Past-paper walk-through

Kirchhoff's Loop Rule ▪ 11.6

eXercise

Questions in this set

- 2024 Set 1 Q2
- 2024 Set 2 Q2
- 2023 Set 2 Q3
- 2019 Set 1 Q2
- 2019 Set 2 Q1
- 2015 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a

Question 2

function of time. The students are required to measure a quantity that decreases with time to determine R.

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that <u>decreases</u> with time.

[Voltmeter Symbol: a circle containing the letter “V”, with two connection leads.]

(a)(ii) Describe a procedure for collecting data that would allow the students to graphically determine the experimental value for R using the measured quantity that <u>decreases</u> with time. Provide enough detail so that another student could replicate the experiment.

Question 2

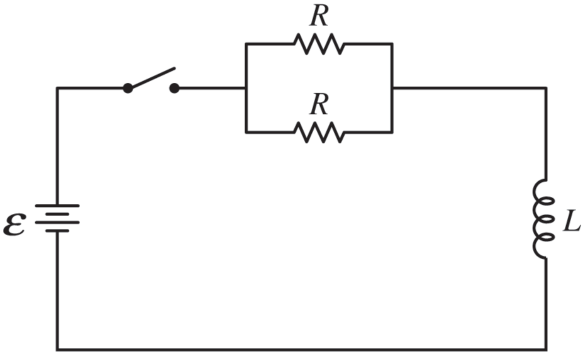

Figure 1

Question 2

Figure 2

Question 2

Voltmeter Symbol

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(b)(i) [Figure 2: A blank graph with a vertical axis (unlabeled) and a horizontal axis (unlabeled, shown as a dashed line), both with arrowheads, meeting at the origin.]
On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

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(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left. From the battery's top terminal, a wire goes right to a switch, then to a node. From that node, two identical resistors, each of resistance R, are connected in parallel between the node and a second node further right (drawn as two parallel horizontal branches, each containing a resistor R with a zigzag symbol, one above the other). From the second node, a wire continues right and down through an inductor of inductance L (coil symbol), and back to the battery's bottom terminal, completing the loop.]

Students are asked to determine the resistance R of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine R.

Question 2

(d) After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
_____$|\Delta V_2| > |\Delta V_1|$ _____$|\Delta V_2| < |\Delta V_1|$ _____$|\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Draw the voltmeter symbol connected in parallel across the two terminals of the inductor L, so that it measures the potential difference across the inductor — the quantity that decreases with time as the circuit approaches steady state after the switch is closed.

Solution 2

(a)(ii) Mark scheme

- Describe a data-collection procedure: after placing the voltmeter across the inductor, close the switch and use the voltmeter to record the potential difference across the inductor as a function of time (1 pt, for a procedure using the voltmeter to measure the potential difference at least once). The measurements must span from immediately after the switch is closed until steady-state conditions are established (or otherwise over a time interval that would allow the time constant to be determined) (1 pt). Example: 'Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.'

Solution 2

(b)(i) Mark scheme

- Sketch the voltmeter reading ΔV (potential difference across the inductor) versus time t, 4 points total (1 pt each): (1) correctly label the vertical axis as potential difference and the horizontal axis as time; (2) draw a concave-up, decreasing curve; (3) the curve should asymptotically approach zero (or start at the origin and approach a horizontal asymptote), consistent with the voltmeter placement across the inductor from part (a)(i); (4) include a vertical intercept (or horizontal asymptote) correctly labeled as $\mathcal{E}$ — the curve begins at $\Delta V = \mathcal{E}$ when $t = 0$ and decays smoothly toward the time axis.

Solution 2

(b)(ii) Mark scheme

- (1) Indicate that the graph should be fit with an exponential function, $V_L = \mathcal{E}\, e^{-tR/2L}$, or correctly relate the area under the curve to the current in the circuit (1 pt). (2) Indicate that the coefficient in front of t in that exponential fit equals $\dfrac{R}{2L}$, or equivalently correctly relate R to L and the current — i.e. that the time constant equals $\dfrac{2L}{R}$ (1 pt). Since $\mathcal{E}$ and L are known, this coefficient/time-constant lets R be calculated. Alternate: identify the time on the graph where $\Delta V \approx 0.37\mathcal{E}$ as the time constant $2L/R$, from which R follows because L is known.

Solution 2

(c) Mark scheme

- Derive the differential equation for the inductor current $I(t)$ using Kirchhoff's loop rule, 3 points: (1) a multi-step derivation that begins with an attempt at the loop rule, e.g. $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ (1 pt); (2) indicate that the potential difference across the parallel combination of the two resistors R is $I\left(\frac{R}{2}\right)$ (1 pt); (3) indicate that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$ (1 pt). Combining: $\mathcal{E} - I\left(\frac{R}{2}\right) - L\frac{dI}{dt} = 0$, which rearranges to $-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$.

Solution 2

(d)

Mark scheme

- Select $|\Delta V_2| > |\Delta V_1|$, with an attempt at a relevant justification (1 pt). The justification must indicate that the potential difference across the original, ideal inductor is zero once steady-state conditions are established (1 pt), and that the potential difference across the new inductor (which has nonnegligible resistance) is nonzero at steady state — equal to the product of the steady-state current and the inductor's resistance (1 pt).

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2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

Students are asked to determine the resistance R of identical resistors R_A and R_B. The resistors are connected in series with each other, a battery of known emf $\mathcal{E}$, an inductor of known inductance L, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R.

Question 2

(a)(i) On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.
[Voltmeter Symbol: a circle containing the letter "V", with a wire lead extending from each side, labeled "Voltmeter Symbol".]
(a)(ii) **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

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long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

______ $|\Delta V_2| > |\Delta V_1|$ ______ $|\Delta V_2| < |\Delta V_1|$ ______ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

Question 2

Figure 2

Question 2

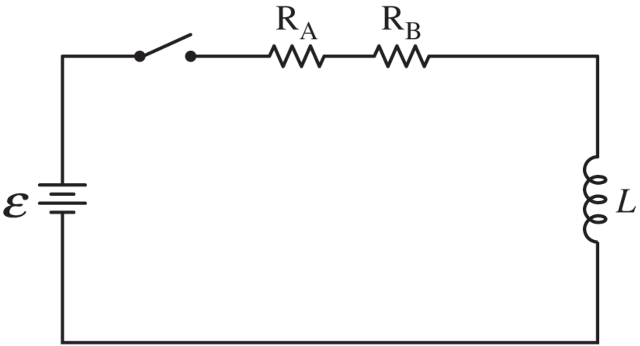

Figure 1

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(b)(i) On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

Question 2

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

[Figure 2: A blank graph with a vertical axis (upward arrow, unlabeled) and a horizontal axis (rightward arrow, unlabeled, drawn dashed). No scale or curve shown; axes are empty for the student to label and sketch.]

(b)(ii) Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the loop. Along the top wire, moving right from the battery, is a switch, then resistor R_A, then resistor R_B (both drawn as zigzag resistor symbols). The right side of the loop contains an inductor of inductance L (drawn as a coil symbol), connecting back down to the battery, completing the rectangular circuit loop.]

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(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the

Question 2

inductor at time t after the switch is closed. Express your answer in terms of R, $\mathcal{E}$, L, t, and physical constants, as appropriate.

Question 2

2024 Set 2 ▪ Q2

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(d) After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has

Question 2

nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.
Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.
$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$
Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Place the voltmeter in parallel with Resistor A, Resistor B, or the series combination of Resistors A and B (not across the inductor) — this location's potential difference increases with time as current builds up after the switch closes, unlike the inductor voltage which decreases with time. 1 point for correctly placing the voltmeter across one of these three valid options.

Solution 2

(a)(ii)

Mark scheme

- Procedure: Close the switch, then use the voltmeter to record the potential difference across the chosen resistor(s) as a function of time, from immediately after the switch is closed until steady-state conditions are established (or until enough time has elapsed that the time constant can be determined from the data). 1 point for a procedure indicating the voltmeter is used to measure the potential difference for at least one time, 1 point for measuring from immediately after the switch closes through steady state (or until the time constant is determinable).

Solution 2

(b)(i) Mark scheme

- Sketch potential difference (vertical axis) vs. time (horizontal axis): a curve that starts at the origin $(0, 0)$, is concave-down and increasing, and asymptotically approaches a nonzero potential-difference value. The labeled horizontal asymptote must be consistent with the voltmeter placement chosen in part (a)(i) — e.g. $\varepsilon/2$ if measuring across one resistor (A or B alone), or ε if measuring across the series combination of A and B. 1 point for correctly labeling potential difference vertically and time horizontally, 1 point for a concave-down increasing curve, 1 point for a curve starting at the origin and asymptotically approaching a nonzero value, 1 point for correctly labeling the horizontal asymptote consistent with part (a)(i).

Solution 2

(b)(ii) Mark scheme

- Fit the data to an exponential function of the form $V_R = \varepsilon\left(1 - e^{-t2R/L}\right)$ (for the series-combination case; for a single resistor the analogous form applies). Since ε and L are known, the coefficient multiplying t in the fit equals $2R/L$, from which R can be calculated. Alternate: read off the time at which the potential difference reaches $\approx 0.63\varepsilon/2$ (one resistor) or $\approx 0.63\varepsilon$ (combination); this time equals the time constant $L/2R$, and since L is known, R can be calculated. 1 point for indicating the curve fit is an exponential function (or, alternately, indicating the 0.63-value time), 1 point for indicating the fitted coefficient of t equals $2R/L$ (or, alternately, that the time constant equals $L/2R$).

Solution 2

(c) Mark scheme

- Apply Kirchhoff's loop rule: $\varepsilon - \Delta V_R - \Delta V_L = 0$. The potential difference across the series combination of the two resistors is $I(2R)$, and the magnitude of the potential difference across the inductor is $L\,dI/dt$. Substituting: $\varepsilon - I(2R) - L\dfrac{dI}{dt} = 0$, which rearranges to $\dfrac{dI}{dt} = \dfrac{\varepsilon}{L} - \dfrac{2IR}{L} = -\dfrac{R}{L}\left(2I - \dfrac{\varepsilon}{R}\right)$ (do not solve further). 1 point for a multi-step derivation beginning with an attempt at Kirchhoff's loop rule, 1 point for indicating the resistor-combination potential difference is $I(2R)$, 1 point for indicating the inductor's potential-difference magnitude is $L\,dI/dt$.

Solution 2

(d) Mark scheme

- $|\Delta V_2| < |\Delta V_1|$. Justification: because the new inductor has nonnegligible resistance, the total resistance of the new circuit is greater than in the original circuit, so at steady state the current is smaller than before, and the potential difference across R_A (which equals IR_A) decreases. (Equivalently: the increased total resistance means more of the source emf is dropped across the inductor's internal resistance at steady state, increasing the potential difference across the inductor and thereby decreasing the potential difference across R_A.) 1 point for selecting $|\Delta V_2| < |\Delta V_1|$ with an attempted justification, 1 point for indicating the total resistance has increased, 1 point for indicating the current decreases due

Solution 2

to the increased resistance (or, alternately, that the inductor's potential difference increases so R_A's must decrease).

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

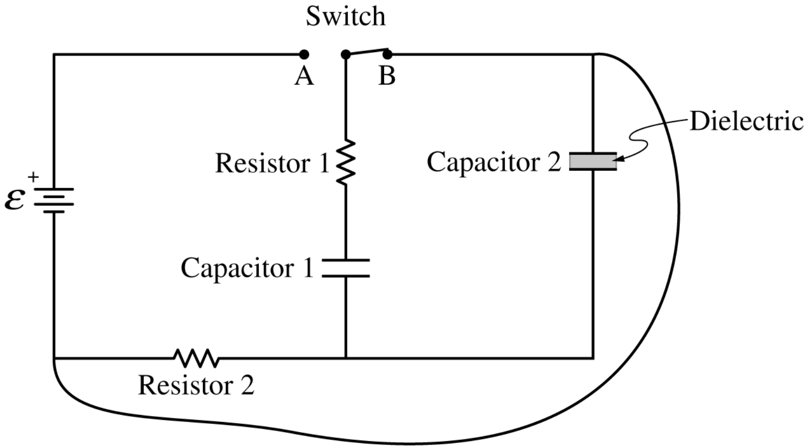
The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

Question 3

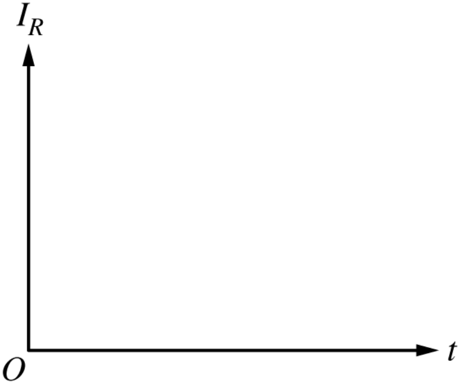
At time $t = 0$, the switch is closed to Position A.

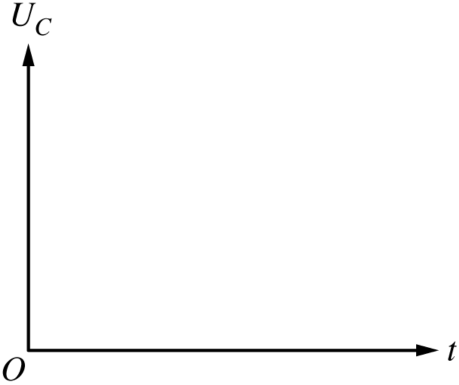
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R, C, Q, t, and fundamental constants, as appropriate.

Question 3

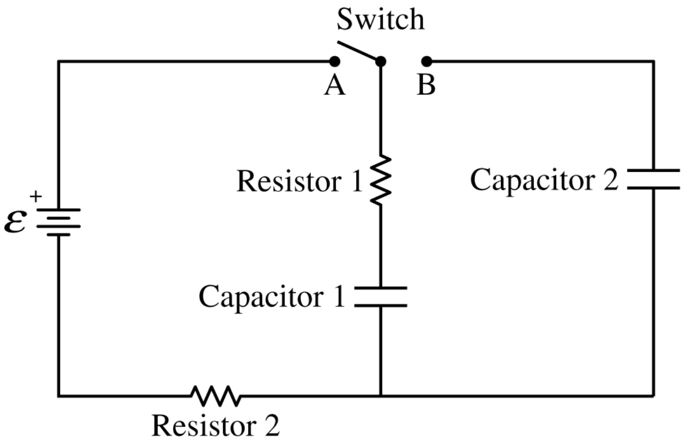

Switch
A
B
Dielectric
Resistor 1
Capacitor 2
ε
+
Capacitor 1
Resistor 2

Question 3





Question 3


Switch
A
B
Resistor 1
Capacitor 2
+
$\mathcal{E}$
Capacitor 1
Resistor 2

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.
[Graph 1: y-axis labeled "I_R", x-axis labeled "t", origin at O, no numeric scale shown.]
[Graph 2: y-axis labeled "U_C", x-axis labeled "t", origin at O, no numeric scale shown.]
A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.
At time t_1, the switch is closed to Position B.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(c)(i) Immediately after t_1, is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.
(c)(ii) Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
(c)(iii) Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C, Q_0, and fundamental constants, as appropriate.
With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.
[Figure: The same circuit as before (battery $\mathcal{E}$, switch positions A/B, Resistor 1, Capacitor 1, Resistor 2, Capacitor 2), now with a dielectric slab labeled "Dielectric" shown inserted between the plates of Capacitor 2. An additional wire of negligible resistance is shown connected between two corners of the circuit (from the node between the switch and Resistor 1/Capacitor 1, curving around to the node below Capacitor 2/Resistor 2), as described below.]
With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

Question 3

2023 Set 2 ▪ Q3

[Figure: A circuit diagram. A battery of emf $\mathcal{E}$ is on the left side of the circuit. A switch at the top has two positions, A and B (A on the left, B on the right). From position A, a branch leads down through Resistor 1 (resistance R) to a node; from position B, a branch leads down through Capacitor 2 (capacitance C) to the same node level on the right side. Capacitor 1 (capacitance C) is connected across the middle, below Resistor 1. Resistor 2 (resistance R) is at the bottom of the circuit, connecting back to the battery. The battery, Resistor 2, and the parallel combination of [Resistor 1 with Capacitor 1 below it] and [the switch branch to Capacitor 2] form the overall loop.]

The circuit shown consists of an ideal battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R, capacitors 1 and 2 each with capacitance C, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

Question 3

(e)(i) Express your answers to part (e)(i) and part (e)(ii) in terms of R, C, Q_0, and fundamental constants, as appropriate.
Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.
(e)(ii) Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

Solution 3

(a) Mark scheme

- Write, but do NOT solve, the differential equation for charge Q on the positive plate of Capacitor 1 vs time t after the switch closes to Position A, in terms of $\mathcal{E}, R, C, Q, t$. (1 point) Loop-rule expression that includes terms for the equivalent resistance $2R$ (the pair of resistors) and the potential difference across the battery. (1 point) An expression that includes charge Q in the term for the potential difference across the capacitor, and charge per unit time $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Result: $\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0 \;\Rightarrow\; \mathcal{E} - \frac{Q}{C} - 2R\frac{dQ}{dt} = 0.$

Solution 3

(b)

Mark scheme

- Sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 vs time t from $t = 0$ until steady state is nearly reached (RC charging curves). (1 point) I_R vs t is a CONCAVE UP, DECREASING curve starting at its maximum value at $t = 0$. (1 point) U_C vs t is a CONCAVE DOWN, INCREASING curve starting at zero. (1 point, can be earned even if the first two are not) Both curves approach a slope of zero (level off / asymptote) as time increases, consistent with exponential charging behavior.

Solution 3

(c)(i) Mark scheme

- Immediately after t_1 (switch moved to Position B, connecting the charged Capacitor 1 through Resistor 1 to the initially uncharged Capacitor 2), the current in Resistor 1 is directed UP. Justification (either form acceptable): positive charge has accumulated on the top plate of Capacitor 1 (and/or negative charge on its bottom plate) from being charged while the switch was at Position A, so at t_1 the higher potential top plate drives current up through Resistor 1 and down through Capacitor 2 to charge it. Equivalently: the top plate of Capacitor 1 is at higher electric potential than the bottom plate, so current flows from high to low potential, i.e. up through Resistor 1, around the loop through Capacitor 2.

Solution 3

(c)(ii) Mark scheme

- Determine the total charge on the positive plate of Capacitor 2 a long time after t_1: the answer is $\dfrac{Q_0}{2}$ (this point can be earned without supporting calculations). Justification: at new steady state, the potential difference across Capacitor 1 equals that across Capacitor 2 (parallel connection via Resistor 1 with no current, so no voltage drop across R1). Since Capacitor 2 has the same capacitance as Capacitor 1, it stores the same charge. By conservation of the total charge Q_0 originally on Capacitor 1, the two capacitors split it evenly, so Capacitor 2 stores $Q_0/2$.

Solution 3

(c)(iii) Mark scheme

- Derive an expression for the total energy dissipated by Resistor 1 from immediately after t_1 until the new steady state, in terms of C, Q_0. (1 point) Indicate that the energy dissipated equals the difference between the initial electric potential energy (at $t = t_1$) and the final electric potential energy (after new steady state): $\Delta E_R = U_C - U_{0C}$. (1 point) Indicate that only Capacitor 1 stores nonzero PE initially, and BOTH capacitors store PE after the new steady state (or an alternate response consistent with part (c)(ii)): $U_{0C} = U_{01}$ AND $U_C = U_1 + U_2$. (1 point) Correct substitution of the charges from part (c)(ii) ($Q_0/2$ on each capacitor). Full solution: $\Delta E_R = \left[\frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2 + \frac{1}{2}\frac{1}{C}\left(\frac{Q_0}{2}\right)^2\right] - \frac{1}{2}\frac{Q_0^2}{C} = \frac{Q_0^2}{4C} - \frac{Q_0^2}{2C} = -\frac{Q_0^2}{4C}$. The

Solution 3

magnitude of energy dissipated by Resistor 1 is $\dfrac{Q_0^2}{4C}$ (the negative sign reflects the decrease in stored electric PE, which is dissipated as heat).

Solution 3

(d) Mark scheme

- Determine the charge on the positive plate of Capacitor 2 a long time after a dielectric ($\kappa = 2$) is inserted between its plates, with the switch still at Position B: $Q_2 = \dfrac{2Q_0}{3}$ (this point can be earned without supporting calculations). Derivation: with $C_1 = C$ and $C_2 = \kappa C = 2C$, the plates are still connected so $\Delta V_{C1} = \Delta V_{C2} \Rightarrow \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow Q_1 = \frac{1}{2}Q_2$. Total charge is conserved: $Q_1 + Q_2 = Q_0 \Rightarrow \frac{1}{2}Q_2 + Q_2 = Q_0 \Rightarrow Q_2 = \dfrac{2}{3}Q_0$.

Solution 3

(e)(i) Mark scheme

- Derive an expression for the current in Resistor 2 immediately after an additional (negligible-resistance) wire is connected into the circuit, in terms of R, C, Q_0. (1 point) Loop-rule equation with terms for the potential difference across Resistor 2 and the potential difference across (dielectric-filled) Capacitor 2: $\Delta V_{R,2} - \Delta V_{C,2} = 0$. (1 point) Correct substitution of IR for the potential difference across Resistor 2: $\Delta V_{R,2} = IR$. (1 point) Correct substitution of $2C$ for Capacitor 2's new (dielectric) capacitance and of the charge from part (d) $(2Q_0/3)$ into the expression for $\Delta V_{C,2} = \dfrac{Q_2}{C_2} = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right)$. Full solution: $IR = \left(\dfrac{2Q_0}{3}\right)\left(\dfrac{1}{2C}\right) = \dfrac{Q_0}{3C} \Rightarrow I = \dfrac{Q_0}{3RC}$.

Solution 3

Solution 3

(e)(ii)

Mark scheme

- Determine the current in Resistor 2 a long time after the wire is connected to the circuit: the current is ZERO. At the new steady state, Capacitor 2 is fully charged (no more charge flow needed to maintain the loop's voltage balance), so no current flows through Resistor 2.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)(i) Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

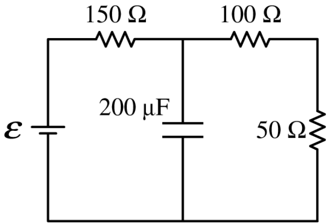
(a)(ii) Calculate the current in the $200\ \Omega$ resistor.

Question 2

(a)(iii) Calculate the power dissipated by the $200\ \Omega$ resistor.
[Figure 2: A circuit with a battery of emf $\mathcal{E}$, a $150\ \Omega$ resistor and $100\ \Omega$ resistor along the top in series from the battery's positive terminal, a $200\ \Omega$ resistor in the middle branch, a voltmeter V connected across the $200\ \Omega$ resistor, and a $50\ \Omega$ resistor in the right branch below the $100\ \Omega$ resistor.]
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

Question 2

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



Question 2

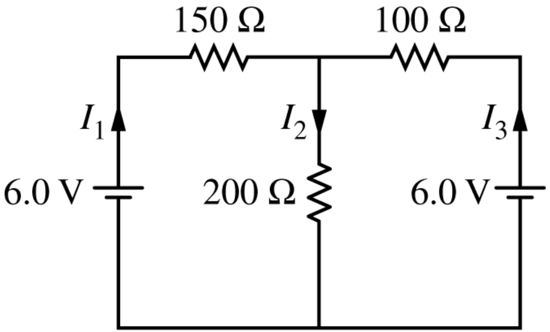


Figure 1

Question 2

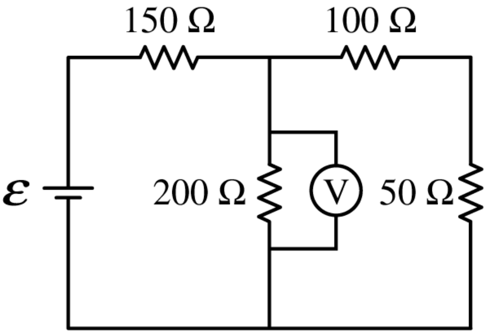


Figure 2

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(b) Calculate the current through the $50\ \Omega$ resistor.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

Question 2

2019 Set 1 ▪ Q2

[Figure 1: A circuit with two 6.0 V batteries and three resistors. The left branch has a $150\ \Omega$ resistor in series carrying current I_1 (arrow pointing up) connected to a 6.0 V battery. The middle branch has a $200\ \Omega$ resistor carrying current I_2 (arrow pointing up). The right branch has a $100\ \Omega$ resistor in series carrying current I_3 (arrow pointing up) connected to a 6.0 V battery. The $150\ \Omega$ and $100\ \Omega$ resistors are along the top, joining at a node above the $200\ \Omega$ resistor.]

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(d)(i) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with a $200\ \mu$F capacitor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

Question 2

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu$F capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

(d)(ii) [Figure: Same circuit as Figure 2 but the $200\ \Omega$ resistor is replaced with an ideal 50 mH inductor in the middle branch, battery $\mathcal{E}$, $150\ \Omega$ and $100\ \Omega$ resistors along the top, $50\ \Omega$ resistor in the right branch.]

The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b)?

$G^{reaterthan}L^{essthan}E^{qualto}$

Justify your answer.

Solution 2

(a)(i)

Mark scheme

- Using Kirchhoff's rules (do not solve): Junction rule at the node joining all three branches: $I_1 - I_2 + I_3 = 0$ (1 pt). Loop rule, left loop (through the left 6.0 V battery, the 150 ohm resistor, and the 200 ohm resistor): $6 - 150I_1 - 200I_2 = 0$ (1 pt). Loop rule, right loop (through the right 6.0 V battery, the 100 ohm resistor, and the 200 ohm resistor): $6 - 100I_3 - 200I_2 = 0$ (1 pt). (Equivalently, the outer loop $6 - 150I_1 + 100I_3 - 6 = 0$ may replace one of these; full credit is also earned for two correct loop equations written using loop currents.)

Solution 2

(a)(ii)

Mark scheme

- Combine the three equations from part (a)(i) (1 pt for combining them / using a calculator's system-solver, e.g. $I_1 - I_2 + I_3 = 0$, $-I_1 - 1.33I_2 = -0.04$, $-I_3 - 2I_2 = -0.06 \Rightarrow -4.33I_2 = -0.10$). Solving: $I_2 = \dfrac{-0.10}{-4.33} = 0.023$ A (1 pt for the correct answer with correct units).

Solution 2

(a)(iii)

Mark scheme

- $P = I_2^2 R = (0.023\ \text{A})^2 (200\ \Omega) = 0.107$ W. 1 point for using a correct power equation and computing this value.

Solution 2

(b) Mark scheme

- In the modified circuit (Figure 2), the 100 ohm and 50 ohm resistors are in series in the right-hand branch, so their combined resistance is $R = 100\ \Omega + 50\ \Omega = 150\ \Omega$ (1 pt). This branch is in parallel with the 200 ohm resistor, across which the voltmeter reads 4.4 V, so that same 4.4 V is the potential difference across the 150 ohm series combination: $I = \frac{V}{R} = \frac{4.4\ \text{V}}{100\ \Omega + 50\ \Omega} = 0.029$ A (1 pt for a correct answer using the correct potential difference).

Solution 2

(c) Mark scheme

- Using $I_1 = I_2 + I_3$ (1 pt for the correct equation determining current through the 150 ohm resistor) and substituting $I_3 = 0.029$ A from part (b) with $I_2 = \dfrac{4.4\text{ V}}{200\ \Omega}$ (1 pt for correct substitution): $I_1 = \dfrac{4.4\text{ V}}{200\ \Omega} + 0.029\text{ A} = 0.051\text{ A}$. Then apply the loop equation through the battery/150-ohm branch (1 pt): $\mathcal{E} = I_1 R_1 + V_{200\Omega\text{ branch}} = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$. (Equivalent alternate method: find the circuit's total equivalent resistance $R_T = 150\ \Omega + \left(\frac{1}{200} + \frac{1}{150}\right)^{-1}\Omega = 236\ \Omega$, giving $\mathcal{E} = I_1 R_T = (0.051\text{ A})(236\ \Omega) \approx 12.0\text{-}12.1\text{ V}$ – also full credit.)

Solution 2

(d)(i) Mark scheme

- At steady state, the fully-charged 200 uF capacitor carries no current, so the 150 ohm, 100 ohm, and 50 ohm resistors form a single series loop carrying the same current (1 pt for correctly finding the total series resistance $R_{tot} = 150 + 100 + 50 = 300\ \Omega$). Using the battery voltage found in part (c), $\mathcal{E} = 12.1$ V (1 pt for substituting this consistent voltage into Ohm's law): $I = \dfrac{\mathcal{E}}{R_{tot}} = \dfrac{12.1\text{ V}}{300\ \Omega} = 40.3$ mA.

Solution 2

(d)(ii)

Mark scheme

- Less than (1 pt for correctly selecting 'Less than' with an attempt at a relevant justification). Justification (1 pt): at steady state an ideal inductor acts as a short circuit (zero resistance / zero back-emf), so essentially all the current from the source flows through the (now zero-resistance) inductor branch and none flows through the 100-ohm + 50-ohm branch – so the current through the 50 ohm resistor is less than the 0.029 A found in part (b) (in fact it approaches zero).

Question 1

2019 Set 2 ▪ Q1

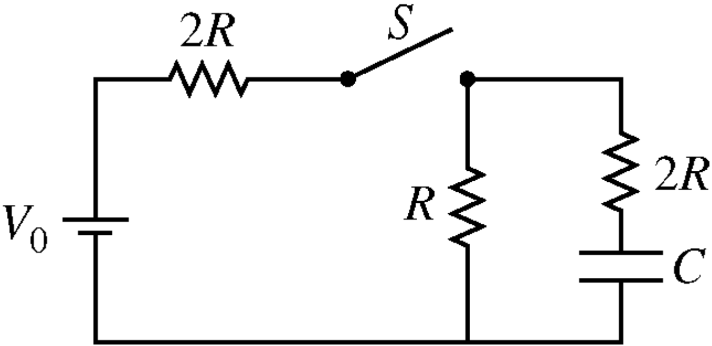
[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) Derive an expression for the steady-state current supplied by the battery.

Question 1

Question 1



Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(b) Derive an expression for the charge on the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(c) Derive an expression for the energy stored in the capacitor.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(d) Now the switch is opened at time $t = 0$.

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(e)(i) Calculate the current in resistor R immediately after the switch is opened.

(e)(ii) On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 1

[Graph with vertical axis labeled "Current" and horizontal axis labeled "Time"; origin labeled O; axes are blank/unscaled for the student to sketch on.]

Question 1

2019 Set 2 ▪ Q1

[Circuit diagram: A battery of voltage V_0 on the left, in series with a resistor labeled $2R$ and a switch S. After the switch, the circuit splits into two parallel branches: a resistor labeled R, and a branch containing a resistor labeled $2R$ in series with a capacitor labeled C.]

The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution 1

(a) Mark scheme

- At steady state the capacitor is fully charged and draws no current, so no current flows through the branch containing the second $2R$ resistor and C. The circuit reduces to the top $2R$ in series with R (effective resistance $3R$). By Ohm's law: $I = \dfrac{V_0}{R_{eff}} = \dfrac{V_0}{2R+R}$, so $I = \dfrac{V_0}{3R}$. (1 pt: using Ohm's law with the correct effective resistance; 1 pt: correct substitution leading to the correct answer.)

Solution 1

(b) Mark scheme

- The stored charge is $q = CV_C$. At steady state no current flows through the $2R$-C branch, so $V_C = V_R$ (the parallel branch's voltage). $V_R = IR = \left(\frac{V_0}{3R}\right) R = \frac{1}{3} V_0$. Therefore $q = CV_R = \frac{1}{3} CV_0$. (1 pt: using the equation relating stored charge to capacitance; 1 pt: correctly determining V_R and substituting.)

Solution 1

(c) Mark scheme

- Energy stored in a capacitor: $U = \dfrac{q^2}{2C}$. Substituting $q = \dfrac{1}{3}CV_0$ from part (b): $U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$. (1 pt: any correct equation for energy stored in a capacitor; 1 pt: correct substitution of charge and/or voltage from part (b).)

Solution 1

(d) Mark scheme

- After the switch opens, current can only flow around the loop containing R, the second $2R$, and C. Voltage loop equation: $V_C - V_R - V_{2R} = 0 \Rightarrow \dfrac{q(t)}{C} = I(R + 2R)$. Since the capacitor is discharging, $I = -\dfrac{dq}{dt}$, giving the differential equation $\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$ (do not solve it). (1 pt: any correct voltage loop equation for when the switch is open; 1 pt: substituting $-dq/dt$ or dq/dt for the current, consistent with the loop equation.)

Solution 1

(e)(i) Mark scheme

- The capacitor voltage cannot change instantaneously, so immediately after the switch opens $V_C(0^+) = V_R(0^+) = \dfrac{V_0}{3}$ (from part (b)). Current now flows through the total resistance $R + 2R = 3R$: $I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$. (1 pt: using a voltage consistent with part (b) in $I = V/R$; 1 pt: substituting the correct total resistance $3R$ into Ohm's law.)

Solution 1

(e)(ii) Mark scheme

- The current starts at its maximum value $I_0 = \dfrac{V_0}{9R}$ (from part (e)(i)) at $t = 0$ and decays exponentially toward zero as the capacitor discharges: $I(t) = \dfrac{V_0}{9R}e^{-t/3RC}$. The sketch should show a curve that is concave up throughout, decreasing monotonically toward the horizontal (time) axis as an asymptote, with the initial value explicitly labeled $V_0/9R$ on the vertical axis at $t = 0$. (1 pt: curve concave up throughout the graph; 1 pt: horizontal axis as an asymptote; 1 pt: labeling the maximum current, consistent with part (e)(i).)

Solution 1

(f) Mark scheme

- Equal to. Justification: after the switch opens, the capacitor discharges all of its stored energy and charge; assuming no energy is lost in the connecting wires, the only circuit elements that dissipate this energy are the two resistors (R and 2R) in series with the capacitor, so the total energy dissipated in the resistors equals the energy that was stored in the capacitor (conservation of energy). (1 pt: selecting \"Equal to\"; 1 pt: a correct justification invoking conservation of energy.) [An alternate full-credit path selects \"Less than\" with justification that some energy is additionally lost due to resistance in the connecting wires and/or losses in the capacitor.]

Question 2

2015 ▪ Q2

[Figure: A circuit diagram. At the top, a voltmeter (circle labeled V) is connected across a variable resistor (labeled R, drawn as a resistor symbol with a diagonal arrow through it, indicating it is adjustable). Below, in a dashed box, a battery is shown as an internal resistance r (resistor symbol) in series with an emf source $\mathcal{E}$. The battery (dashed box) is connected in series with the variable resistor R, and the voltmeter is connected across R.]

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

Question 2

Trial	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)(i) Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

(a)(ii) Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

Question 2

shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The shown in the table below.

Trial #	Resistance (Ω)	Voltage (V)	$1/R$ $(1/\Omega)$	$1/V$ $(1/\text{V})$
1	0.50	5.6	2.00	0.179
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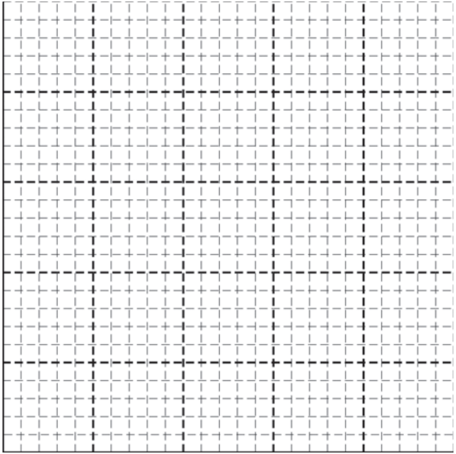
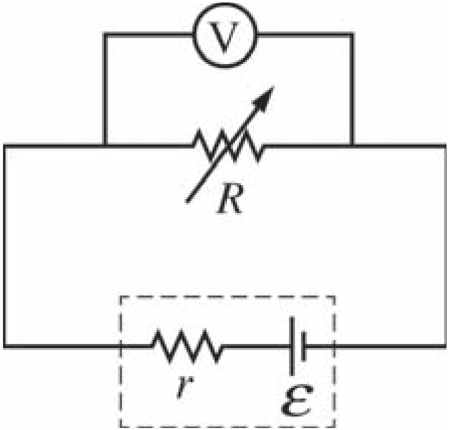
Question 2



Question 2



Question 2

2015 ▪ Q2

[Figure: A circuit diagram. At the top, a voltmeter (circle labeled V) is connected across a variable resistor (labeled R, drawn as a resistor symbol with a diagonal arrow through it, indicating it is adjustable). Below, in a dashed box, a battery is shown as an internal resistance r (resistor symbol) in series with an emf source $\mathcal{E}$. The battery (dashed box) is connected in series with the variable resistor R, and the voltmeter is connected across R.]

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

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6	10	11.4	0.10	0.088

Question 2

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.
[Figure: A blank grid (approximately 30 x 30 gridlines with heavier gridlines every 6 squares) for plotting $1/V$ vs. $1/R$.]

Question 2

2015 ▪ Q2

[Figure: A circuit diagram. At the top, a voltmeter (circle labeled V) is connected across a variable resistor (labeled R, drawn as a resistor symbol with a diagonal arrow through it, indicating it is adjustable). Below, in a dashed box, a battery is shown as an internal resistance r (resistor symbol) in series with an emf source $\mathcal{E}$. The battery (dashed box) is connected in series with the variable resistor R, and the voltmeter is connected across R.]

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

Trial	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ ($1/\text{V}$)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(c)(i) Use the straight line from part (b) to obtain a value for $\mathcal{E}$.

Question 2

(c)(ii) Use the straight line from part (b) to obtain a value for r.

Question 2

2015 ▪ Q2

[Figure: A circuit diagram. At the top, a voltmeter (circle labeled V) is connected across a variable resistor (labeled R, drawn as a resistor symbol with a diagonal arrow through it, indicating it is adjustable). Below, in a dashed box, a battery is shown as an internal resistance r (resistor symbol) in series with an emf source $\mathcal{E}$. The battery (dashed box) is connected in series with the variable resistor R, and the voltmeter is connected across R.]

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5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

Question 2

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

Question 2

2015 ▪ Q2

[Figure: A circuit diagram. At the top, a voltmeter (circle labeled V) is connected across a variable resistor (labeled R, drawn as a resistor symbol with a diagonal arrow through it, indicating it is adjustable). Below, in a dashed box, a battery is shown as an internal resistance r (resistor symbol) in series with an emf source $\mathcal{E}$. The battery (dashed box) is connected in series with the variable resistor R, and the voltmeter is connected across R.]

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

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5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

Question 2

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

______ The voltmeter with low resistance will provide the most accurate measurement.

______ The voltmeter with high resistance will provide the most accurate measurement.

______ The two voltmeters will provide equal accuracy.

Justify your answer.

Solution 2

(a)(i) Mark scheme

- Apply Kirchhoff's loop rule around the circuit containing the battery (emf $\mathcal{E}$, internal resistance r) and the variable resistor R: $\mathcal{E} = Ir + IR$ (1 point for a correct application of Kirchhoff's loop rule), so $I = \dfrac{\mathcal{E}}{r + R}$. The voltmeter reads the voltage across the variable resistor, so by Ohm's law $V = IR = \dfrac{\mathcal{E}}{r + R}R = \dfrac{\mathcal{E}R}{r + R}$ (1 point for a correct expression for the measured voltage across the variable resistor).

Solution 2

(a)(ii) Mark scheme

- Take the reciprocal of $V = \dfrac{\mathcal{E}R}{r+R}$ from part (a)-i and rearrange into a form linear in $1/R$: $\dfrac{1}{V} = \dfrac{r+R}{\mathcal{E}R} = \left(\dfrac{r}{\mathcal{E}}\right)\dfrac{1}{R} + \dfrac{1}{\mathcal{E}}$ (1 point; the expression must be consistent with the answer from part (a)-i). This is a straight line in $1/R$ with slope $r/\mathcal{E}$ and y-intercept $1/\mathcal{E}$.

Solution 2

(b) Mark scheme

- Plot $1/V$ (vertical axis, units of $1/\text{V}$) versus $1/R$ (horizontal axis, units of $1/\Omega$) using the given data table. Credit: 1 point for correctly labeling both axes with variables and units; 1 point for correctly scaling both axes with an acceptable and appropriate scale (spanning the data, e.g. $1/R$ from 0 to about $2.5\ \Omega^{-1}$ and $1/V$ from 0 to about $0.25\ \text{V}^{-1}$); 1 point for correctly plotting the given data points; 1 point for drawing a single straight best-fit line through the data (not simply connecting the dots). The reference line in the scoring guidelines has a y-intercept near $0.080\ \text{V}^{-1}$ and rises to about $0.20\ \text{V}^{-1}$ near $1/R = 2.3\ \Omega^{-1}$.

Solution 2

(c)(i) Mark scheme

- Read the y-intercept of the best-fit line drawn in part (b): $b \approx 0.080\ \mathsf{V}^{-1}$ (1 point; the value must be consistent with the line drawn in part (b)). From the linear form in part (a)-ii, $\frac{1}{V} = \left(\frac{r}{\mathcal{E}}\right)\frac{1}{R} + \frac{1}{\mathcal{E}}$, the intercept equals $b = 1/\mathcal{E}$ (1 point for correctly substituting into the equation from part (a)-ii), so $\mathcal{E} = \frac{1}{b} = \frac{1}{0.080\ \mathsf{V}^{-1}} = 12.5\ \mathsf{V}$.

Solution 2

(c)(ii)

Mark scheme

- Calculate the slope of the best-fit line from part (b) using two points that lie on the line itself, not the raw data points, e.g. $(0.40,\ 0.100)$ and $(1.50,\ 0.151)$: $m = \dfrac{0.151 - 0.100}{1.50 - 0.40} = 0.0463\ \Omega/\mathrm{V}$ (1 point for calculating the slope using the line, not data points). From part (a)-ii, the slope equals $r/\mathcal{E}$, so $r = m\mathcal{E}$ (1 point for correctly substituting into the equation from part (a)-ii); using $\mathcal{E} = 12.5$ V from part (c)-i, $r = (0.0463\ \Omega/\mathrm{V})(12.5\ \mathrm{V}) = 0.58\ \Omega$.

Solution 2

(d)

Mark scheme

- The maximum current the battery can supply occurs when $R = 0$ (short circuit), so Ohm's law applied to the internal resistance alone gives $\mathcal{E} = Ir$ (1 point for using Ohm's law). Substituting the values found in part (c): $(12.5\ \text{V}) = I(0.58\ \Omega)$ (1 point for a correct substitution of the values from part (c)), giving $I = 21.6$ A.

Solution 2

(e) Mark scheme

- Select the voltmeter with high internal resistance (1 point). Justification: a real voltmeter behaves like a resistor placed in the circuit with the battery, and it measures the potential difference across its own internal resistance. The higher that internal resistance, the smaller the current it draws from the battery, and the closer the voltmeter's reading comes to the actual emf of the battery (1 point for a correct justification). Example from the scoring guidelines: 'The voltmeter acts like a resistor in a circuit with the battery. It will measure the potential difference across its own internal resistance. The higher its internal resistance, the closer its potential difference will be to the emf of the battery.'