

Week 21

AP Physics C: Electricity and Magnetism

Homework bundle for this week

本周作业合集

[exercises.lol](#)

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1. Outside a bar magnet, the field lines run:
 - ☐ from S to N
 - ☐ from N to S
 - ☐ straight up
 - ☐ in no particular direction
2. A magnetic field exerts a force on a charge only when the charge is:
 - ☐ at rest
 - ☐ moving
 - ☐ positive
 - ☐ very large
3. The magnetic field around a long straight wire forms:
 - ☐ straight lines pointing away
 - ☐ circles around the wire
 - ☐ a uniform field
 - ☐ no field at all
4. In Ampère's law, the field summed around a loop equals:
 - ☐ the enclosed charge over ϵ_0
 - ☐ μ_0 times the enclosed current
 - ☐ zero always
 - ☐ the total current in the universe
5. The SI unit of magnetic field strength is the _____.

6. A 3 C charge moves at 4 m/s across a 2 T field ($\theta = 90^\circ$). Find F (in N).
_____ N
7. The straight-wire field strength falls off with distance as:
 - ☐ $1/r^2$
 - ☐ $1/r$

☐ it stays constant

☐ $1/r^3$

8. A current outside an Amperian loop contributes nothing to the line integral of B .

☐ True ☐ False

9. A charge moving exactly along the field ($\theta = 0$) feels no magnetic force.

☐ True ☐ False

10. A current-carrying wire produces its own magnetic field.

☐ True ☐ False

AP Physics C: Electricity and Magnetism

1. **from N to S**

Outside, field lines run from the north pole to the south pole.

2. **moving**

The magnetic force $F = qvB$ needs a nonzero velocity.

3. **circles around the wire**

Field lines circle the wire; $B = \mu_0 I / 2\pi r$.

4. μ_0 **times the enclosed current**

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}.$$

5. **tesla**

Magnetic field B is measured in tesla (T).

6. **24 N**

$$F = qvB = 3 \times 4 \times 2 = 24 \text{ N}.$$

7. $1/r$

$B = \mu_0 I / 2\pi r$ — a $1/r$ falloff.

8. **True**

Only the enclosed current counts; outside current cancels around the loop.

9. **True**

$$\sin 0 = 0, \text{ so } F = qvB \sin \theta = 0.$$

10. **True**

Oersted showed a current deflects a compass — it makes a field.

12.1 Magnetic Fields

Name: _____ Class: _____ Date: _____

Total: 8 marks

KEY WORDS magnetic field 磁场 • ferromagnetic 铁磁性 • magnetic field lines 磁感线 • magnetic monopoles 磁单极子
 • diamagnetic 抗磁性

Objective

Build the skills to answer exam questions on **magnetic fields**.

You must be able to:

- describe a **magnetic field** 磁场 $\vec{B}$ as a vector field acting on moving charges and magnets
- represent fields with **field lines** that form closed loops from north to south
- explain that **magnetic monopoles** 磁单极子 have never been observed
- compare **ferromagnetic**, **paramagnetic**, and **diamagnetic** materials

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The magnetic field

$\vec{B}$ exerts forces on moving charges and magnets; its **field lines** form closed loops running from a north pole to a south pole.

■ No monopoles

Isolated poles have never been found, so field lines **never begin or end** — cutting a magnet just makes two smaller magnets.

■ Magnetic materials

Ferromagnetic (iron) — strongly magnetized; **paramagnetic** — weakly attracted; **diamagnetic** — weakly repelled. The behaviour reflects the alignment of atomic magnetic dipoles.

2 Practice

2.1 State the shape of magnetic field lines. [1]

2.2 State why magnetic field lines never begin or end. [1]

2.3 Name the type of material that is strongly magnetic, like iron. [1]

3 Exam-style questions

3.1 Magnetic field lines form [1]

- **A** open curves
 - **B** closed loops
 - **C** straight lines only
 - **D** isolated dots
-

3.2 Iron is an example of a _____ material. [1]

- **A** diamagnetic
 - **B** paramagnetic
 - **C** ferromagnetic
 - **D** non-magnetic
-

3.3 A bar magnet is cut in half.

(a) State what each piece becomes. [1]

(b) Explain your answer in terms of magnetic monopoles. [2]

Solutions

2.1 closed loops.

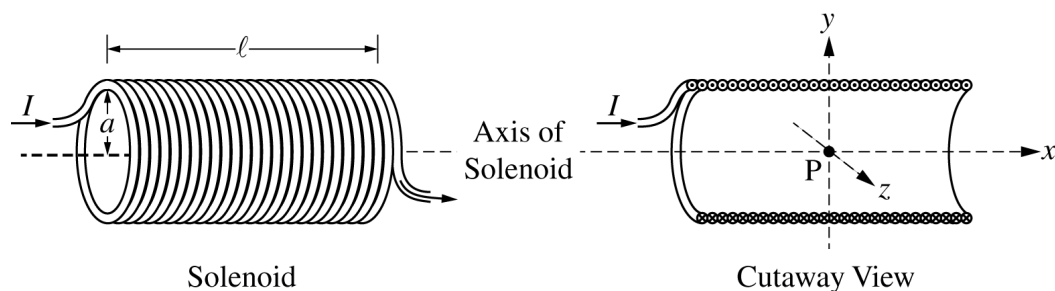
2.2 because there are no magnetic monopoles for them to start or end on.

2.3 ferromagnetic.

3.1 B.

3.2 C.

3.3 (a) each piece is a complete magnet with its own north and south pole. (b) magnetic poles only exist in pairs — no monopoles — so a new north-south pair forms at the cut.



Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a , length ℓ , and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I , as shown in the figure on the left above. The x -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz -coordinate system, as shown in the cutaway view on the right above. Assume $\ell \gg a$.

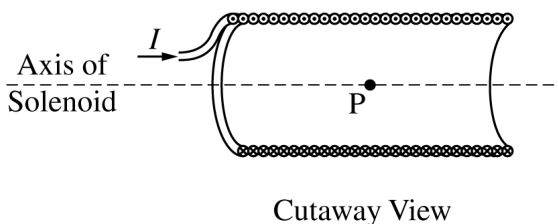
(a) Select the correct direction of the magnetic field at point P.

_____ $+x$ -direction _____ $+y$ -direction _____ $+z$ -direction
 _____ $-x$ -direction _____ $-y$ -direction _____ $-z$ -direction

Justify your selection.

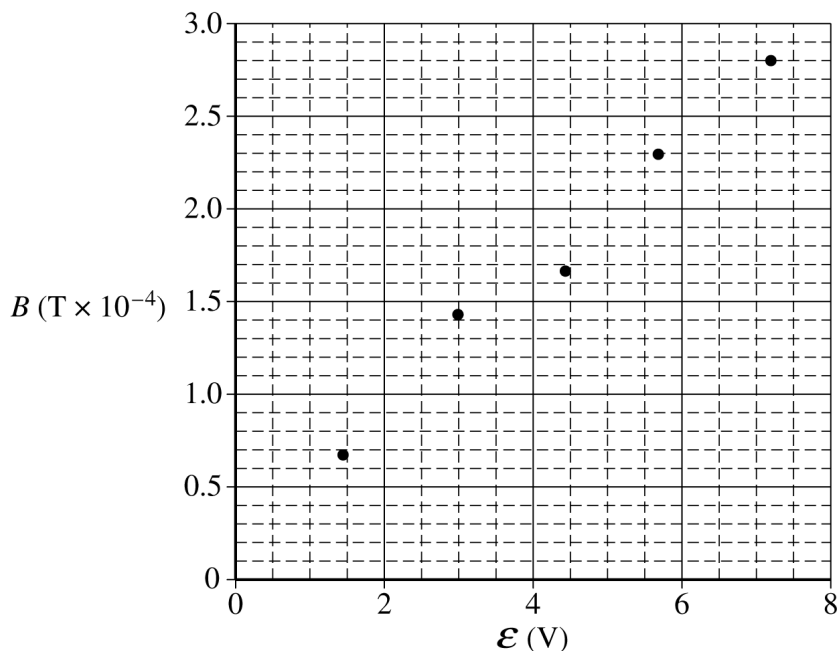
(b)

- i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.

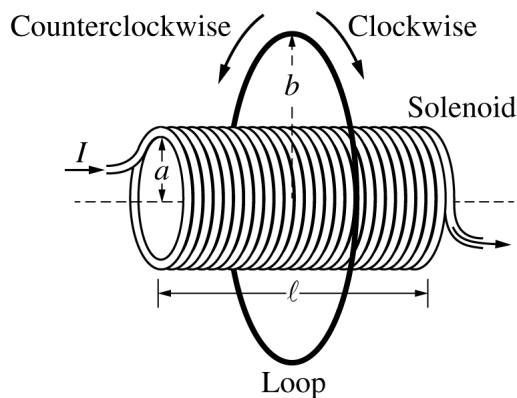


- ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I , ℓ , N , a , and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



- (c)
- On the graph above, draw a best-fit line for the data.
 - Use the straight line to determine the resistance R_S of the solenoid used in the experiment.
- (d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.
- Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?
☐ Yes ☐ No
Justify your answer.
 - Would the horizontal orientation of the solenoid affect the calculated value for R_S ?
☐ Yes ☐ No
Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt .

(e)

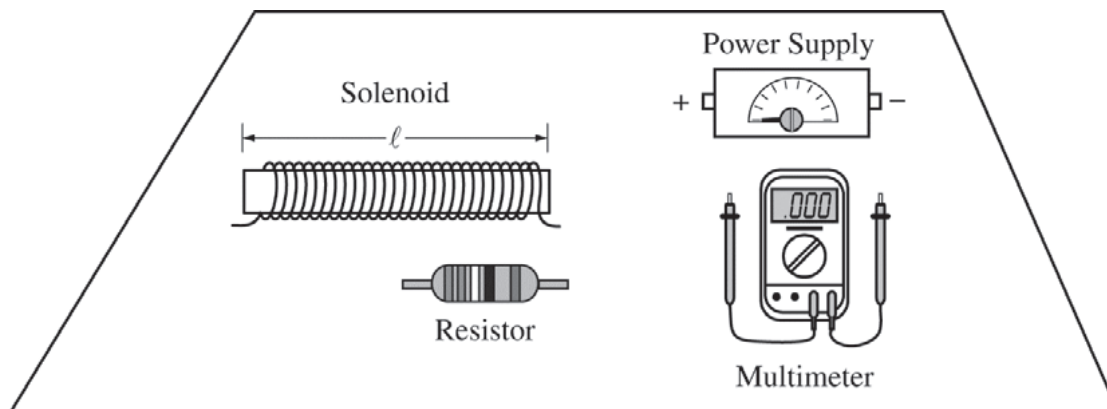
- i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

- ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I , ℓ , N , a , b , R_L , R_S , Δt , and physical constants, as appropriate.

STOP
END OF EXAM



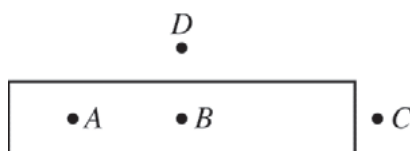
3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0 . The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

- i. On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.
- ii. Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

☐ Toward the top of the page	☐ To the left	☐ Out of the page
☐ Toward the bottom of the page	☐ To the right	☐ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A , B , and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B .

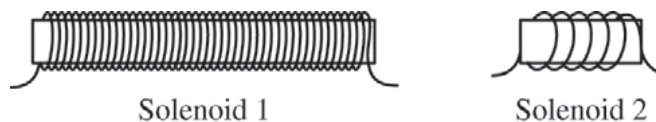


- iii. From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0 .

☐ A ☐ B ☐ C ☐ D

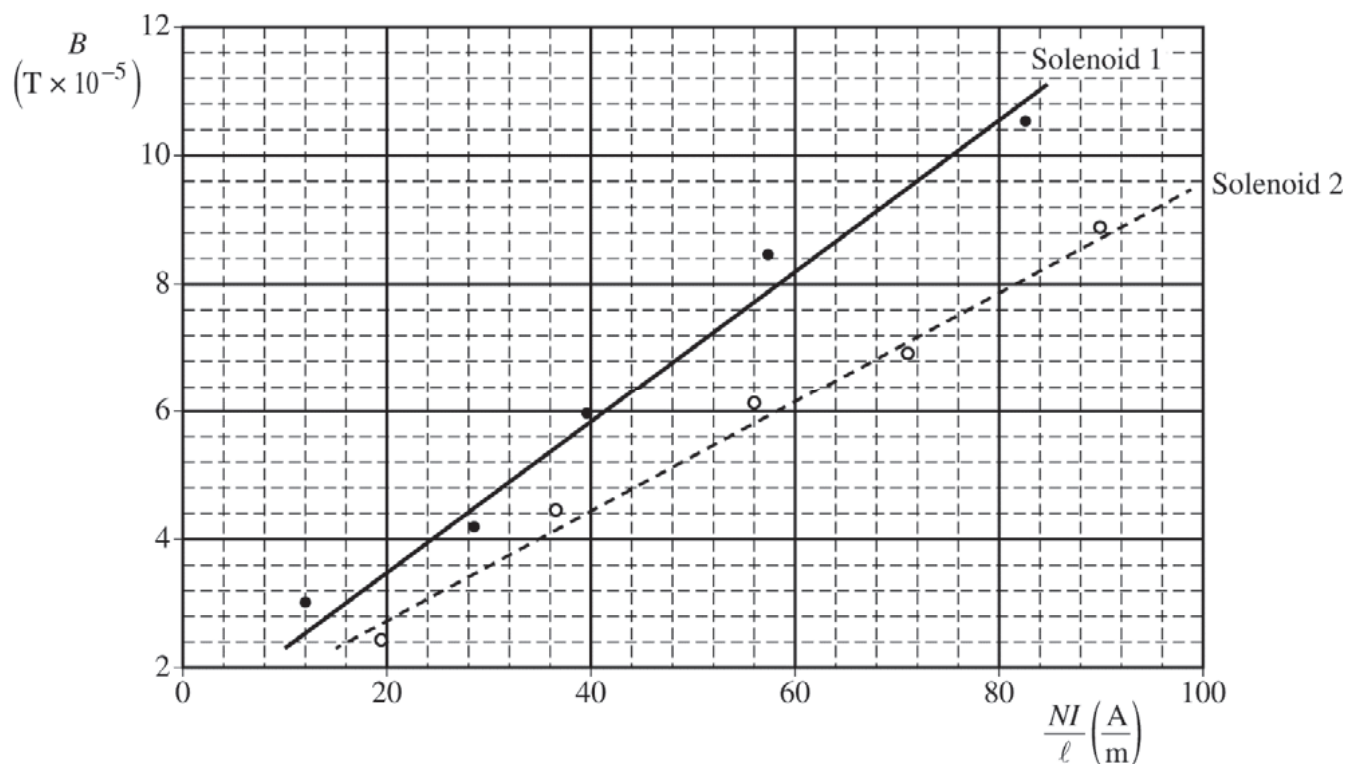
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



- (b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0 ?

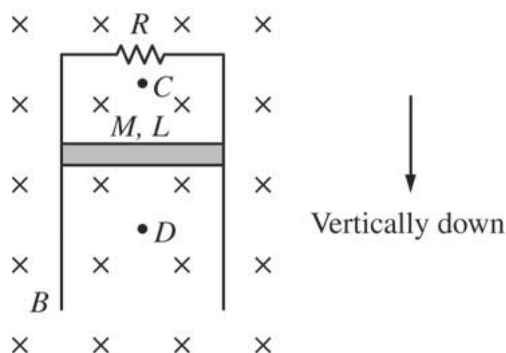
_____ Solenoid 1 _____ Solenoid 2

Justify your answer.

- (c)
- Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0 .
 - Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

- (d)
- i. What is a reasonable physical explanation for a best-fit line that does not pass through the origin?
 - ii. Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP
END OF EXAM



E&M.3.

A conducting bar of mass M , length L , and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

_____ Left _____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D , is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M , L , R , B , and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t .

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T , M , L , R , B , and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t .

STOP

END OF EXAM

12.2 Magnetism and Moving Charges

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS magnetic force 磁力 • perpendicular 垂直 • right-hand rule 右手定则 • centripetal force 向心力
 • magnetic field 磁场

Objective

Build the skills to answer exam questions on **magnetism and moving charges**.

You must be able to:

- calculate the **magnetic force** 磁力 on a moving charge, $\vec{F} = q\vec{v} \times \vec{B}$
- use the **right-hand rule** 右手定则 for the direction of $\vec{v} \times \vec{B}$
- explain why the magnetic force does **no work**
- analyse the **circular motion** of a charge in a uniform field, $r = \frac{mv}{qB}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Force on a moving charge

$\vec{F} = q\vec{v} \times \vec{B}$, with magnitude $F = qvB \sin \theta$ and direction from the right-hand rule.

■ No work

Because $\vec{F} \perp \vec{v}$, the magnetic force does **no work** — it changes only the **direction** of the velocity, not the speed.

■ Circular motion

For $\vec{v} \perp \vec{B}$, the force provides the centripetal force: $qvB = \frac{mv^2}{r}$, so $r = \frac{mv}{qB}$.

2 Practice

2.1 A charge of 3.0×10^{-6} C moves at 2.0×10^4 m s⁻¹ perpendicular to a 0.40 T field. Find the force. [2]

2.2 Explain why the magnetic force does no work on a moving charge. [2]

2.3 Write the formula for the radius of a charge's circular path in a magnetic field. [1]

3 Exam-style questions

3.1 The magnetic force on a moving charge is [1]

- **A** $q\vec{E}$
- **B** $q\vec{v} \times \vec{B}$
- **C** $I\vec{L}$
- **D** $m\vec{a}$

3.2 The magnetic force on a moving charge does [1]

- **A** positive work
- **B** negative work
- **C** no work
- **D** maximum work

3.3 A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) moves at 4.0×10^6 m s⁻¹ perpendicular to a 0.50 T field.

(a) Find the magnetic force on it. [2]

(b) Find the radius of its circular path. [2]

Solutions

2.1 $F = qvB = (3.0 \times 10^{-6})(2.0 \times 10^4)(0.40) = 2.4 \times 10^{-2} \text{ N}.$

2.2 the force is perpendicular to the velocity, so $\vec{F} \cdot \vec{v} = 0$ and no work is done — only the direction changes.

2.3 $r = \frac{mv}{qB}.$

3.1 B.

3.2 C.

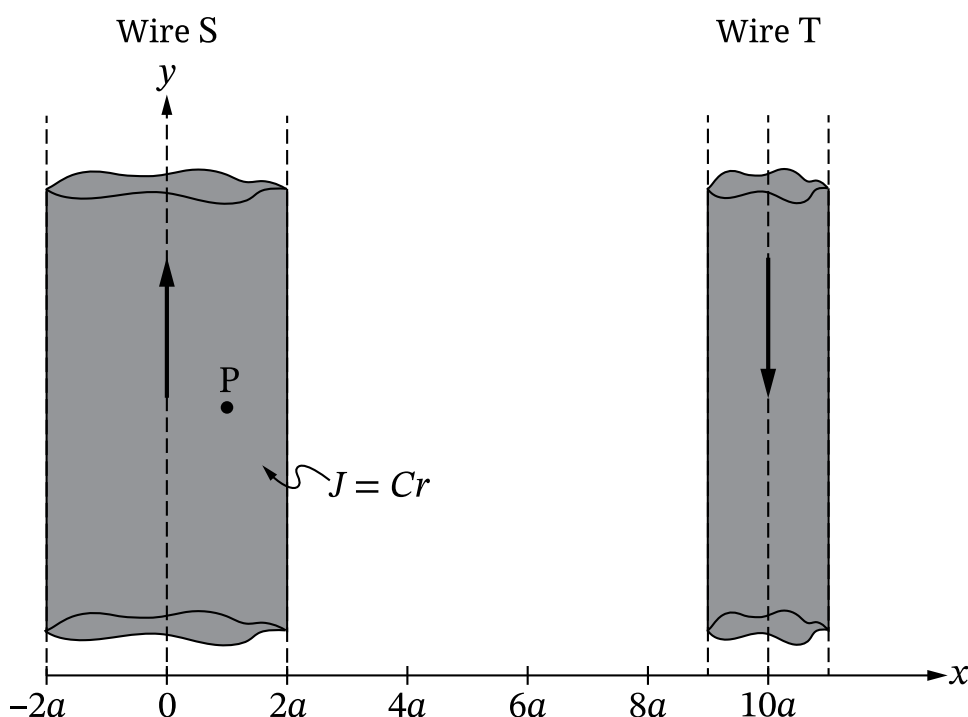
3.3 (a) $F = qvB = (1.6 \times 10^{-19})(4.0 \times 10^6)(0.50) = 3.2 \times 10^{-13} \text{ N}.$ (b) $r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27})(4.0 \times 10^6)}{(1.6 \times 10^{-19})(0.50)} = 0.084 \text{ m}.$

Question 1: Version J

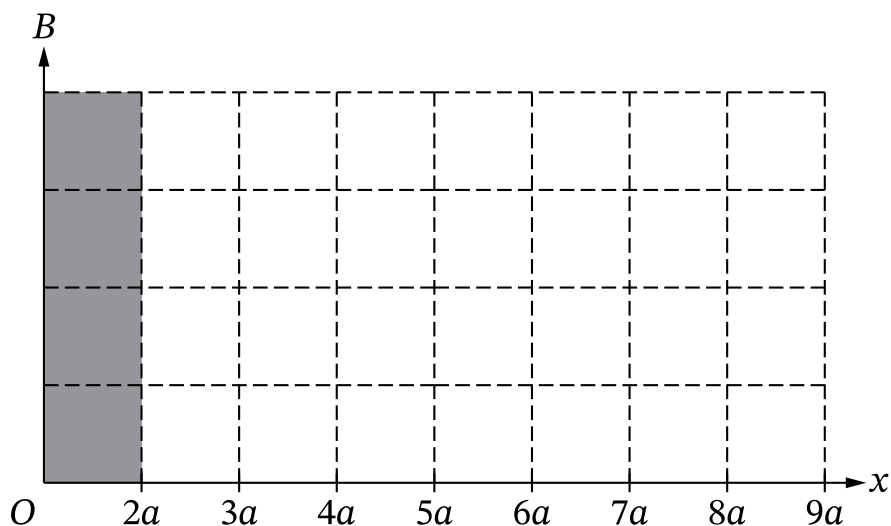
1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y -axis, as shown in Figure 1. The centers of the wires are in the xy -plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$ -direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$ -direction.

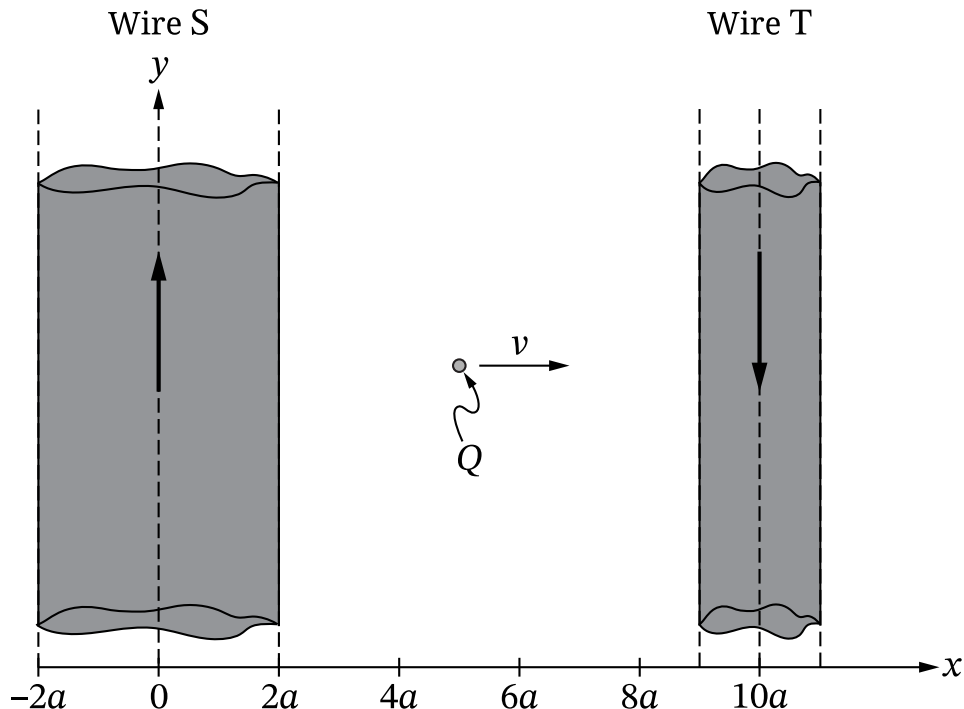
Figure 1

**Part A**

- Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C , a , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x -axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

Figure 2**Part B**

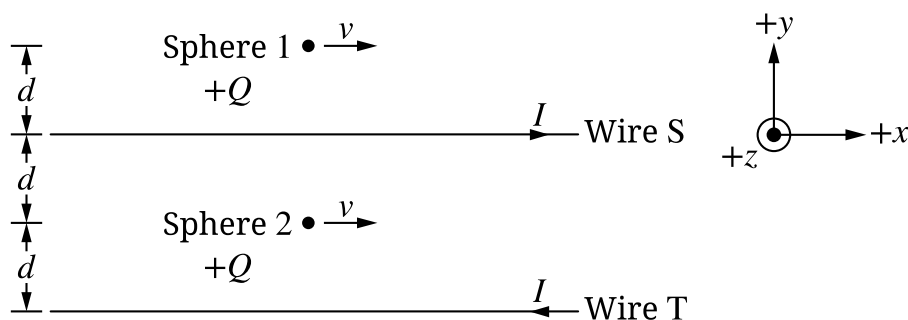
At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy -plane with speed v in the $+x$ -direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F .

Figure 3

Derive an expression for F in terms of a , Q , I_0 , v , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I , but the currents are in opposite directions. Both wires are parallel to the x -axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$ -direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1

- A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

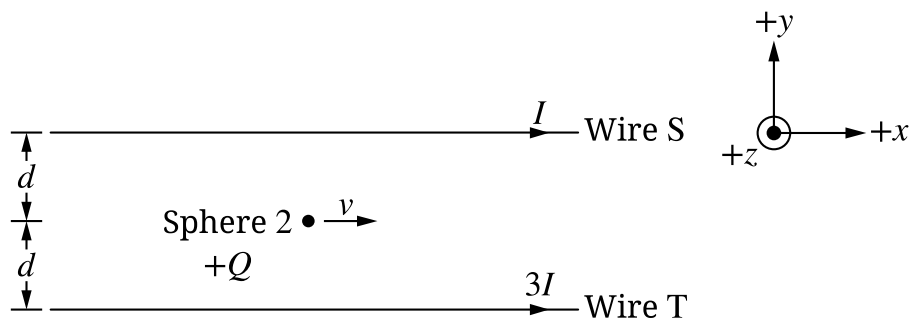
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

- B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d , I , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. Later, Wire T carries current $3I$ in the $+x$ -direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$ -direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2

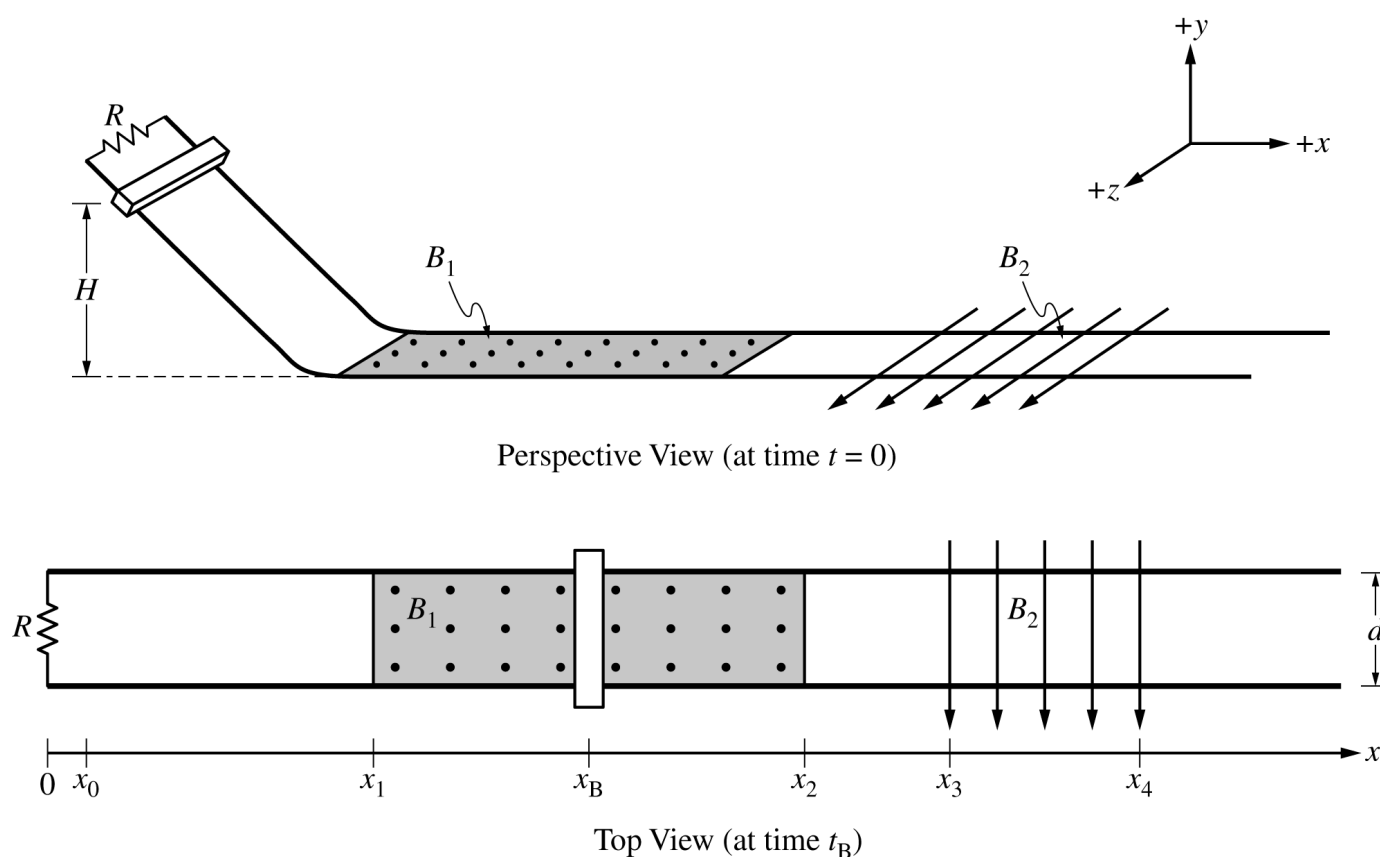


Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP
END OF EXAM



Two parallel conducting rails are separated by distance $d = 0.30$ m. A resistor of resistance $R = 0.20\ \Omega$ connects the rails. A conducting bar is placed on a sloped section of the rails at height H above the horizontal section of the rails. Frictional forces and the resistances of the bar and rails are negligible.

- At time $t = 0$, the bar is released from rest from position x_0 and slides down the sloped section of the rails, as shown in the Perspective View.
- At time t_1 , the bar reaches position x_1 and smoothly transitions to the horizontal section of the rails and enters a uniform magnetic field of magnitude $B_1 = 0.40$ T that is directed in the $+y$ -direction.
- At time t_2 , the bar reaches position x_2 and enters a region with no magnetic field.
- At time t_3 , the bar reaches position x_3 and enters a uniform magnetic field of magnitude $B_2 = 0.60$ T that is directed in the $+z$ -direction.
- At time t_4 , the bar reaches position x_4 and enters a region with no magnetic field.

The bar is at position x_B (shown in Top View) at time t_B such that $t_1 < t_B < t_2$.

GO ON TO THE NEXT PAGE.

- (a) On the following diagram of the bar, as observed from the Top View, draw an arrow indicating the direction of the net force F_{net} exerted on the bar at time t_B . If the net force is zero, write $F_{\text{net}} = 0$.

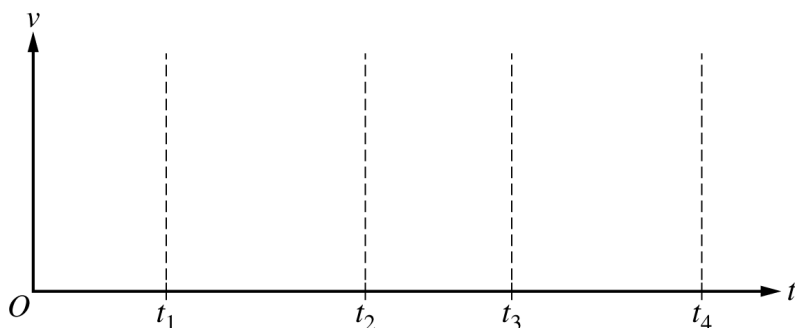


- (b) At time t_B , the speed of the bar is $v = 2.5$ m/s.

i. Calculate the magnitude of the current in the bar at time t_B .

ii. Calculate the magnitude of the net force F_{net} exerted on the bar at time t_B .

- (c) On the following axes, sketch a graph of the speed v of the bar as a function of time t between $t = 0$ and t_4 .



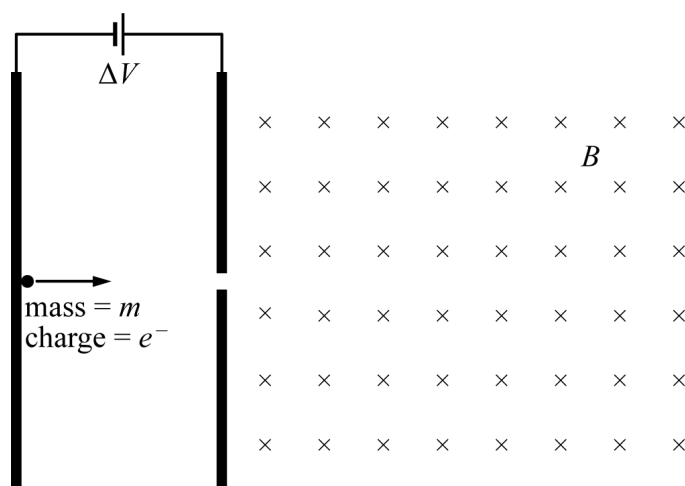
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- (d) The original scenario is repeated but with a new bar that has the same mass but with a nonnegligible resistance $R = 0.20 \, \Omega$. The new bar is released from rest and smoothly transitions to the horizontal section of the rails and enters the first uniform magnetic field.
- i. Determine the total resistance of the closed circuit.

ii. In the original scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{original} . In the new scenario, the magnitude of the acceleration of the bar immediately after the bar enters the first uniform magnetic field is a_{new} . Is a_{new} greater than, less than, or equal to a_{original} ? Justify your answer.

- (e) Describe a modification to H , B_1 , or d that will result in a larger induced current in the new bar immediately after the bar enters the first uniform magnetic field. Justify your answer.

GO ON TO THE NEXT PAGE.



3. Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V , m , e , B , and fundamental constants, as appropriate.

(a)

- i. What is the initial direction of the force on the sphere as it enters the magnetic field?

_____ Into the page

_____ Out of the page

_____ Toward the top of the page

_____ Toward the bottom of the page

- ii. Describe the path taken by the sphere after it enters the magnetic field.

(b) Derive an expression for the speed of the sphere as it passes through the small opening.

(c) Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field ($T \times 10^{-3}$)	2.62	2.78	3.39	3.54	3.78	3.99

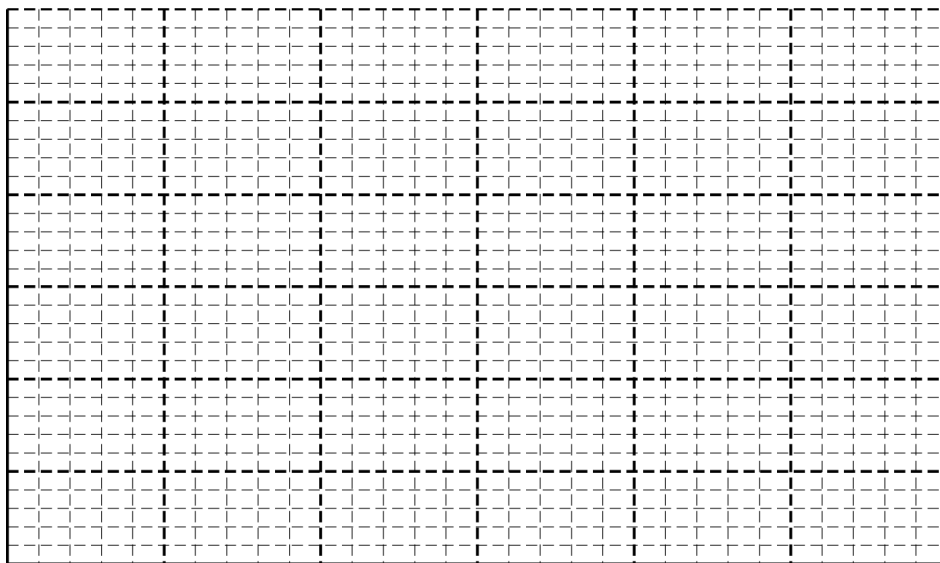
- (d) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis: _____

Horizontal axis: _____

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

- (e) On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



- (f) Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

STOP
END OF EXAM

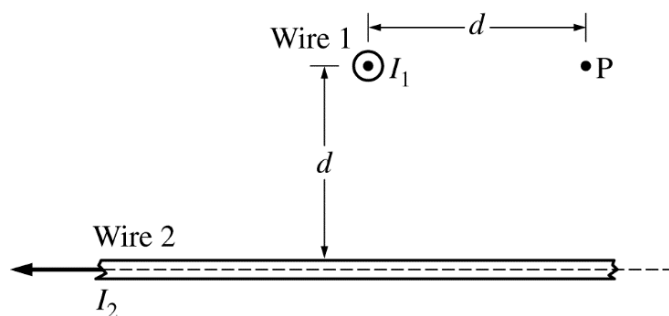


Figure 1. Side view

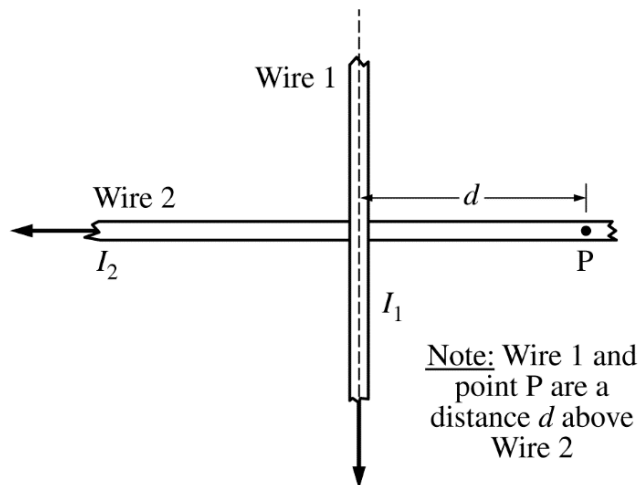


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.
- (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.
- (b) Derive an expression for the magnitude of the net magnetic field at point P.
- (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.
- (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.
- ☐ Wire 1 will not move.
- ☐ Wire 1 will move upward as viewed in Figure 1.
- ☐ Wire 1 will move downward as viewed in Figure 1.
- ☐ Wire 1 will rotate clockwise as viewed in Figure 2.
- ☐ Wire 1 will rotate counterclockwise as viewed in Figure 2.
- Justify your answer.

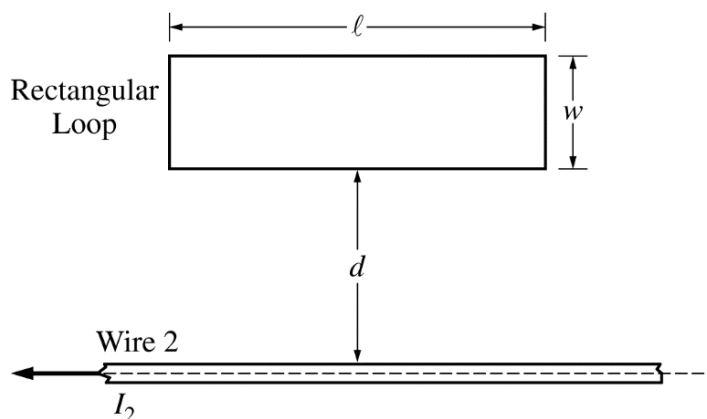


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation

$$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right),$$

derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3?

_____ Clockwise _____ Counterclockwise

_____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP
END OF EXAM

12.3 Magnetic Fields of Current-Carrying Wires and the Biot-Savart Law

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS biot-savart law 毕奥 • magnetic field 磁场

Objective

Build the skills to answer exam questions on **magnetic fields of current-carrying wires and the Biot-Savart law**.

You must be able to:

- state the **Biot-Savart law** 毕奥-萨伐尔定律, $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$
- calculate the field of a **long straight wire**, $B = \frac{\mu_0 I}{2\pi r}$
- find the field at the **centre of a circular loop**, $B = \frac{\mu_0 I}{2R}$
- use the **right-hand rule** for field direction

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The Biot-Savart law

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$ gives the field of a small current element; integrate over the wire for the total field ($\mu_0 = 4\pi \times 10^{-7}$).

■ Standard results

- Long straight wire: $B = \frac{\mu_0 I}{2\pi r}$.
- Centre of a circular loop of radius R : $B = \frac{\mu_0 I}{2R}$.

■ Direction

Curl the right hand around the wire with the thumb along the current; the fingers give the field direction.

2 Practice

2.1 Find the field 0.050 m from a long wire carrying 8.0 A ($\mu_0 = 4\pi \times 10^{-7}$). [2]

2.2 State the rule used to find the direction of the field around a straight wire. [1]

2.3 Write the field at the centre of a circular loop of radius R carrying current I . [1]

3 Exam-style questions

3.1 The magnetic field of a long straight wire is [1]

- **A** $\frac{\mu_0 I}{2\pi r}$
 - **B** $\frac{\mu_0 I}{2R}$
 - **C** $\mu_0 n I$
 - **D** BIL
-

3.2 The Biot-Savart law gives the field of a [1]

- **A** current element
 - **B** point charge
 - **C** capacitor
 - **D** resistor
-

3.3 A circular loop of radius 0.10 m carries 5.0 A ($\mu_0 = 4\pi \times 10^{-7}$).

(a) Find the magnetic field at its centre. [2]

(b) State how the field changes if the current is doubled.

[1]

Solutions

2.1 $B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(8.0)}{2\pi(0.050)} = 3.2 \times 10^{-5} \text{ T}.$

2.2 the right-hand rule (grip rule).

2.3 $B = \frac{\mu_0 I}{2R}.$

3.1 A.

3.2 A.

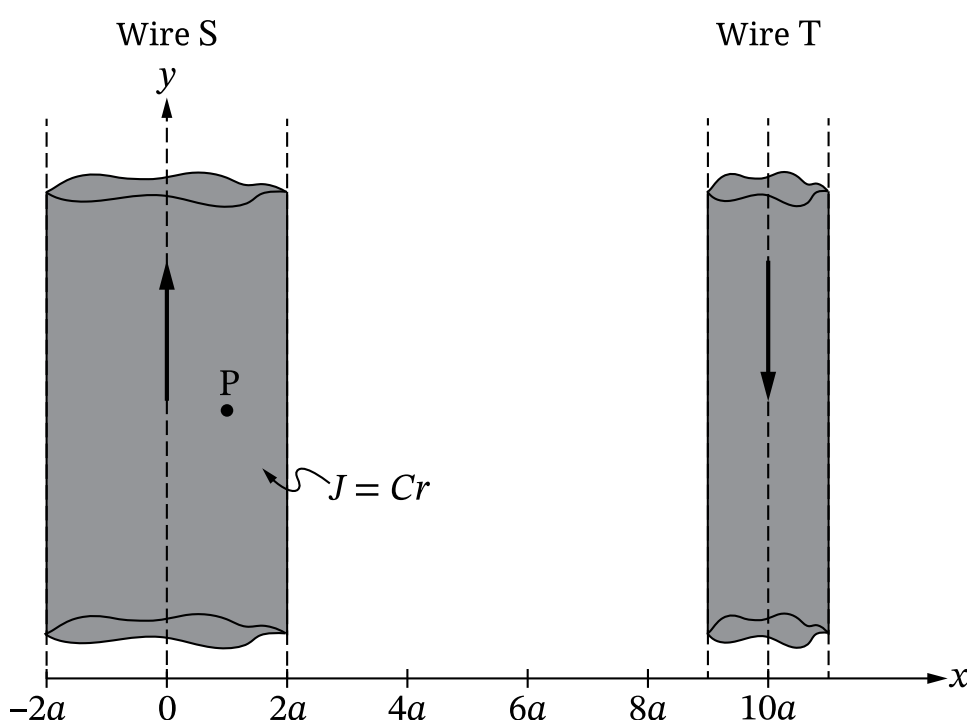
3.3 (a) $B = \frac{\mu_0 I}{2R} = \frac{(4\pi \times 10^{-7})(5.0)}{2(0.10)} = 3.1 \times 10^{-5} \text{ T}.$ (b) it doubles ($B \propto I$).

Question 1: Version J

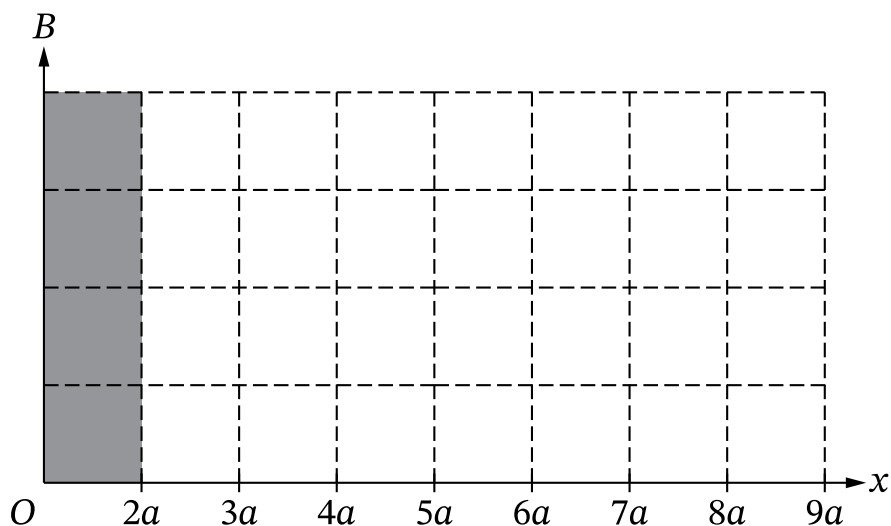
1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y -axis, as shown in Figure 1. The centers of the wires are in the xy -plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$ -direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$ -direction.

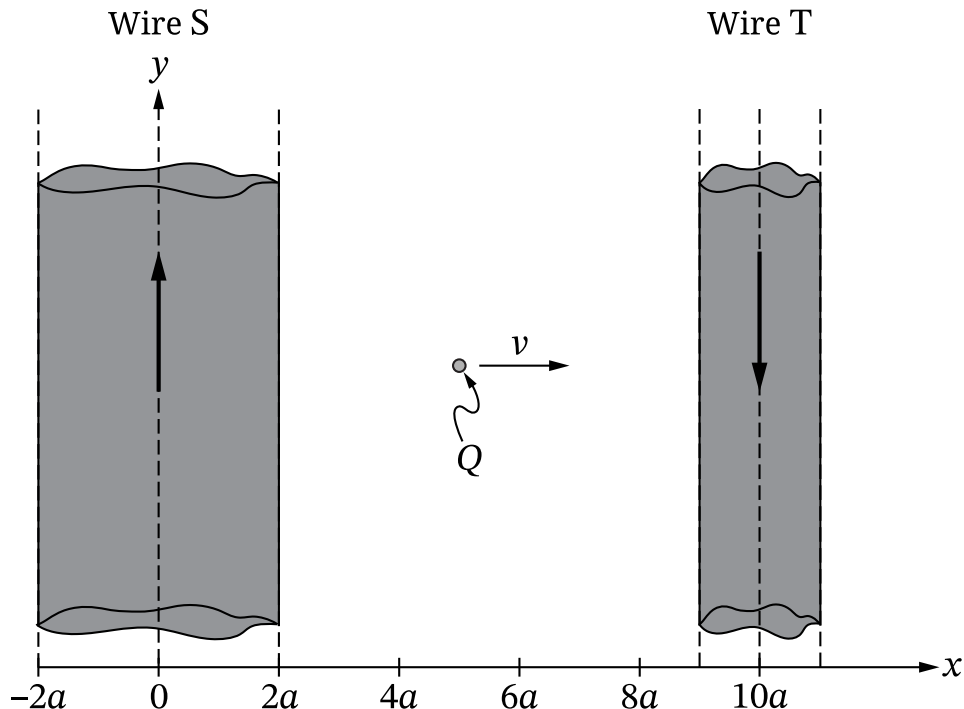
Figure 1

**Part A**

- Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C , a , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x -axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

Figure 2**Part B**

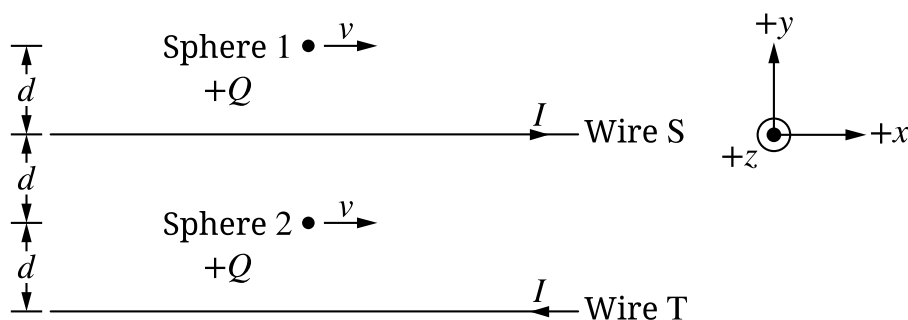
At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy -plane with speed v in the $+x$ -direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F .

Figure 3

Derive an expression for F in terms of a , Q , I_0 , v , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I , but the currents are in opposite directions. Both wires are parallel to the x -axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$ -direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1

- A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

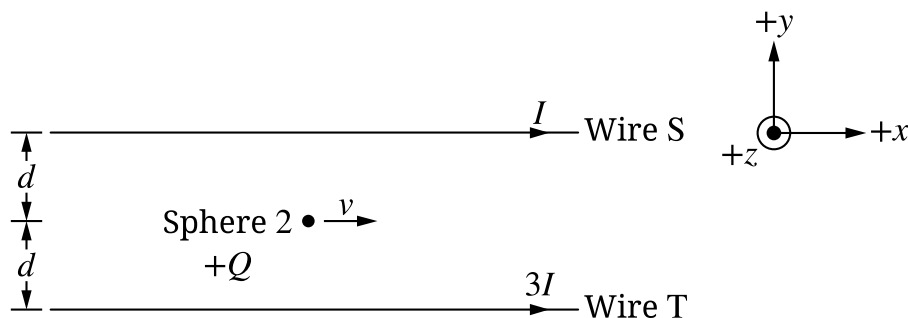
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

- B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d , I , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. Later, Wire T carries current $3I$ in the $+x$ -direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$ -direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP
END OF EXAM

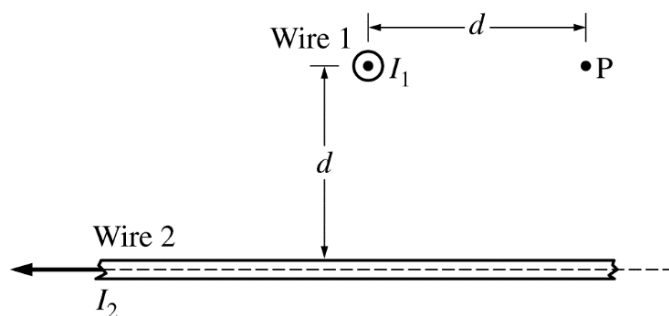


Figure 1. Side view

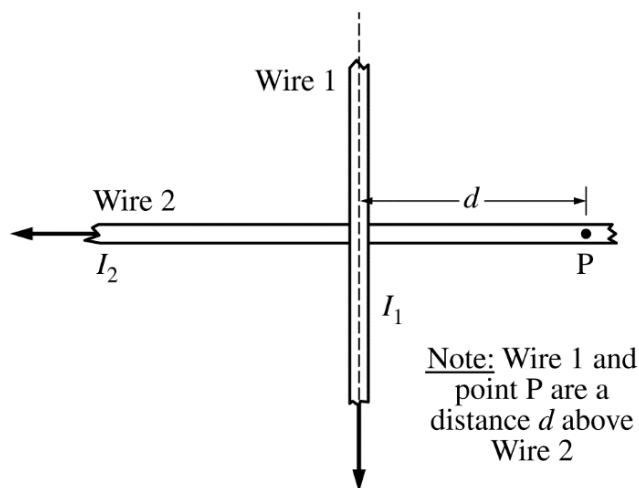


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.
- (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.
 - (b) Derive an expression for the magnitude of the net magnetic field at point P.
 - (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.
 - (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.
 - ☐ Wire 1 will not move.
 - ☐ Wire 1 will move upward as viewed in Figure 1.
 - ☐ Wire 1 will move downward as viewed in Figure 1.
 - ☐ Wire 1 will rotate clockwise as viewed in Figure 2.
 - ☐ Wire 1 will rotate counterclockwise as viewed in Figure 2.
- Justify your answer.

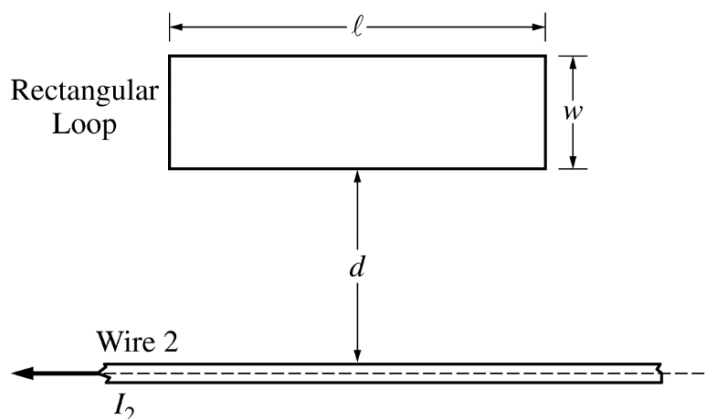


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3?

_____ Clockwise _____ Counterclockwise

_____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP
END OF EXAM

12.4 Ampère's Law

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS solenoid 螺线管 • amperian loop 安培环路 • ampère's law 安培定律

Objective

Build the skills to answer exam questions on **Ampère's law**.**You must be able to:**

- state **Ampère's law** 安培定律, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$
- choose an **Amperian loop** 安培环路 whose symmetry makes $\vec{B}$ constant and parallel to the path
- apply it to a long straight wire and a cylindrical conductor
- derive the field inside a **solenoid** 螺线管, $B = \mu_0 n I$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Ampère's law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$: the field integrated around a closed loop equals μ_0 times the enclosed current.

■ Choose the symmetry

Pick a loop on which B is constant and parallel to the path, so $\oint \vec{B} \cdot d\vec{l} = B \times (\text{length})$.

■ Solenoid

Inside an ideal solenoid the field is uniform: $B = \mu_0 n I$, where n is the number of turns per metre. Ampère's law is easier than Biot-Savart whenever such symmetry exists.

2 Practice

2.1 State Ampère's law. [1]

2.2 Find the field inside a solenoid of 500 turns per metre carrying 2.0 A ($\mu_0 = 4\pi \times 10^{-7}$). [2]

2.3 State when Ampère's law is easier to use than the Biot-Savart law. [1]

3 Exam-style questions

3.1 Ampère's law relates the field around a loop to the [1]

- **A** enclosed charge
 - **B** enclosed current
 - **C** magnetic flux
 - **D** voltage
-

3.2 The field inside an ideal solenoid is [1]

- **A** $\frac{\mu_0 I}{2\pi r}$
 - **B** $\mu_0 n I$
 - **C** $\frac{\mu_0 I}{2R}$
 - **D** zero
-

3.3 A solenoid has 1000 turns over a length of 0.50 m and carries 3.0 A ($\mu_0 = 4\pi \times 10^{-7}$).

(a) Find the number of turns per metre, n . [1]

(b) Find the field inside.

[3]

Solutions

2.1 the line integral of $\vec{B}$ around a closed loop equals μ_0 times the enclosed current.

2.2 $B = \mu_0 n I = (4\pi \times 10^{-7})(500)(2.0) = 1.3 \times 10^{-3} \text{ T}$.

2.3 when the current has enough symmetry to make B constant and parallel along a chosen loop.

3.1 B.

3.2 B.

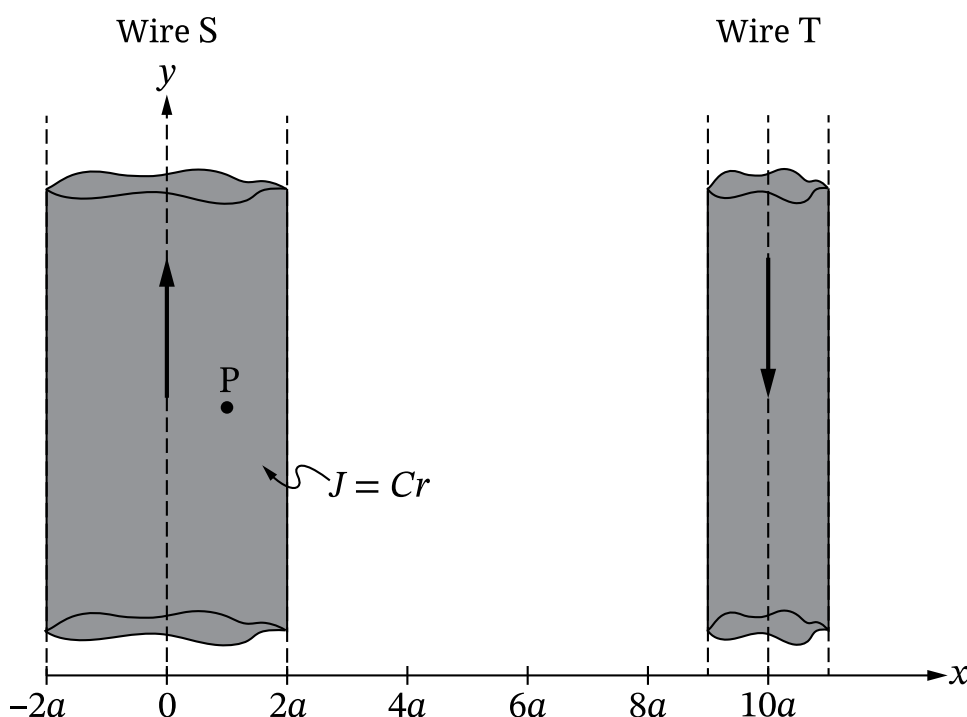
3.3 (a) $n = \frac{1000}{0.50} = 2000 \text{ m}^{-1}$. (b) $B = \mu_0 n I = (4\pi \times 10^{-7})(2000)(3.0) = 7.5 \times 10^{-3} \text{ T}$.

Question 1: Version J

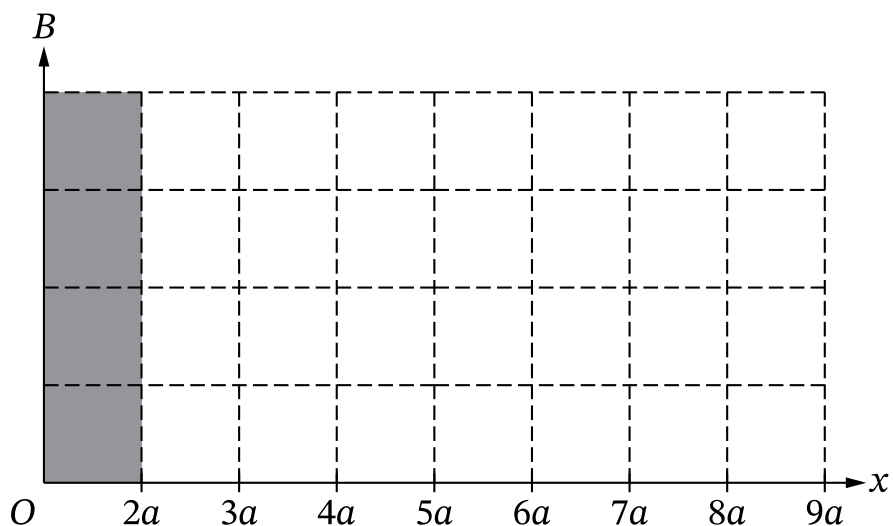
1. Two very long, cylindrical, current-carrying Wires S and T are oriented parallel to the y -axis, as shown in Figure 1. The centers of the wires are in the xy -plane. The wires are described as follows.

- Wire S has radius $2a$ and is centered at $x = 0$. Wire S has nonuniform current density and carries total current I_0 in the $+y$ -direction. The magnitude of the current density in Wire S as a function of the radial distance r from the center of the wire is described by $J = Cr$, where C is a positive constant.
- Wire T has radius a and is centered at $x = 10a$. Wire T has uniform current density and carries total current I_0 in the $-y$ -direction.

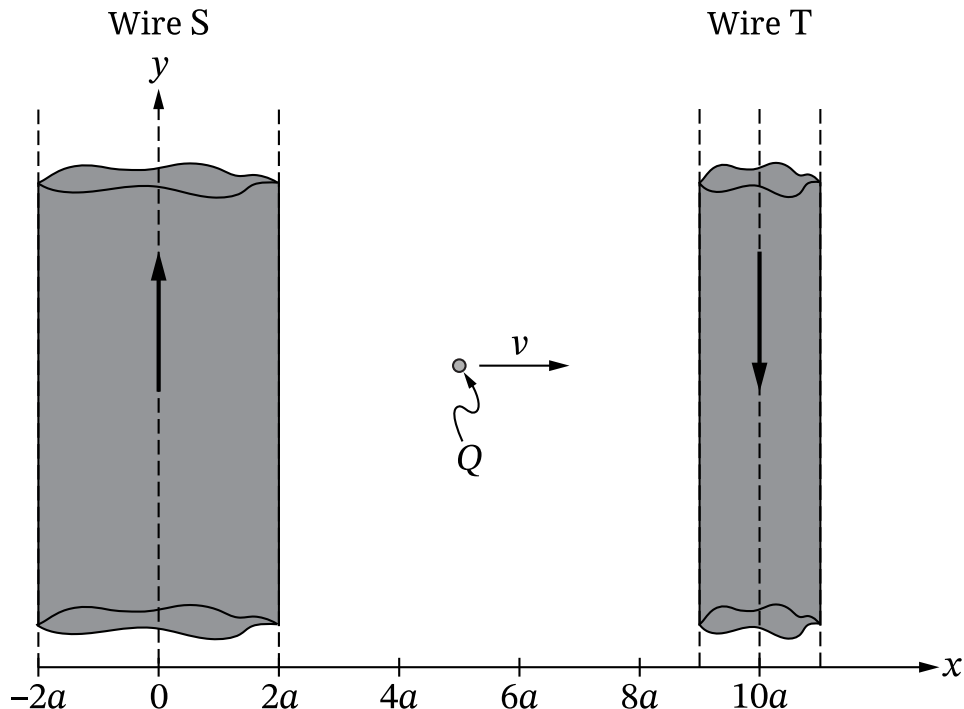
Figure 1

**Part A**

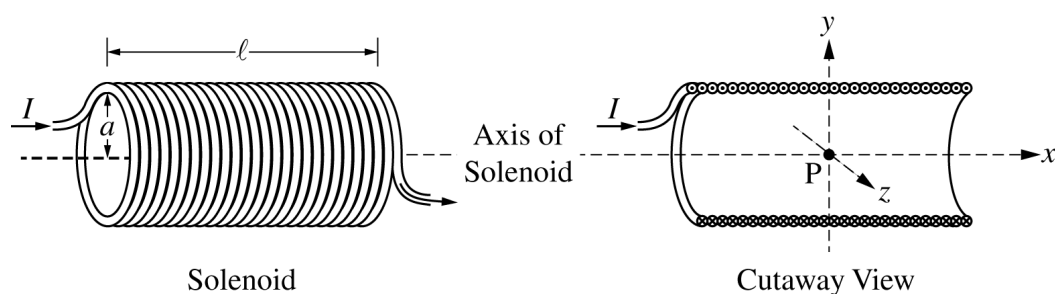
- Point P is located inside of Wire S at horizontal position $x = a$. **Derive** an expression for the magnitude of the magnetic field at Point P due only to the current in Wire S in terms of C , a , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.
- On the axes shown in Figure 2, **sketch** a graph of the magnitude B of the magnetic field due to the currents in both wires as a function of x along the x -axis in the region $2a \leq x \leq 9a$. Sketches made in the shaded region will not be graded.

Figure 2**Part B**

At the instant shown in Figure 3, a small sphere with charge Q is at horizontal position $x = 5a$ and moving in the xy -plane with speed v in the $+x$ -direction. At this instant, the net magnetic force exerted on the sphere due to the currents in Wires S and T has magnitude F .

Figure 3

Derive an expression for F in terms of a , Q , I_0 , v , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



Note: Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a , length ℓ , and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I , as shown in the figure on the left above. The x -axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz -coordinate system, as shown in the cutaway view on the right above. Assume $\ell \gg a$.

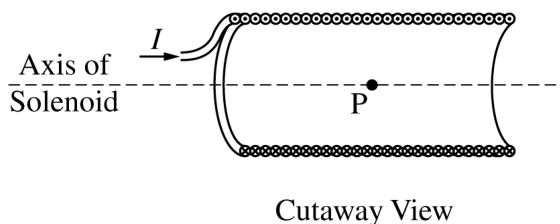
- (a) Select the correct direction of the magnetic field at point P.

____ $+x$ -direction ____ $+y$ -direction ____ $+z$ -direction
____ $-x$ -direction ____ $-y$ -direction ____ $-z$ -direction

Justify your selection.

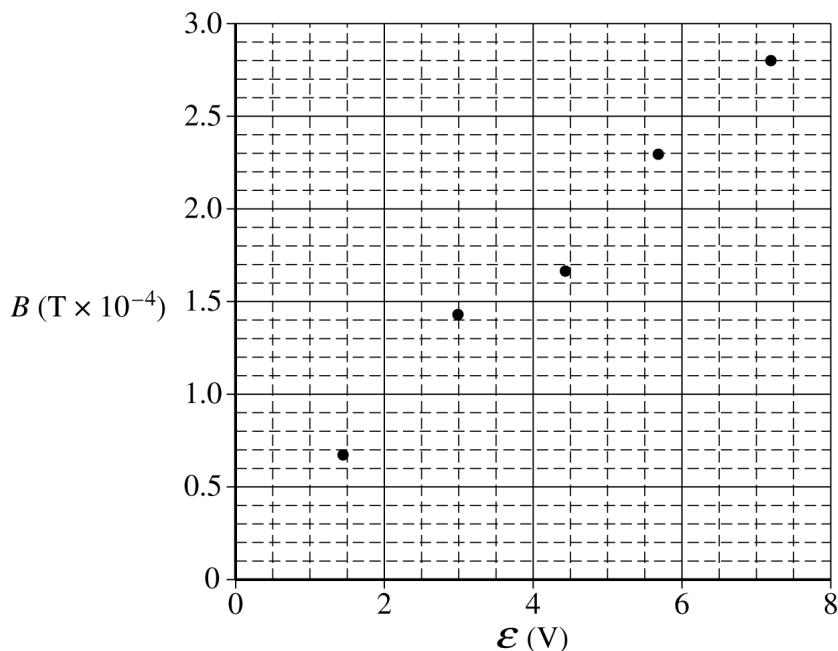
- (b)

- i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



- ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I , ℓ , N , a , and physical constants, as appropriate.

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.



(c)

- On the graph above, draw a best-fit line for the data.
- Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

- Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

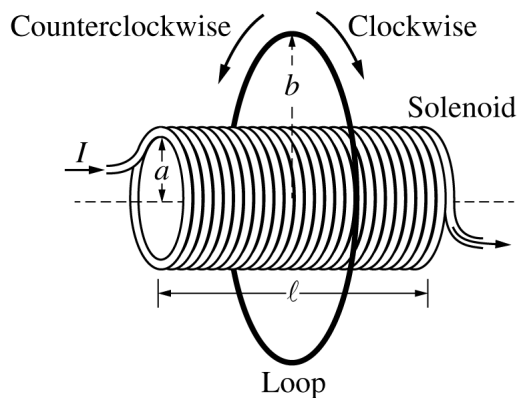
____ Yes ____ No

Justify your answer.

- Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt .

(e)

- i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

_____ Clockwise _____ Counterclockwise

Justify your answer.

- ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I , ℓ , N , a , b , R_L , R_S , Δt , and physical constants, as appropriate.

STOP
END OF EXAM

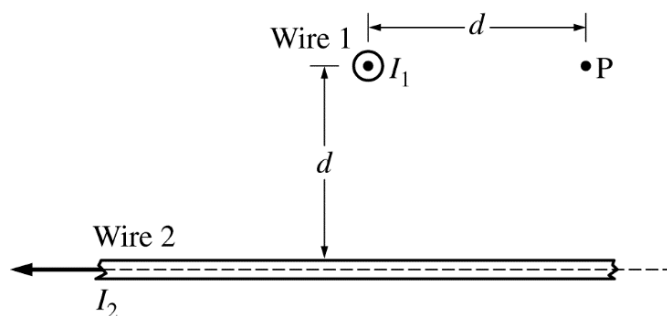


Figure 1. Side view

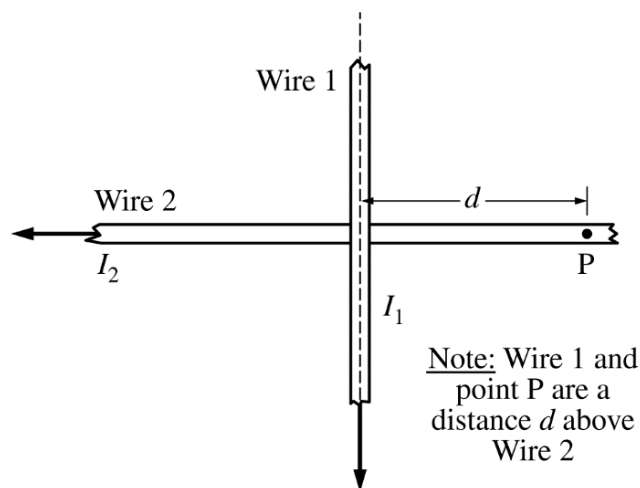


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.
- (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.
- (b) Derive an expression for the magnitude of the net magnetic field at point P.
- (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.
- (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.
- ☐ Wire 1 will not move.
- ☐ Wire 1 will move upward as viewed in Figure 1.
- ☐ Wire 1 will move downward as viewed in Figure 1.
- ☐ Wire 1 will rotate clockwise as viewed in Figure 2.
- ☐ Wire 1 will rotate counterclockwise as viewed in Figure 2.
- Justify your answer.

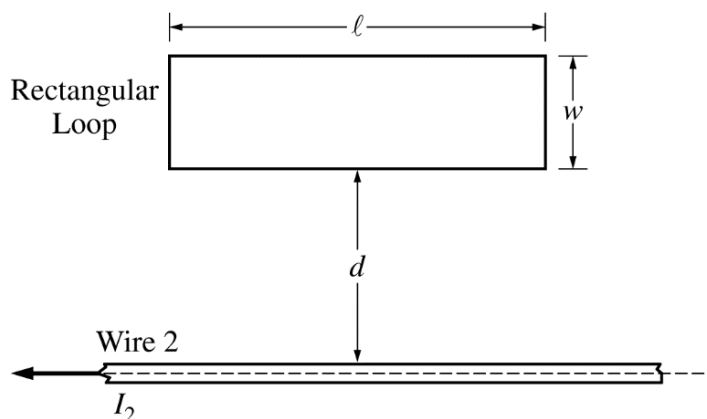


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

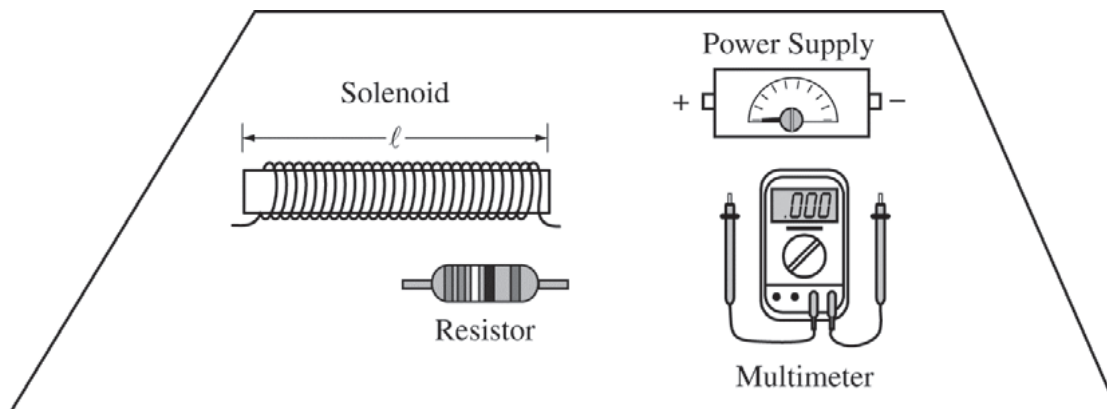
(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_____ Clockwise _____ Counterclockwise

_____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP
END OF EXAM



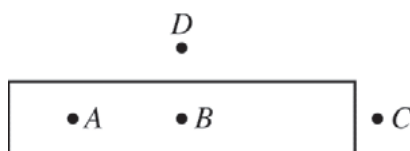
3. When studying Ampere's law, students collect data on the magnetic field of two different solenoids in order to determine the magnetic permeability of free space μ_0 . The solenoids are created by wrapping wire around a hollow plastic tube. The solenoids of length ℓ with N turns of wire will be connected in series to a power supply and resistor. A multimeter will be used as an ammeter to measure the magnitude of the current I through the solenoids. The main components for the setup with one of the solenoids are shown in the figure above.

(a)

- On the figure above, draw wire connections between the solenoid, power supply, resistor, and multimeter that will complete the circuit and allow students to measure the magnitude of the current through the solenoid.
- Using the connections you made in part (a)i above, what will be the direction of the magnetic field inside the solenoid?

☐ Toward the top of the page	☐ To the left	☐ Out of the page
☐ Toward the bottom of the page	☐ To the right	☐ Into the page

The rectangle shown below represents the solenoid (the loops of wire are not shown). Points A , B , and C are along the central axis of the solenoid with point B at the middle of the solenoid. Point D is directly above point B .

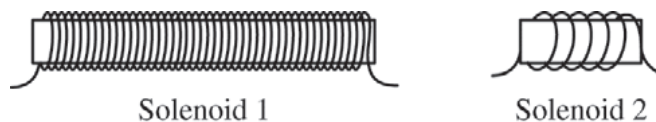


- From the choices below, select the point where you would place a magnetic field probe (a probe that can measure the magnitude of the magnetic field) to best measure the strength of the magnetic field of the solenoid in order to determine the magnetic permeability of free space μ_0 .

☐ A ☐ B ☐ C ☐ D

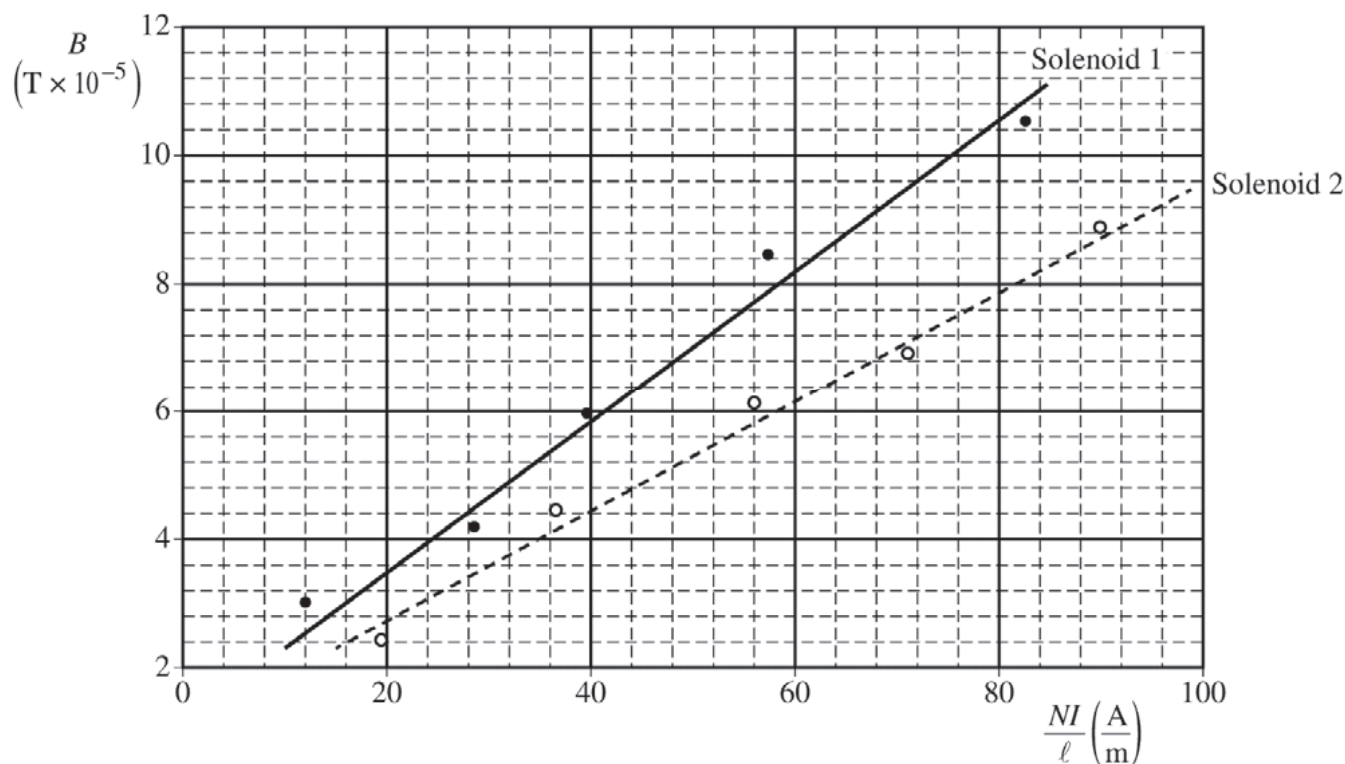
Justify your answer based on the model for a simple solenoid.

The figures below show two different solenoids that will be connected in the circuit above. Solenoid 1 has a length $\ell = 25$ cm with $N = 100$ turns. Solenoid 2 has a length $\ell = 5.0$ cm with $N = 5$ turns.



Note: Figures not drawn to scale.

A graph of the magnitude of the magnetic field B as a function of NI/ℓ is shown below. The best-fit lines for the data are shown as a solid line for solenoid 1 and as a dashed line for solenoid 2.



- (b) Which solenoid's best-fit line would give the best results for determining a value for the magnetic permeability of free space μ_0 ?

_____ Solenoid 1 _____ Solenoid 2

Justify your answer.

- (c)
- Use the slope of the best-fit line for the solenoid chosen in part (b) to calculate the magnetic permeability of free space μ_0 .
 - Calculate the percent error for the experimental value of the magnetic permeability of free space μ_0 determined in part (c)i.

- (d)
- What is a reasonable physical explanation for a best-fit line that does not pass through the origin?
 - Suppose a student connects the solenoid in a closed circuit similar to the circuit in part (a)i but without the resistor. The student notices the multimeter stops functioning after the power supply is turned on. Explain what causes the failure of the multimeter.

STOP
END OF EXAM