Exercise walk-through

Electric Fields of Charge Distributions ▪ 8.4

eXercise

You must be able to

- treat a **continuous charge distribution 连续电荷分布** as many elements dq
- relate dq to **linear**, **surface**, and **volume charge density** (λ, σ, ρ)
- set up an **integral** for the field of a symmetric distribution
- use **symmetry 对称性** to see which field components cancel

Method — From point charge to integral

Split the charge into elements dq, each giving $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$, then $\vec{E} = \int d\vec{E}$.

Method — Charge densities

$$\lambda = \frac{dq}{dl} \text{ (per length)}, \; \sigma = \frac{dq}{dA} \text{ (per area)}, \; \rho = \frac{dq}{dV} \text{ (per volume)}.$$

Method — Symmetry and the far field

By symmetry, components perpendicular to the axis cancel; only the axial part survives. Far away, any finite distribution looks like a **point charge** ($E \to kQ/r^2$).

Practice 2.1 ▪ 1 mark

Write the charge dq on a length dl of a rod with linear charge density λ.

Solution 2.1

Method: Charge densities

Worked steps

$dq = \lambda \, dl$ **B1**.

Practice 2.2 ▪ 1 mark

State the SI units of surface charge density σ.

Solution 2.2

Method: Charge densities

Worked steps

$\mathrm{C\ m^{-2}}$ **B1**.

Practice 2.3 ▪ 1 mark

State what the field of any finite charge distribution looks like from very far away.

Solution 2.3

Method: Symmetry and the far field

Worked steps

it approaches the field of a point charge B1.

Exam-style 3.1 ▪ 1 mark

The linear charge density λ is defined as

A charge per unit volume

B charge per unit area

C charge per unit length

D charge $\times$ length

Solution 3.1

Method: Charge densities

A charge per unit volume

B charge per unit area

C charge per unit length ✓

D charge $\times$ length

Worked steps

C.

Exam-style 3.2 ▪ 1 mark

To find the field of a continuous distribution you

A add two point charges

B integrate $d\vec{E}$ over all the charge elements dq

C use $V = IR$

D ignore symmetry

Solution 3.2

Method: Symmetry and the far field

A add two point charges

B integrate $d\vec{E}$ over all the charge elements dq ✓

C use $V = IR$

D ignore symmetry

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A ring of radius a carries total charge Q. On its axis at distance x the field is $E = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$.

(a) State the field at the centre of the ring ($x = 0$).

(b) State what E approaches when $x \gg a$, and explain.

Solution 3.3

Method: Symmetry and the far field

Worked steps

(a) $E = 0$ (the contributions cancel by symmetry) **B1**. (b) $E \to \dfrac{kQ}{x^2}$ **B1** — far away the ring looks like a point charge Q **B1**.