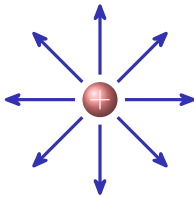


Electric Charges, Fields, & Gauss's Law

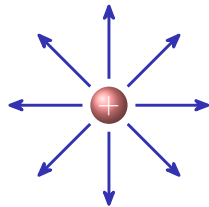
AP Physics C: E&M ■ Unit 8

AP Physics C: E&M



What we will cover

- 1 Charge and Coulomb's law
- 2 The process of charging
- 3 Electric fields and field lines
- 4 Fields of charge distributions
- 5 Electric flux
- 6 Gauss's law





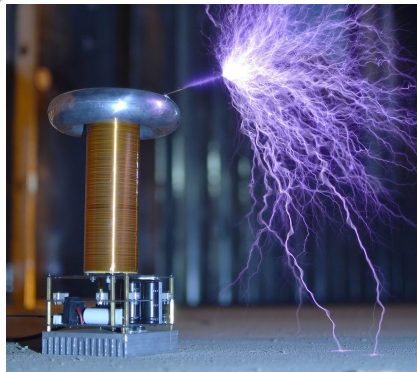
Charge and Coulomb's law

Charge is **quantized** ($q = ne$) and **conserved** :

A vector, inverse-square force

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}, \quad k = \frac{1}{4\pi\epsilon_0}$$

Forces from many charges add by **superposition** – the vector sum.



Tesla coil discharge

Charge is quantized, and it is conserved

quantized

every charge is a whole-number multiple of the **elementary charge** $e = 1.6 \times 10^{-19} \text{ C}$

conservation of charge

charge is never created or destroyed, only moved – the total of an isolated system is fixed

point charge

a model that ignores the size of the object; Coulomb's law is exact only for these

 ϵ_0

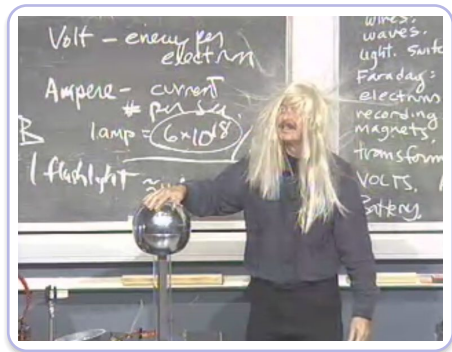
the **permittivity of free space** ; $k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$

Charge is a **fundamental property** of matter, like mass. Like charges repel, opposites attract – and no experiment has ever produced a fraction of e in isolation.



The process of charging

- a **conductor** lets charge move; an **insulator** holds it fixed
- **friction** – electrons rubbed across
- **conduction** – touch: **same** sign
- **induction** – near + **ground** : **opposite** sign



Static charge builds on a Van de Graaff generator

Three ways to charge an object

charging by friction

rubbing two materials transfers electrons from one to the other – they end up equal and opposite

charging by conduction

a charged body *touches* a neutral one; charge flows until both share it, so **same sign** results

charging by induction

a charged body is brought *near*, the neutral body is grounded then ungrounded – the result is the **opposite** sign, with no contact

polarization

an **insulator** cannot pass charge, but its molecules stretch or turn so one face goes slightly positive – which is why a charged rod attracts paper

An **electroscope** shows all of this: its leaves diverge when like charge spreads onto both of them.



Electric fields and field lines

The **electric field** is force per unit positive **test charge** :

Field of a point charge

$$\vec{E} = \frac{\vec{F}}{q_0}, \quad \vec{E} = k \frac{q}{r^2} \hat{r}$$

- lines leave +, enter -; denser = stronger
- lines never cross; $\vec{F} = q\vec{E}$ on any charge



dipole field

Worked example – Coulomb's law with numbers

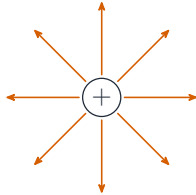
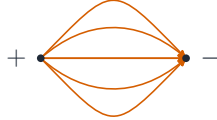
Two $+2.0 \mu\text{C}$ point charges sit 0.30 m apart. Find the force on each.

- $F = \frac{kq_1q_2}{r^2}$
- $= \frac{(9.0 \times 10^9)(2.0 \times 10^{-6})^2}{(0.30)^2}$
- $= 0.40 \text{ N, repulsive}$

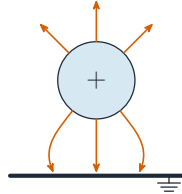
Both charges feel 0.40 N : Newton's third law, whatever the sizes of q_1 and q_2 . Get the **direction** from the signs, never from the algebra.

Electric field-line patterns for parallel plates

a) parallel plates (uniform field) b) dipole (+ and -)



c) point charge (radial)



d) sphere above earthed plate

Electric field-line patterns for parallel plates, a dipole, and a point charge



Fields of charge distributions

Split a spread-out charge into pieces dq and **integrate**:

Continuous superposition

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

Densities : $\lambda = \frac{dq}{d\ell}$, $\sigma = \frac{dq}{dA}$, $\rho = \frac{dq}{dV}$. Use **symmetry** to cancel components.

Two standard results

$$\text{ring (axis)} : E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$\text{infinite line} : E = \frac{2k\lambda}{r}$$

Point $\sim 1/r^2$, line $\sim 1/r$, sheet $\sim$ uniform.

Choose the density that matches the geometry

linear λ

$dq = \lambda dl$ – a rod, a ring, an arc

surface σ

$dq = \sigma dA$ – a disc, a sheet, a shell

volume ρ

$dq = \rho dV$ – a solid sphere or cylinder

then integrate

$\vec{E} = \int \frac{k dq}{r^2} \hat{r}$, and use **symmetry** to kill the components that
cancel

Picking λ , σ or ρ correctly is the **first marked step** of every distribution FRQ. Write dq before you write the integral.

Charge densities, and the symmetry to match

Linear charge density Linear charge density λ in C/m – for a rod or a wire.

Surface charge density Surface charge density σ in C/m² – for a sheet or a conductor's surface.

Volume charge density Volume charge density ρ in C/m³ – for a solid insulating ball.

Match the symmetry Gauss's law is only useful with symmetry: a Gaussian cylinder for **cylindrical symmetry**, a sphere for **spherical symmetry**, a pillbox for **planar symmetry**.

Choose the surface so that $\vec{E}$ is constant and either parallel or perpendicular to it everywhere. Then the integral becomes multiplication.



Electric flux

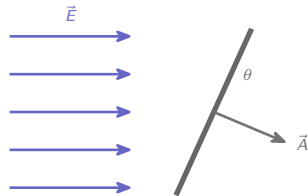
Electric flux counts the field lines through a surface:

Uniform, then general

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Largest when $\vec{E} \perp$ the surface ($\theta = 0$); zero when parallel.



Worked example – the field on a ring's axis

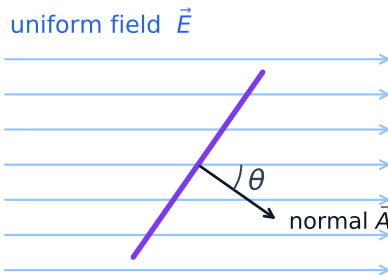
**A ring of radius R carries total charge Q .
Find E at a point a distance z along its axis.**

- every element is the same distance $\sqrt{R^2 + z^2}$ away
- the perpendicular components **cancel** by symmetry; only the axial part survives
- $\cos \theta = \frac{z}{\sqrt{R^2 + z^2}}$, so $E = \frac{kQz}{(R^2 + z^2)^{3/2}}$

Check the limits: at $z = 0$, $E = 0$ (the centre); for $z \gg R$ it becomes kQ/z^2 , the point charge again.

State the cancellation – that sentence is the mark.

Electric flux through a surface depends on



$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta \quad \cdot \quad \text{max when } \vec{E} \parallel \vec{A}$$

Electric flux through a surface depends on the angle between the field and the surface normal



Gauss's law

One of Maxwell's equations – flux out of a closed surface reveals the charge inside:

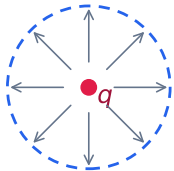
Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

Choose a **Gaussian surface** matching the [symmetry](#) so E comes out of the integral.

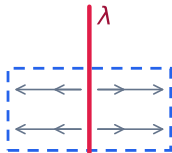
Gauss's law, in detail

Spherical
(point / sphere)



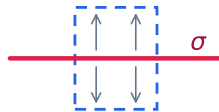
E radial, constant on r · $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

Cylindrical
(line / cylinder)



E radial from line · $E = \frac{\lambda}{2\pi\epsilon_0 r}$

Planar
(sheet / pillbox)



$E \perp$ sheet, both sides · $E = \frac{\sigma}{2\epsilon_0}$

Gauss: $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\epsilon_0$

Choose a Gaussian surface so E is constant (or zero) on each face — then

$$\Phi_E = Q_{\text{enc}}/\epsilon_0$$

Sphere, cylinder, or pillbox – matched to the symmetry.

A Gaussian surface is chosen to match the symmetry of the charge

A full-credit Gauss's-law answer has four parts

1. name the surface

say which **Gaussian surface** you chose and where it sits

2. state the symmetry

why E is **constant** on it and **perpendicular** to it – the **area vector** $d\vec{A}$ points along the outward normal

3. compute the flux

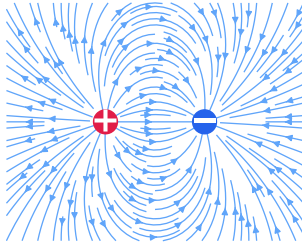
$\Phi_E = \oint \vec{E} \cdot d\vec{A} = EA$, with A the surface's own area

4. equate and solve

$EA = \frac{Q_{\text{enc}}}{\epsilon_0}$, then say clearly *which* charge is enclosed

Three symmetries qualify and no others: **spherical**, **cylindrical** and **planar**. Anything else needs integration.

Electric Fields

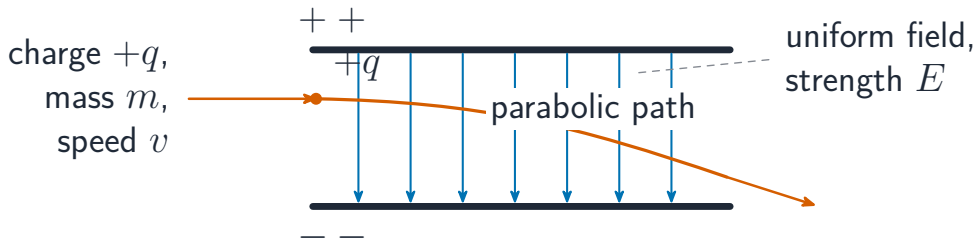


$$\vec{p} = q\vec{d} \cdot \text{lines leave } + \text{ enter } - \cdot$$

denser $\Leftrightarrow$ stronger $|\vec{E}|$

Electric field lines of a dipole point from the positive to the negative charge

Electric Fields, continued



A charge crossing a uniform field follows a parabolic path

In a uniform field, a charge is a projectile

the force

$\vec{F} = q\vec{E}$ is **constant**, so the acceleration $a = qE/m$ is constant too

launched along the field

straight-line motion with constant acceleration – the SUVAT equations, unchanged

launched *across* the field

constant velocity one way, constant acceleration the other – a **parabola**

the analogy

exactly **projectile motion** under gravity, with qE/m in place of g

Every capacitor-deflection question is a projectile question wearing different symbols. Split the motion into the two perpendicular directions and reuse the mechanics you already have.

Exam tips

- Use **Coulomb's law** $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ and superpose forces as **vectors** (components, not magnitudes).
- For a continuous charge, integrate $d\vec{E}$ over $dq = \lambda dl, \sigma dA, \rho dV$; use symmetry to cancel components.
- The field points **away from** positive and **toward** negative charge; draw field lines correctly.
- Distinguish the force on a charge ($\vec{F} = q\vec{E}$) from the field the charge creates.
- Keep the constant $k = \frac{1}{4\pi\epsilon_0}$ straight and check units.



Unit 8 – key takeaways

1

Coulomb

$$\vec{F} = kq_1q_2\hat{r}/r^2; \text{ superpose vectors}$$

2

Field

$$\vec{E} = kq\hat{r}/r^2; \text{ integrate } \int k dq\hat{r}/r^2 \text{ for distributions}$$

3

Flux

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}; \text{ lines through a surface}$$

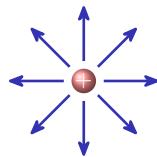
4

Gauss

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\epsilon_0; \text{ use symmetry}$$

Check yourself

- 1 Do charges outside a closed surface contribute net flux through it?
- 2 What integral gives the field of a continuous charge distribution?
- 3 When is the flux through a flat surface zero?
- 4 Where does all excess charge sit on a conductor?



Answers 1. no (zero net) 2. $\int k dq \hat{r}/r^2$ 3. when $\vec{E}$ is parallel to it 4. on its outer surface