point
Example: The field at point C, which is above the bar, is less than the original magnetic field, and point D is on the other side of the bar. Therefore, the direction of the magnetic field from the bar at point D is the opposite of the direction at point C, so the net magnetic field at D when the bar is falling must be greater than the original magnetic field.		
No points are earned if the wrong answer is selected.		
(4 points		
For correctly applying Newton’s second law to the motion of the bar		1 point
$F_{\text{net}} = Mg - F_M = Mg - BIL$		
$I = \mathcal{E}/R$		
For attempting to use Faraday’s law to obtain an expression for the emf in the bar		1 point
$\mathcal{E} = d\Phi/dt = BL(dx/dt)$		
For correctly using the expression for emf to obtain an expression for the current		1 point
$I = \frac{BL(dx/dt)}{R} = \frac{BLv}{R}$		
$Ma = Mg - B\frac{BLv}{R}L$		
$a = g - \frac{B^2L^2v}{MR}$		
Writing the acceleration as dv/dt		1 point
$\frac{dv}{dt} = g - \frac{B^2L^2v}{MR}$		

SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

2 points

For setting the net force at terminal velocity equal to zero

1 point

$$0 = Mg - \frac{B^2 L^2 v_T}{R}$$

$$\frac{B^2 L^2 v_T}{R} = Mg$$

in answer consistent with part (c)

1 point

$$v_T = \frac{MgR}{B^2 L^2}$$

(1 point

For a correct or consistent substitution in an appropriate power equation

1 point

$$P = I^2 R \quad \text{or} \quad V = IR$$

$$\frac{BLv_T}{R} = I$$

$$P = \frac{B^2 L^2 v_T^2}{R}$$

*Alternate Solution**Alternate points**For a correct or consistent substitution in an appropriate power equation**1 point*

$$P = I^2 R$$

$$Mg = BIL$$

$$I = \frac{Mg}{BL}$$

$$P = \frac{M^2 g^2 R}{B^2 L^2}$$

SCORING GUIDELINES

Question 3 (continued)

Distribution
of points

) 3 points

Using the equation from part (c)

$$\frac{dv}{dt} = g - \frac{B^2 L^2 v}{MR}$$

$$\frac{dv}{dt} = \frac{-B^2 L^2}{MR} \left(v - \frac{MRg}{B^2 L^2} \right)$$

For attempting separation of variables

1 point

$$\frac{dv}{v - \frac{MRg}{B^2 L^2}} = \frac{-B^2 L^2}{MR} dt$$

For attempting to integrate with the correct limits or the correct constant of integration

1 point

$$\int_{v'=0}^{v'=v(t)} \frac{1}{\left(v' - \frac{MRg}{B^2 L^2} \right)} dv' = \int_{t'=0}^{t'=t} \frac{-B^2 L^2}{MR} dt'$$

$$\ln \left[v' - \frac{MRg}{B^2 L^2} \right]_{v'=0}^{v'=v(t)} = \frac{-B^2 L^2}{MR} t$$

$$\ln \left(\frac{v(t) - \frac{MRg}{B^2 L^2}}{\frac{MRg}{B^2 L^2}} \right) = \frac{-B^2 L^2}{MR} t$$

For a correct answer

1 point

$$v(t) = \frac{MRg}{B^2 L^2} \left(1 - e^{-\frac{B^2 L^2}{MR} t} \right)$$

ng a trial solution in the differential equation and verifying its correctness is also acceptable.