HOLIDAY HOMEWORK 假期作业

AP Physics C: Electricity and Magnetism

Exercise sheets and past-paper practice, topic by topic.

每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

Contents 目录

8 Electric charge and Coulomb's law – Part 1

Name: ______________ Class: __________ Date: ____________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Electric charge	电荷			
Coulomb's law	库仑定律			

Recall

You have seen electric charge in everyday life:

- Rub a balloon on your hair and it sticks to a wall.
- Like charges push apart; opposite charges pull together.
- Static shocks are charge jumping across a gap.

This sheet makes the force between charges exact. Each idea has a worked example.

New ideas

1 Electric charge

Electric charge 电荷 comes in two kinds, positive and negative. Like charges **repel**; opposite charges **attract**. Charge is carried by electrons moving between objects.

2 Coulomb's law

The force between two point charges is **Coulomb's law** 库仑定律:

$$F = \frac{kq_1q_2}{r^2},$$

with $k = 9.0 \times 10^9$. It is an inverse-square law – like gravity, but it can push or pull.

Worked example. Two charges of 3.0 μC and 2.0 μC, 0.10 m apart: $F =$

$$\frac{9.0 \times 10^9 \times (3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.10)^2} = 5.4 \text{ N}.$$

■ 3 Conductors and insulators

A **conductor** lets charge move freely (metals); an **insulator** holds it in place (plastic, rubber).

■ 4 Charge is conserved

Charge is never created or destroyed, only moved from one object to another.

Watch out: Coulomb's law is inverse-square – **doubling** the distance makes the force **four times smaller**, not two times. Always square the distance.

Practice

Try these in order. The methods are all above.

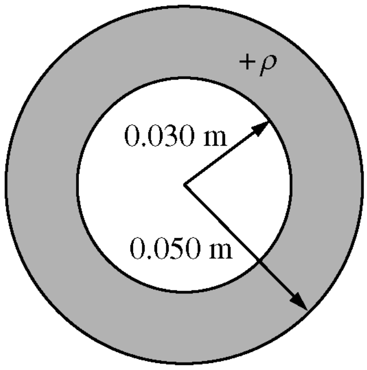
2.1 State whether the force is a push or a pull between (a) two negative charges, (b) a positive and a negative charge. [2]

2.2 Two charges of 2.0 μC and 2.0 μC are 0.20 m apart. Find the force ($k = 9.0 \times 10^9$). [3]

2.3 Give one example of a conductor and one of an insulator. [2]

2.4 Two charges are moved from 0.10 m to 0.30 m apart. State the factor by which the force changes. [2]

2.5 State what "charge is conserved" means. [1]



2. A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \quad \rho = 0$$

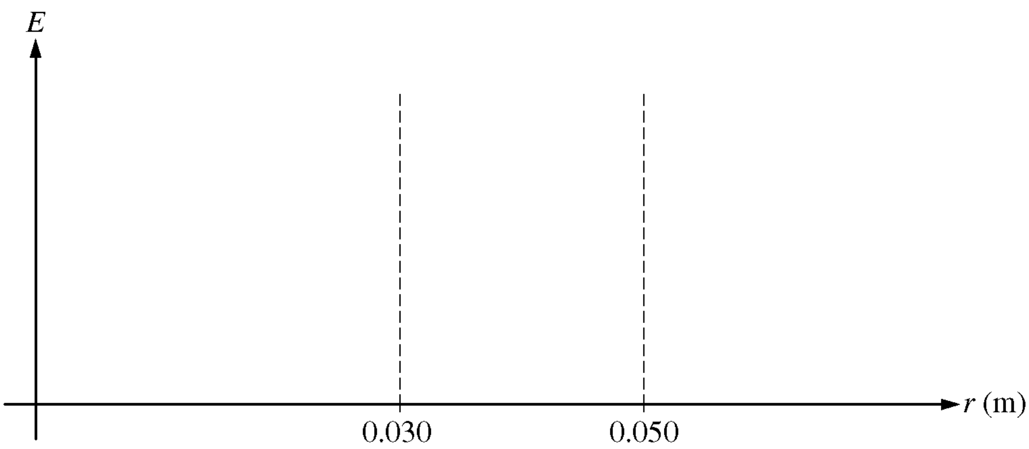
$$0.030 \text{ m} < r < 0.050 \text{ m}: \quad \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \quad \rho = 0$$

(a) Calculate the total charge of the sphere.

(b) Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

(c) On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.

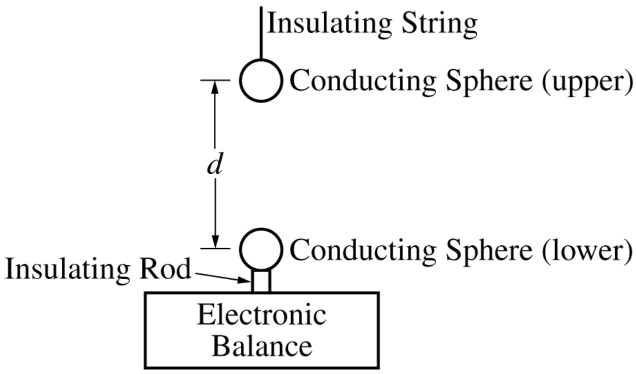


(d) Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

(e) A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

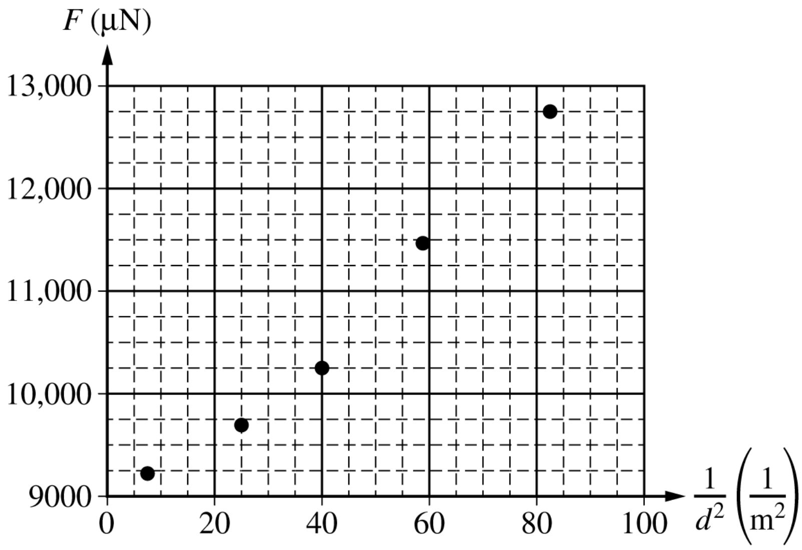
i. Calculate the magnitude of the initial acceleration of the proton.

ii. Calculate the speed of the proton after a long time.



Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



(a)

i. Draw a line that represents the best fit to the points shown.

ii. Use the graph to calculate the charge Q.

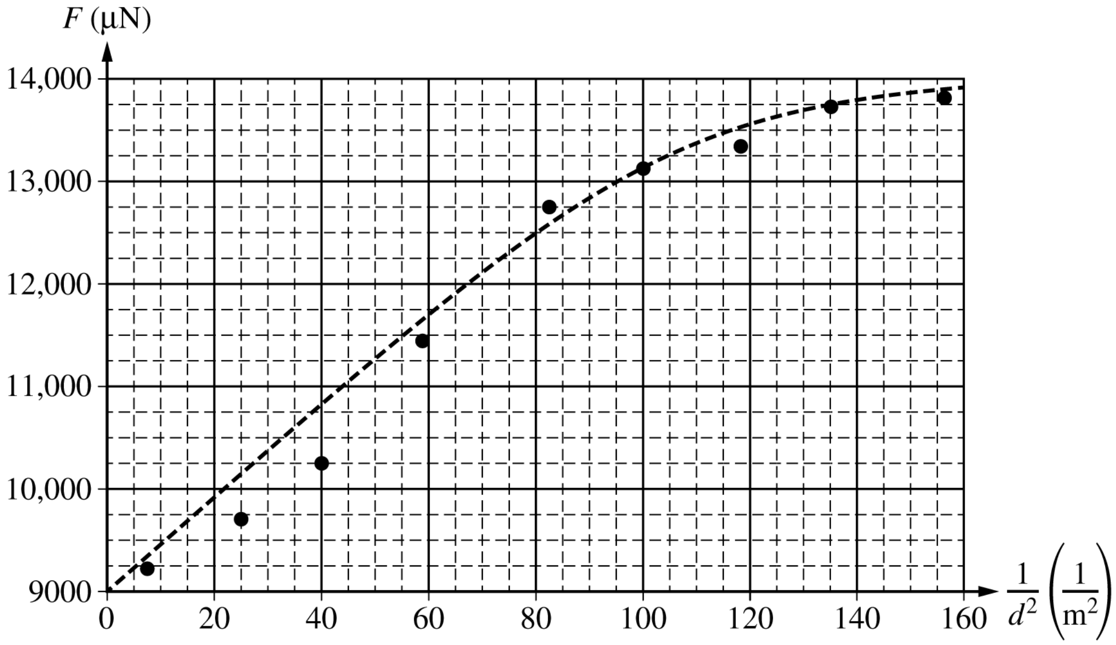
GO ON TO THE NEXT PAGE.

iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

iv. Determine the distance between the centers of the spheres for the data point indicated above.

v. What physical quantity does the vertical intercept represent?

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

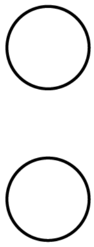
____ Yes ______ No

Justify your answer.

GO ON TO THE NEXT PAGE.

) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

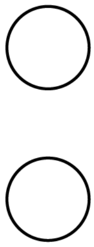
i. On the circles representing the spheres below, use a single "+" sign on each sphere to represent the locations of highest concentration of the excess positive charges.



ii. Explain how this rearrangement could be responsible for the discrepancy.

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single "+" sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single "–" sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

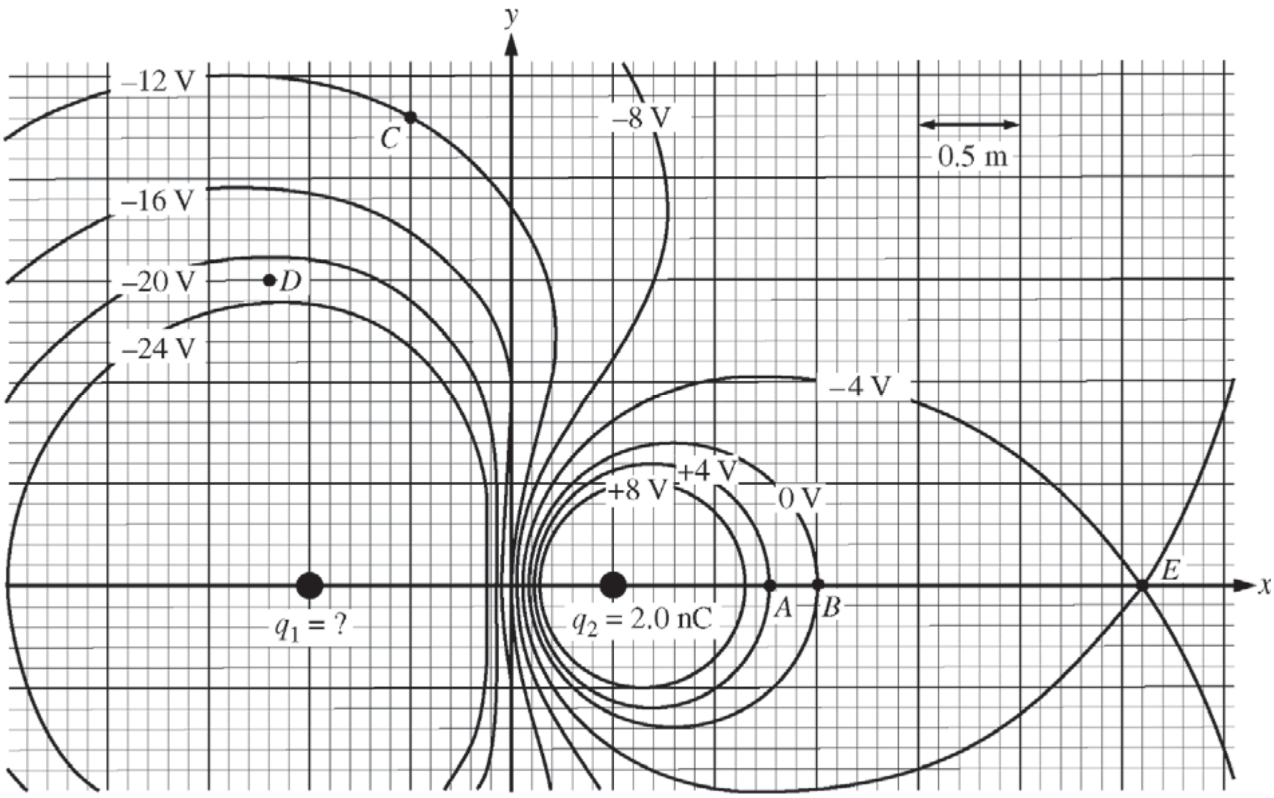
_____ Greater than   _____ Less than   _____ Equal to

Justify your answer.

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

8 The electric field – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
electric field	电场	____________ ____________ ____________

Recall

In Part A you met the force between charges. Now the **field** that carries that force:

- A charge seems to reach across empty space to push another.
- We describe this with an “electric field” filling the space around a charge.

Each idea has a worked example before the Practice.

New ideas

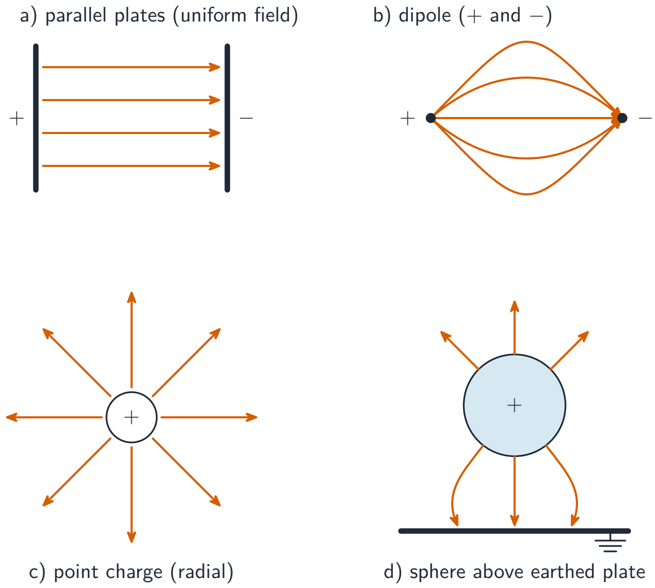
■ 1 The electric field

An **electric field** 电场 E is the force felt per unit of charge at a point:

$$E = \frac{F}{q}, \qquad \text{so a charge in a field feels} \qquad F = qE.$$

■ 2 Field lines

Field lines point **away** from positive charges and **toward** negative ones. Where the lines are denser, the field is stronger.



3 Force on a charge

Worked example. A field of 2000 N/C acts on a 3.0×10^{-6} C charge: $F = qE = 3.0 \times 10^{-6} \times 2000 = 6.0 \times 10^{-3}$ N.

4 The field of a point charge

A single point charge Q makes a field $E = \dfrac{kQ}{r^2}$ – stronger close in, weaker far away (inverse-square, like the force).

Watch out: the field points the way a **positive** test charge would be pushed. A negative charge in the same field feels a force in the **opposite** direction to the field.

Practice

Try these in order. The methods are all above.

2.1 State what an electric field is, in words. [2]

2.2 State which way field lines point around a positive charge. [1]

2.3 A field of 500 N/C acts on a 4.0×10^{-6} C charge. Find the force. [2]

2.4 State what a denser pattern of field lines means. [1]

2.5 State the direction of the force on a negative charge placed in a field. [2]

PHYSICS C: ELECTRICITY AND MAGNETISM

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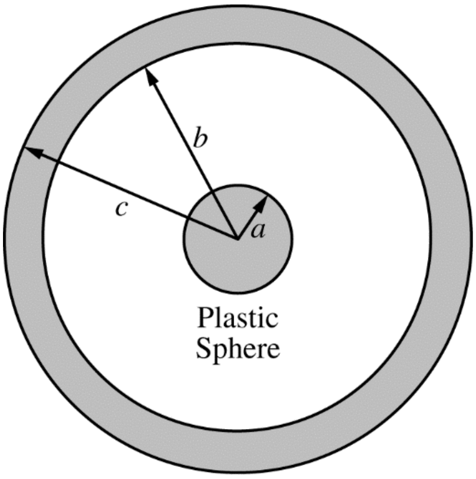


Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

      i. $r < a$

      ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

      i. Determine the charge on the inner surface of the conducting shell.

      Justify your answer.

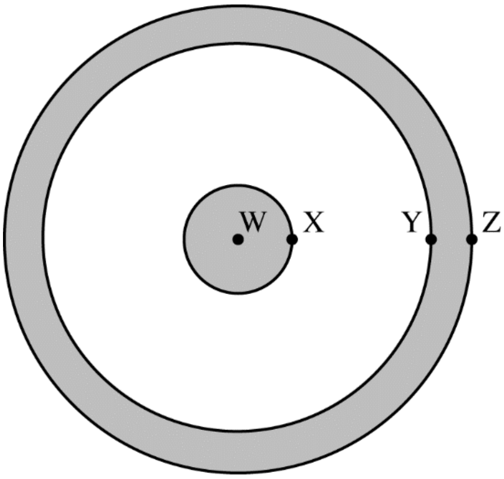
      ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



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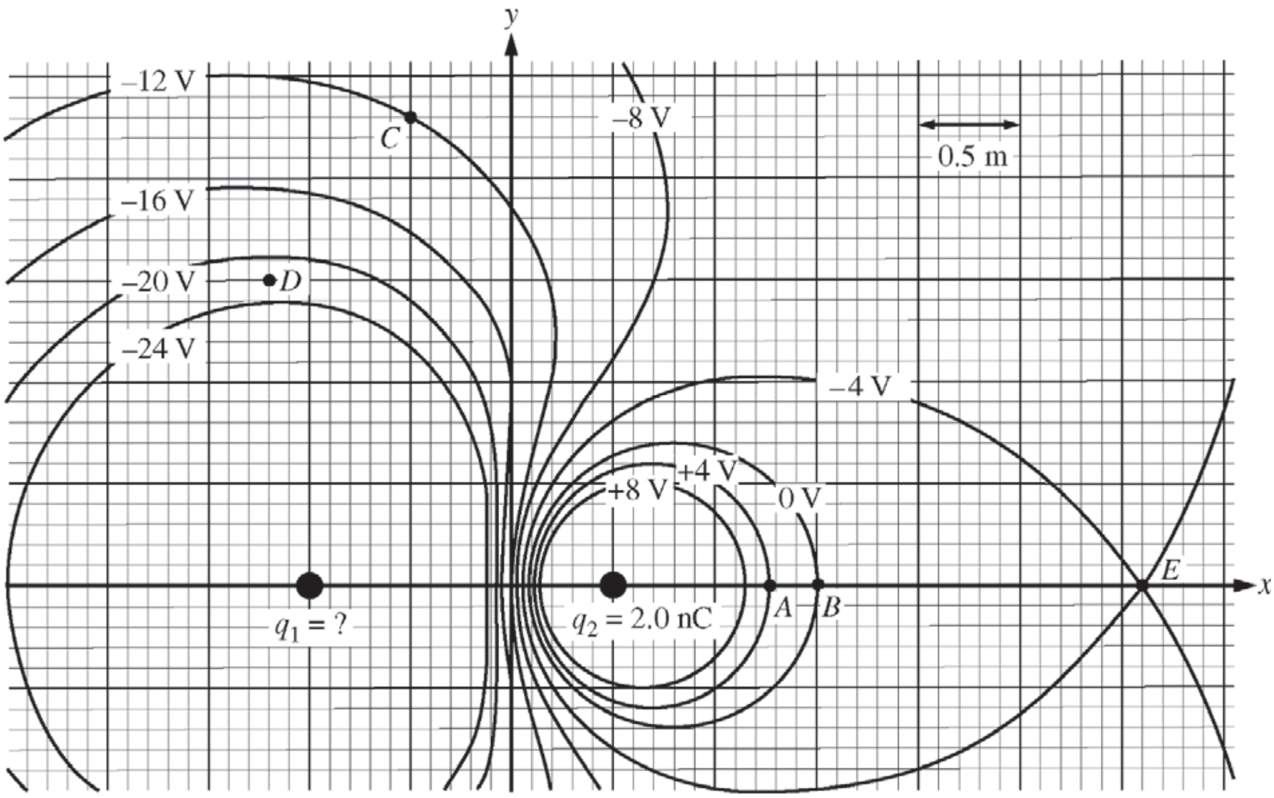

E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

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Justify your answer.

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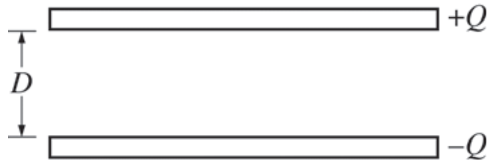


E&M.1.

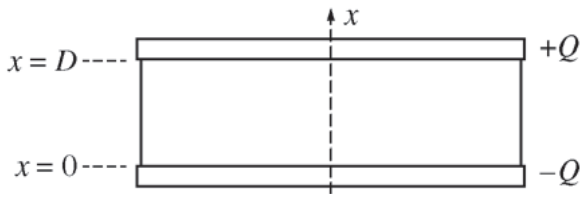
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

9 Electric potential and voltage – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)
Electric potential	电势	
voltage	电压	

Recall

You use “voltage” all the time:

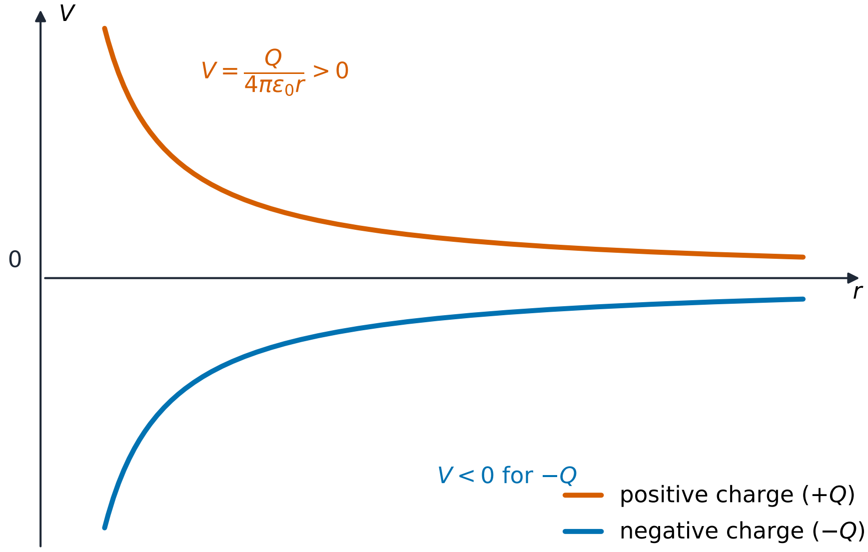
- Batteries are labelled 1.5 V, 9 V, and so on.
- Voltage is what pushes current around a circuit.
- It is really a measure of electric energy.

This sheet lines up electric potential and voltage. Each idea has a worked example.

New ideas

■ 1 Electric potential

Electric potential 电势 V is the electric potential energy per unit of charge, measured in volts.



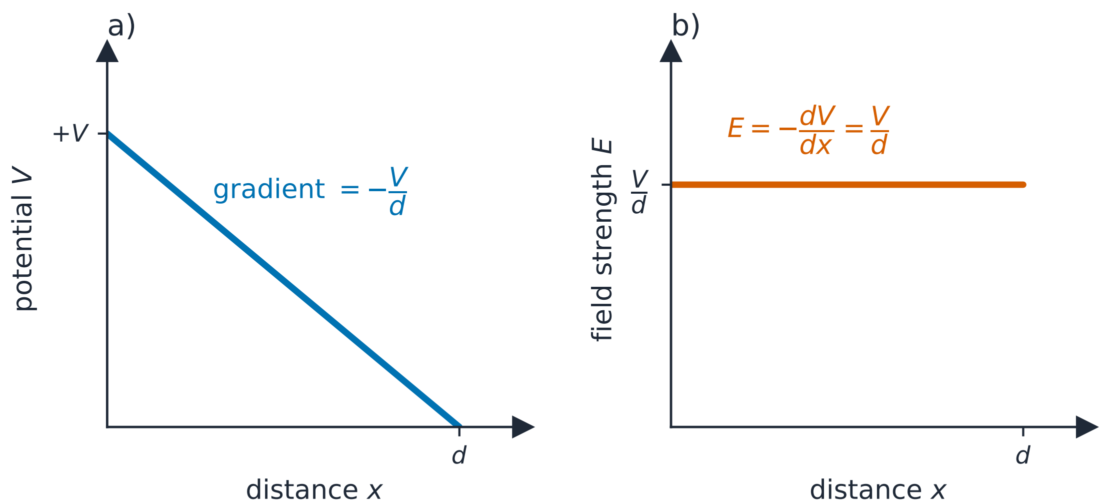
2 Voltage

The **voltage** 电压 (potential difference) between two points is the work done per unit charge to move a charge between them: $U = qV$.

Worked example. Moving a charge of 2.0 C through 6.0 V transfers $U = qV = 2.0 \times 6.0 = 12$ J.

3 Downhill for positive charge

A positive charge moves from **high** to **low** potential on its own – like a ball rolling downhill.



■ **4 A scalar**

Potential is just a number (a scalar), so potentials from several charges simply **add** – much easier than adding field vectors.

Watch out: electric potential (V, a scalar) and electric field (E, a vector) are different things. Potentials add as plain numbers; fields must be added by direction as well as size.

Practice

Try these in order. The methods are all above.

2.1 State what electric potential measures. [2]

2.2 Moving a 3.0 C charge through 4.0 V – find the energy transferred. [2]

2.3 State which way a positive charge moves on its own (high to low potential, or low to high). [1]

2.4 Moving a 0.5 C charge through 12 V – find the energy transferred. [2]

2.5 State why potential is easier to add than the electric field. [2]

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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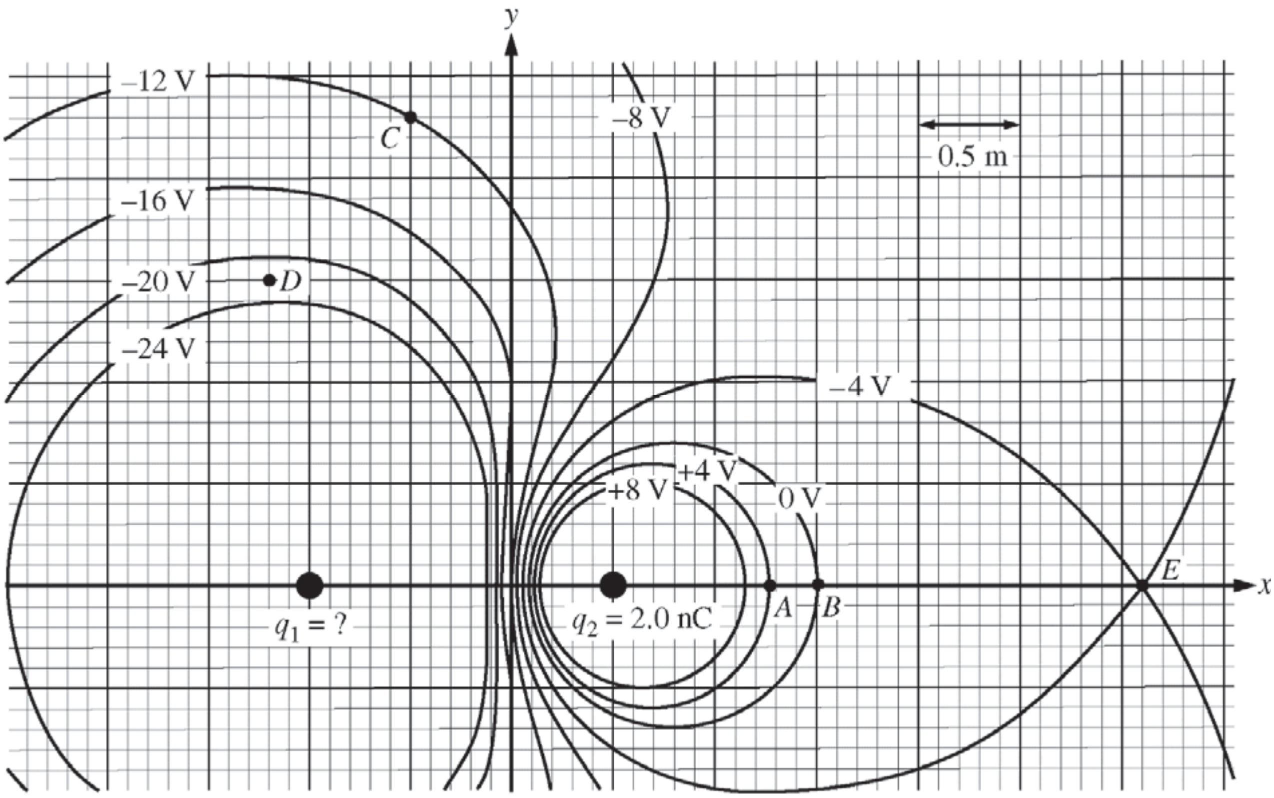

E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

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(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

9 Accelerating charges with voltage – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

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English	中文	Write it 3 times (English)
electric potential energy	电势能	____________ ____________ ____________

Recall

In Part A you met potential and voltage. Now use voltage to give charges energy:

- A voltage can speed up a charged particle.
- This is how old TV tubes and electron microscopes work.

Each idea has a worked example before the Practice.

New ideas

■ 1 Electric potential energy

Two charges have **electric potential energy** 电势能 stored in their arrangement – like a stretched spring stores energy.

■ 2 Energy from voltage

A charge released across a voltage speeds up, turning electric energy into kinetic energy:

$$qV = \frac{1}{2}mv^2.$$

■ 3 A worked example

Worked example. A charge of 2.0 C falls through 10 V, gaining $qV = 2.0 \times 10 = 20$ J of kinetic energy.

■ **4 Conservation of energy**

This is just energy conservation for charges: the electric potential energy lost equals the kinetic energy gained.

Watch out: a **positive** charge gains energy moving from high to low potential (downhill), while a **negative** charge gains energy moving the opposite way. The sign of the charge decides which way is "downhill".

Practice

Try these in order. The methods are all above.

2.1 State what electric potential energy is stored in. [1]

2.2 A charge of 3.0 C falls through 5.0 V. Find the kinetic energy it gains. [2]

2.3 A charge of 0.20 C is accelerated through 50 V. Find the energy gained. [2]

2.4 State the energy conversion when a charge is accelerated through a voltage. [2]

2.5 State which way a positive charge must move to gain energy. [1]

PHYSICS C: ELECTRICITY AND MAGNETISM
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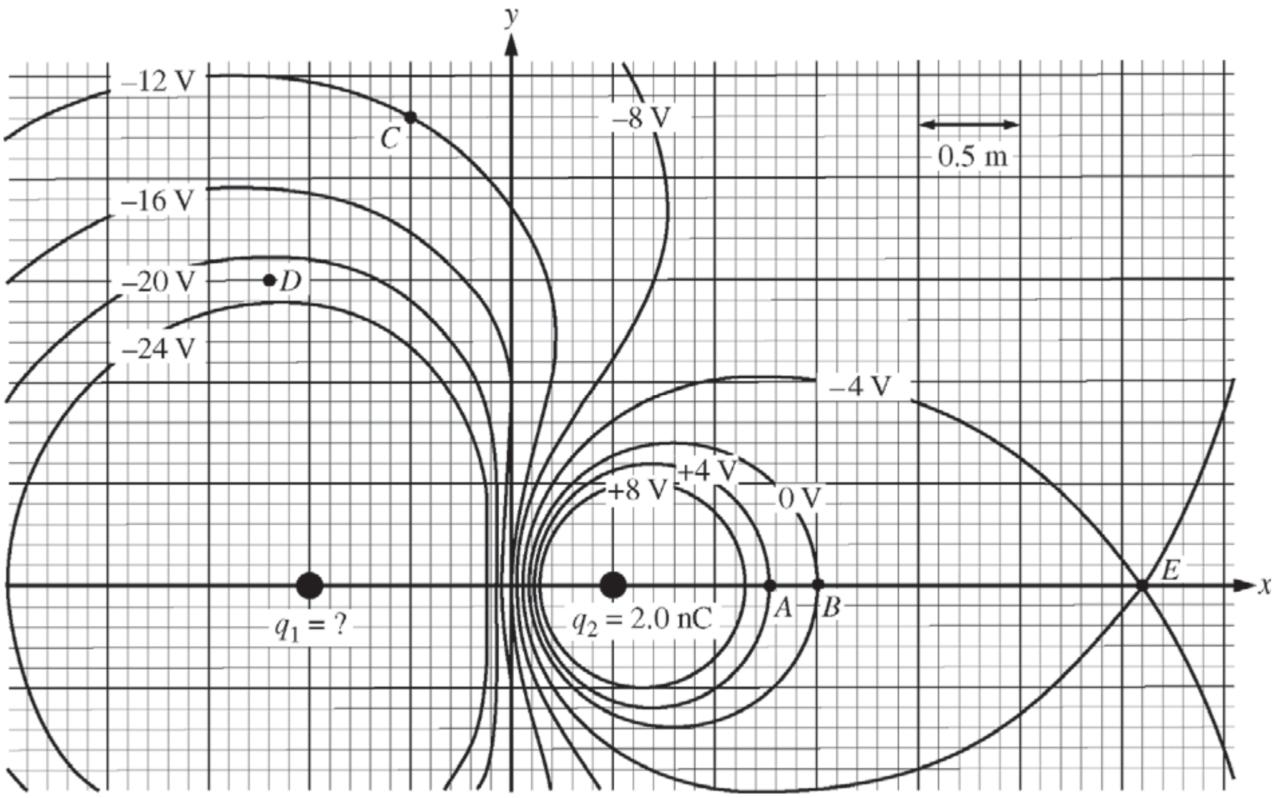

E&M.1.

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(a) Calculate the value of q_1.

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Justify your answer.

PHYSICS C: ELECTRICITY AND MAGNETISM

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3 Questions

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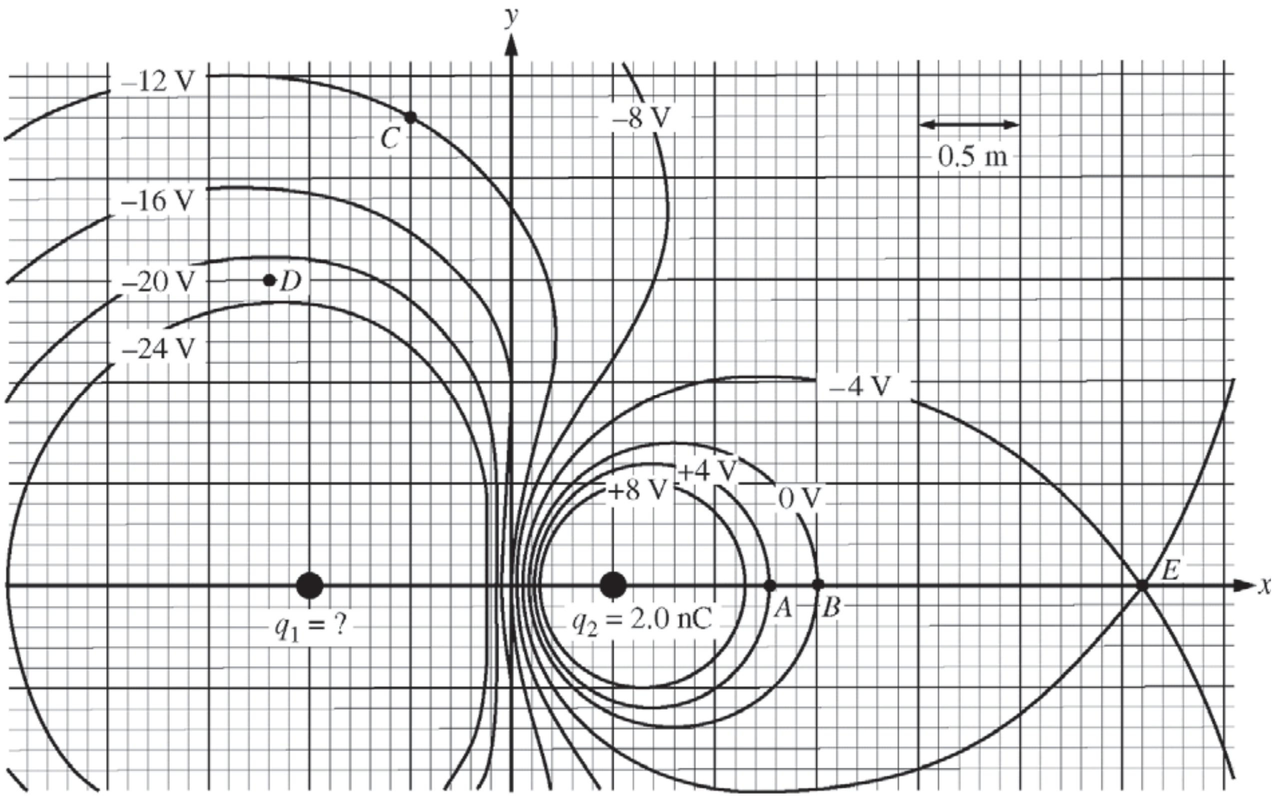


E&M.1.

Two point charges, q_1 and q_2, are fixed in place on the x-axis at positions $x_1 = -1.00$ m and $x_2 = +0.50$ m, respectively. Charge q_2 has a value of +2.0 nC. Values of electric potential are illustrated by the given equipotentials in the diagram shown above, which is drawn to scale.

(a) Calculate the value of q_1.

(b) At point C on the diagram, draw a vector representing the direction of the electric field at that point.

(c) Calculate the approximate magnitude of the electric field strength at point D on the diagram.

(d) The equipotential labeled 0 V is the cross section of a nearly spherical surface. Calculate the electric flux for this surface.

(e) A proton is placed at point A and then released from rest.

i. Calculate the work done by the electric field on the proton as it moves from point *A* to point *E*.

ii. Calculate the speed of the proton when it reaches point *E*.

(f) An electron is released from rest at point B. Which of the following indicates the direction of the initial acceleration, if any, of the electron?

____ Up ____ Down

____ Left ____ Right

____ Into the page ____ Out of the page

____ The direction is undefined since the acceleration is zero.

Justify your answer.

10 Conductors and capacitors – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
capacitor	电容器	
capacitance	电容	

Recall

Many devices store electric charge:

- A camera flash charges up, then releases energy in a burst.
- Phones and computers are full of tiny components called capacitors.

This sheet lines up conductors and capacitors. Each idea has a worked example.

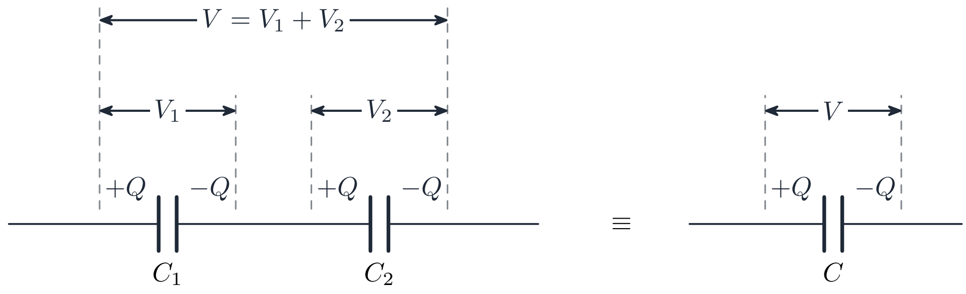
New ideas

■ 1 Conductors

In a **conductor**, charge moves freely. Any extra charge spreads out over the **surface**, and inside the metal the field is zero.

■ 2 Capacitors

A **capacitor** 电容器 stores charge on two conducting plates separated by a gap.



3 Capacitance

The **capacitance** 电容 links the charge stored to the voltage across it:

$$C = \frac{Q}{V}.$$

Worked example. A capacitor holds $Q = 1.2 \times 10^{-3}$ C at $V = 12$ V, so $C = \dfrac{Q}{V} = \dfrac{1.2 \times 10^{-3}}{12} = 1.0 \times 10^{-4}\ \text{F} = 100\ \mu\text{F}$.

4 Finding the charge

Rearranged, $Q = CV$: a bigger capacitance or a bigger voltage stores more charge.

Watch out: capacitance depends on the plates' **size and spacing**, not on how much charge you put on. Adding charge raises the voltage in step, keeping $C = Q/V$ fixed.

Practice

Try these in order. The methods are all above.

2.1 State where extra charge sits on a conductor. [1]

2.2 State what a capacitor does. [2]

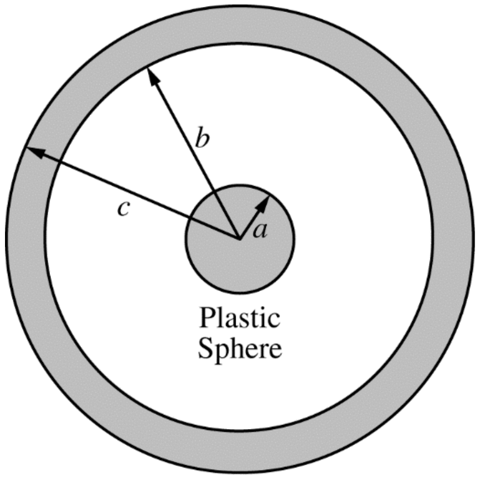
2.3 A capacitor stores 6.0×10^{-4} C at 12 V. Find its capacitance. [2]

2.4 A 50 μF capacitor is charged to 10 V. Find the charge stored ($Q = CV$). [2]

2.5 State what capacitance depends on. [2]

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   ii. $a < r < b$

   (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

   i. $r < a$

   ii. $a < r < b$

   (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

   i. Determine the charge on the inner surface of the conducting shell.

   Justify your answer.

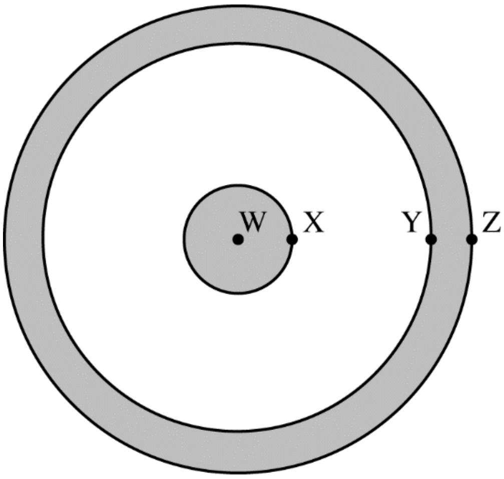
   ii. Determine the charge on the outer surface of the conducting shell.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



Begin your response to **QUESTION 1** on this page.

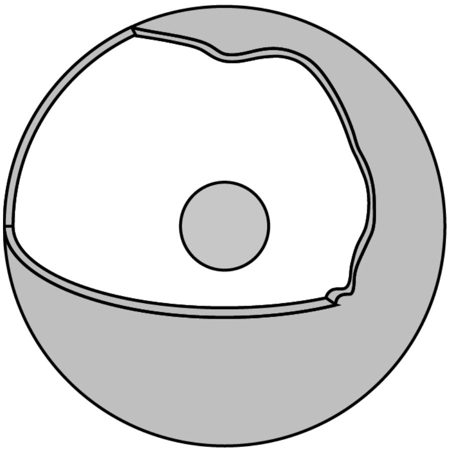
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

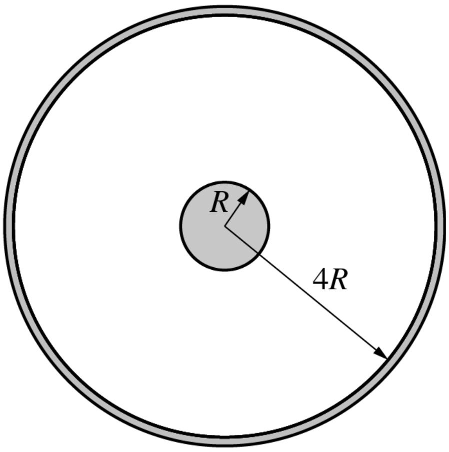
Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View



Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

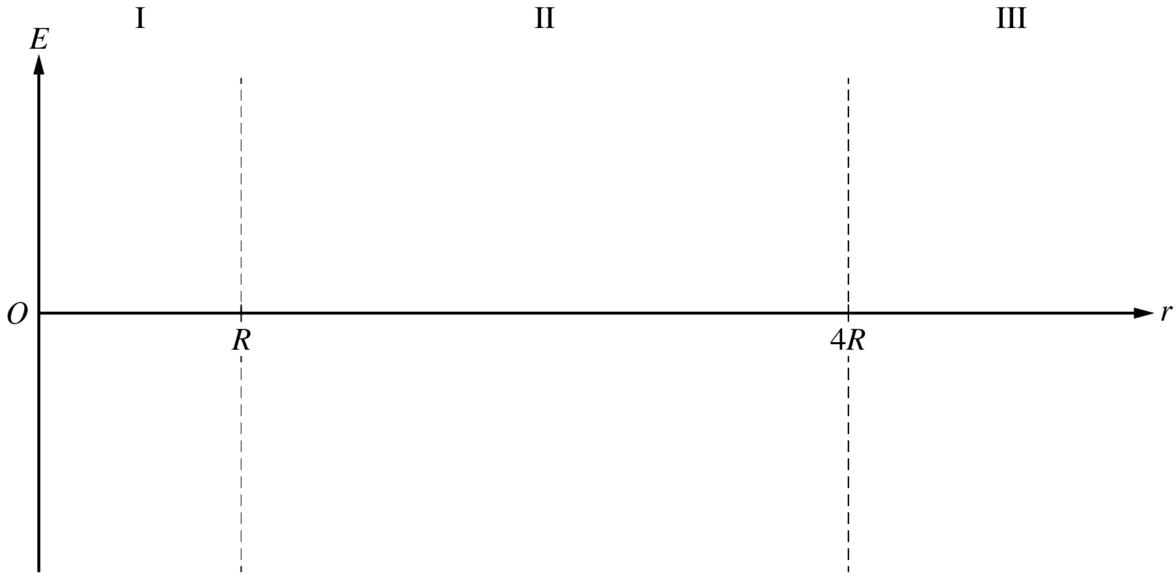
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

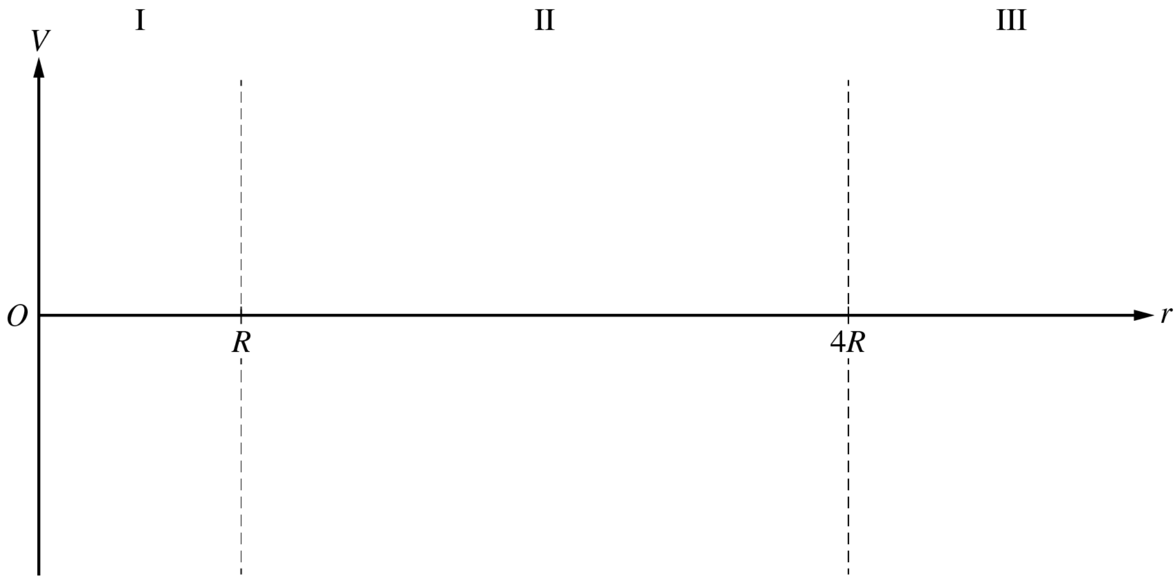
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



GO ON TO THE NEXT PAGE.

10 Capacitor energy and dielectrics – Part 2

Name: ______________ Class: __________ Date: ____________

Recall

In Part A you saw a capacitor stores charge. It also stores **energy**:

- A charged capacitor can release energy in a quick burst (a camera flash).
- The more you charge it, the more energy it holds.

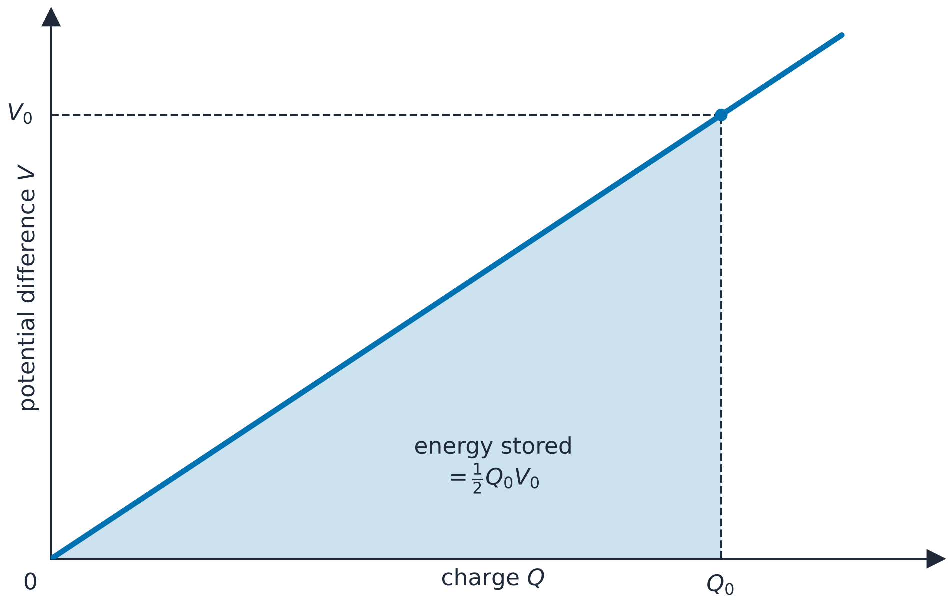
Each idea has a worked example before the Practice.

New ideas

■ 1 Stored energy

A charged capacitor stores energy

$$U = \tfrac{1}{2}CV^2.$$



Worked example. A 100 μF capacitor at 12 V stores $U = \frac{1}{2} \times 100 \times 10^{-6} \times 12^2 = 7.2 \times 10^{-3}$ J.

■ **2 Energy depends on V squared**

Because of the V^2, **doubling** the voltage stores **four times** the energy.

■ **3 A quick release**

A capacitor can release its stored energy very quickly – far faster than a battery – which is why it powers a camera flash.

■ **4 Dielectrics**

Putting an insulating material (a **dielectric**) between the plates increases the capacitance, so more charge and energy can be stored at the same voltage.

Watch out: the stored energy depends on the **square** of the voltage ($U = \frac{1}{2}CV^2$), so a small rise in voltage gives a large rise in energy – double the voltage, quadruple the energy.

Practice

Try these in order. The methods are all above.

2.1 Write the formula for the energy stored in a capacitor. [1]

2.2 A 200 μF capacitor is charged to 10 V. Find the energy stored. [3]

2.3 A capacitor's voltage is doubled. State the factor by which the stored energy changes. [1]

2.4 State one everyday use of a capacitor's quick energy release. [1]

2.5 State what adding a dielectric between the plates does. [2]

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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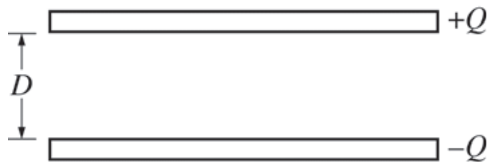


E&M.1.

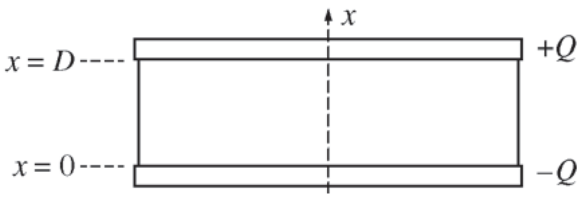
A parallel-plate capacitor is constructed of two parallel metal plates, each with area A and separated by a distance D. The plates of the capacitor are each given a charge of magnitude Q, as shown in the figure above. Ignore edge effects.

(a)

i. On the figure above, draw an arrow to indicate the direction of the electric field between the plates.

ii. On the figure above, draw an appropriate Gaussian surface that will be used to derive an expression for the magnitude of the electric field E between the plates.

iii. Using Gauss's law and the Gaussian surface from part (a)-ii, derive an expression for the magnitude of the electric field E between the plates. Express your answer in terms of A, D, Q, and physical constants, as appropriate.



The space between the plates is now filled with a dielectric material that is engineered so that its dielectric constant varies with the distance from the bottom plate to the top plate, defined by the x-axis indicated in the diagram above. As a result, the electric field between the plates is given by $\vec{E} = -\dfrac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\hat{i}$, where κ_0 is a positive constant. Express all algebraic answers to the remaining parts in terms of A, D, Q, κ_0, x, and physical constants, as appropriate.

(b) Determine an expression for the dielectric constant κ as a function of x.

(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

ii. Using the equation from part (c)-i, derive an expression for the potential difference $V_D - V_0$, where V_D is the potential of the top plate and V_0 is the potential of the bottom plate.

(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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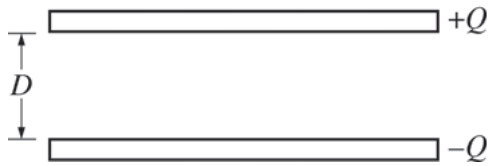

E&M.1.

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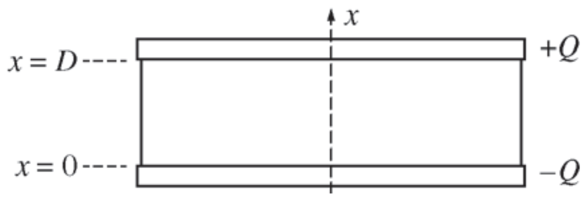

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(c)

i. Write, but do NOT solve, an equation that could be used to determine the potential difference V between the plates of the capacitor.

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(d) Determine the capacitance of the capacitor.

(e) The energy stored in the capacitor that has a varying dielectric is U_V. A second capacitor that has a constant dielectric of value κ_0 is also given a charge Q. The energy stored in the second capacitor is U_C. How do the values of U_V and U_C compare?

____ $U_V < U_C$ ____ $U_V > U_C$ ____ $U_V = U_C$

Justify your answer.

11 Current, resistance, and power – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Electric current	电流			
Resistance	电阻			
Ohm's law	欧姆定律			
Electric power	电功率			

Recall

You have built simple circuits before:

- A battery pushes a current around a loop to light a bulb.
- More voltage pushes more current; more resistance holds it back.
- You have wired components in a line and side by side.

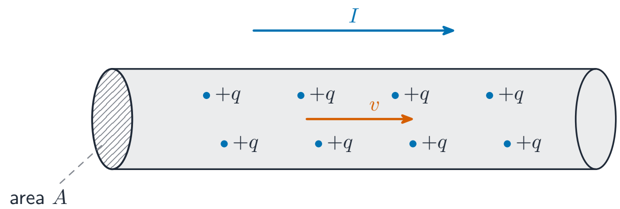
This sheet lines these up with formulas. Each idea has a worked example.

New ideas

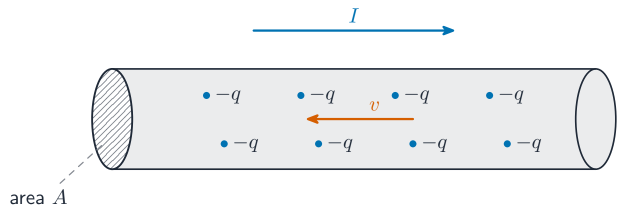
■ 1 Electric current

Electric current 电流 is the rate at which charge flows past a point, measured in amperes (A):

$$I = \frac{\Delta q}{\Delta t}.$$



(a) positive carriers drift with I



(b) electrons drift against I

Worked example. If 6.0 C flows past a point in 3.0 s, the current is $I = \dfrac{6.0}{3.0} = 2.0$ A.

2 Resistance and Ohm's law

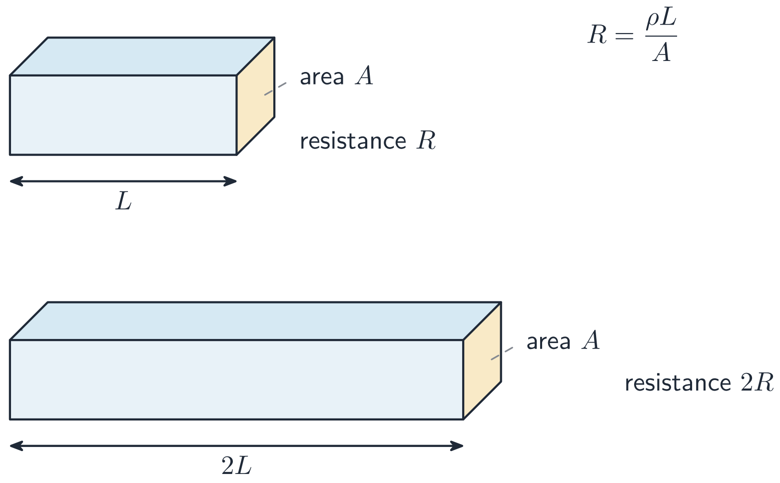
Resistance 电阻 R opposes the current. **Ohm's law** 欧姆定律 links voltage, current, and resistance:

$$V = IR.$$

Worked example. A 2.0 A current through a 6.0 Ω resistor needs $V = IR = 2.0 \times 6.0 = 12$ V.

3 What sets the resistance

A wire's resistance grows with its **length** and shrinks with its **thickness** (cross-section).



twice the length ⇒ twice the resistance

■ 4 Electric power

Electric power 电功率 is how fast a component uses energy: $P = IV = I^2R$.

Watch out: in a **series** circuit the current is the same everywhere; in a **parallel** circuit the voltage is the same across each branch. Mixing these up is the most common circuit mistake.

Practice

Try these in order. The methods are all above.

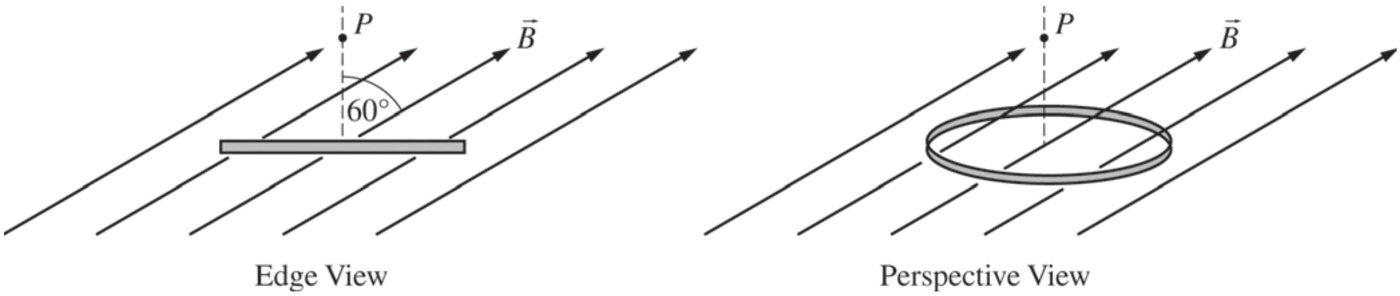
2.1 A charge of 10 C flows past a point in 5.0 s. Find the current. [2]

2.2 A 3.0 A current flows through a 4.0 Ω resistor. Find the voltage across it. [2]

2.3 State how a wire's resistance depends on its length and thickness. [2]

2.4 A resistor carries 0.50 A with 6.0 V across it. Find the power it uses. [2]

2.5 State what is the same for every component in a series circuit. [1]



E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

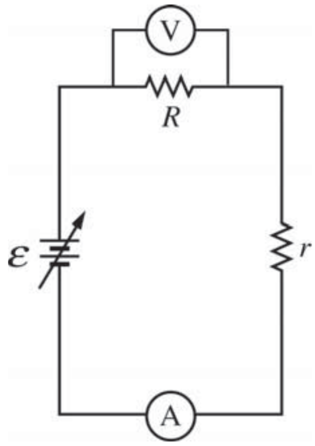
Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM



E&M.2.

The circuit shown above consists of a source of variable emf $\mathcal{E}$, an ideal ammeter A, an ideal voltmeter V, a resistor of resistance R, and a sample of wire with resistance r.

(a) How does the current through the wire sample compare with the current through the resistor R ?

_____ It is greater through R. _____ It is greater through the sample.

_____ It is the same through both. _____ It depends on the resistance of the sample.

Justify your answer.

(b) How does the potential difference across the wire sample compare with the potential difference across the resistor R ?

_____ It is greater across R. _____ It is greater across the sample.

_____ It is the same across both. _____ It depends on the resistance of the sample.

Justify your answer.

With the sample of wire in place, the emf of the source is set to a given value. The current through and potential difference across the resistor R are measured. This is repeated for several values of emf, and the data are recorded in the table below.

$\mathcal{E}$ (V)	V_R (V)	I_R (A)		
0.250	0.179	0.162		
0.500	0.335	0.327		
0.750	0.520	0.490		
1.000	0.670	0.687		

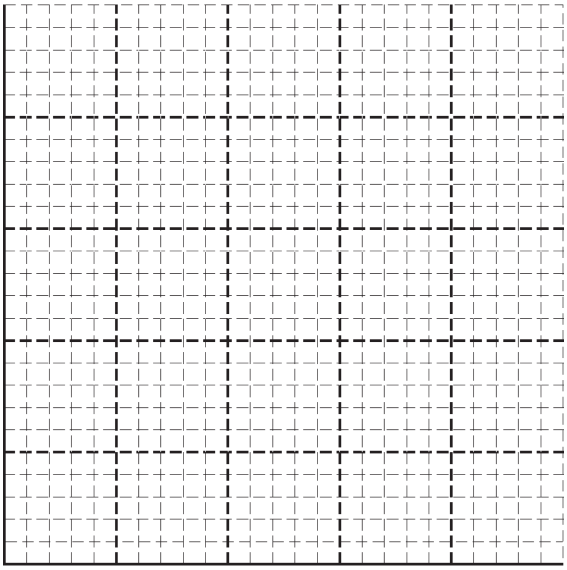
(c) Indicate below which quantities should be graphed to yield a straight line that could be used to calculate a numerical value for the resistance of the wire sample.

Horizontal axis: _____________

Vertical axis: _____________

You may use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given.

(d) On the grid below, plot the straight line data points from part (c). Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(e) Use your straight line to calculate the value of the resistance of the wire sample.

(f) The wire sample has a length of 3.00 m and a radius of 1.00×10^{-3} m . Calculate the resistivity of the material from which the wire sample is made.

(g)

i. Suppose the ammeter used to collect these data was not ideal. Would the actual value of the resistance of the wire sample be greater than, less than, or equal to that calculated in part (e) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. If the ideal voltmeter is replaced by a voltmeter that is not ideal and the experiment is repeated, would the readings of the ideal ammeter be greater than, less than, or equal to those in the data chart before part (c) ?

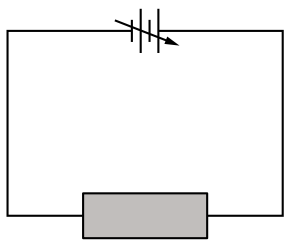
____ Greater than ____ Less than ____ Equal to

Justify your answer.

Question 3

3. In Experiment 1, students are asked to use a graph to determine the resistivity ρ_1 of a circuit element that is connected to a variable power supply, as shown in Figure 1. The circuit element is cylindrical and has uniform resistivity. The students have access to a voltmeter, an ammeter, and a ruler.

Figure 1



Circuit Element

A. Describe a procedure for collecting data that would allow the students to use a graph to determine ρ_1, including any steps necessary to reduce experimental uncertainty.

B. Describe how the collected data could be graphed and how that graph would be analyzed to determine ρ_1.

In Experiment 2, the students are asked to use a graph to determine the resistivity ρ_2 of solid, cylindrical resistors made of the same material but of different lengths L. The cross-sectional area of each resistor is $5.0\times10^{-6}\ \text{m}^2$. The students directly measure the resistance R between the ends of each resistor. Table 1 provides L and R for each resistor.

Table 1

L (m)	R (Ω)
0.010	0.90
0.020	1.6
0.030	2.5
0.040	3.2
0.050	4.0

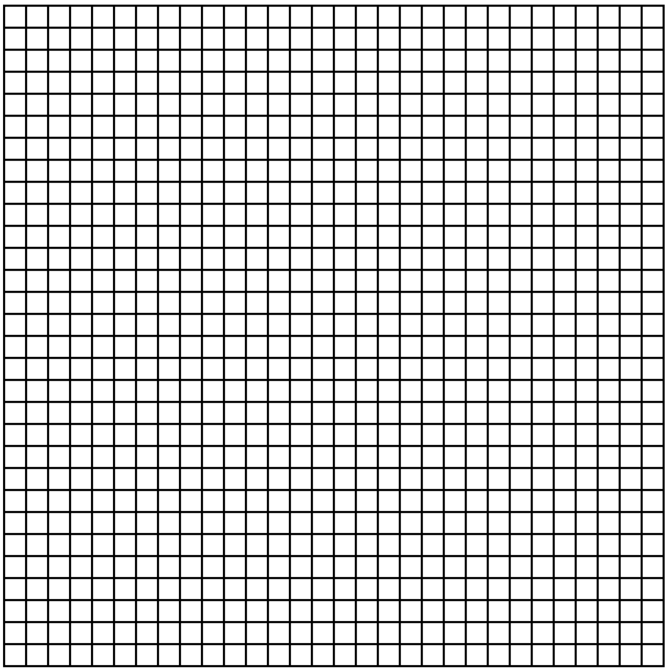
C.

i. **Indicate** two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine ρ_2.

Vertical axis: ______ Horizontal axis: ______

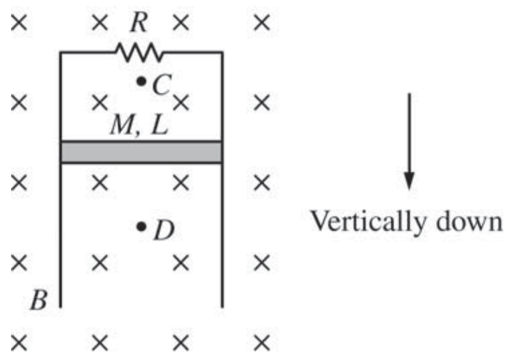
ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.



iii. **Draw** a best-fit line for the data graphed in part C (ii).

D. Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for ρ_2.



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM

11 Combining resistors and circuit rules – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you used $V = IR$ for a single resistor. Now handle circuits with **several**:

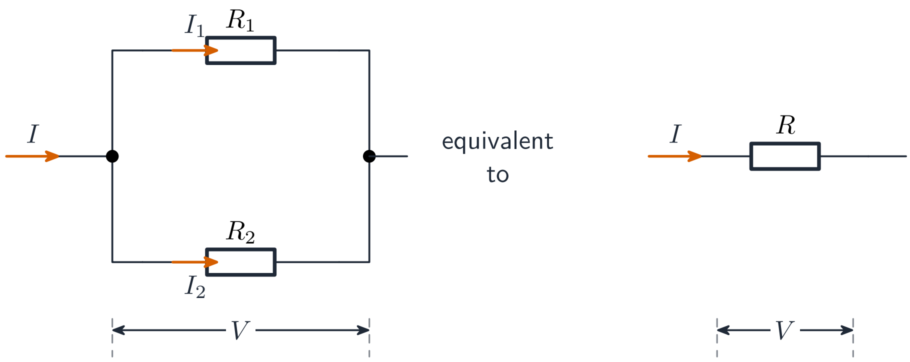
- Resistors in a line add up; resistors side by side share the current.
- Whatever current flows into a junction flows out again.

Each idea has a worked example before the Practice.

New ideas

1 Combining resistors

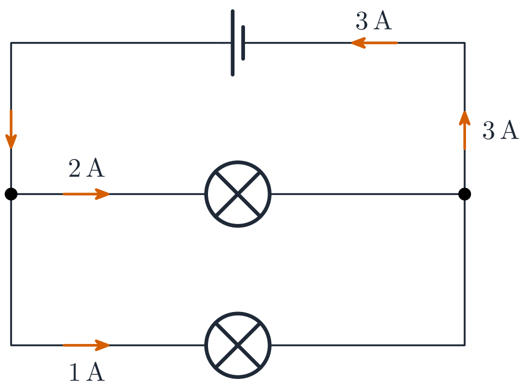
- **Series:** $R_{eq} = R_1 + R_2 + \cdots$ (they add).
- **Parallel:** $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots$ (the total is **less** than the smallest).



Worked example. A 4.0 Ω and a 12 Ω in parallel: $\dfrac{1}{R_{eq}} = \dfrac{1}{4} + \dfrac{1}{12} = \dfrac{4}{12}$, so $R_{eq} = 3.0\ \Omega$.

2 The junction rule

The current flowing **into** a junction equals the current flowing **out** (charge is conserved).



at each junction: $3\,\mathrm{A} = 2\,\mathrm{A} + 1\,\mathrm{A}$

■ 3 The loop rule

Going around any complete loop, the voltage gains (from the battery) and drops (across resistors) add up to **zero** (energy is conserved).

■ 4 Solving a circuit

Worked example. If 5.0 A enters a junction and one branch carries 2.0 A, the other carries $5.0 - 2.0 = 3.0$ A.

Watch out: putting resistors in **parallel** makes the total resistance **smaller** than the smallest one – because you have opened extra paths for the current. Series makes it larger.

Practice

Try these in order. The methods are all above.

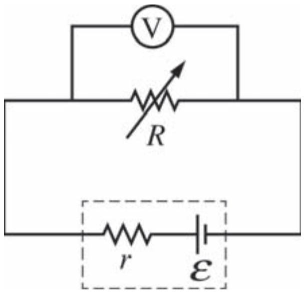
2.1 Find the equivalent resistance of a 3.0 Ω and a 5.0 Ω resistor in **series**. [2]

2.2 Find the equivalent resistance of a 6.0 Ω and a 3.0 Ω resistor in **parallel**. [3]

2.3 A current of 4.0 A enters a junction; one branch carries 1.5 A. Find the current in the other branch. [2]

2.4 State the conservation idea behind the loop rule. [1]

2.5 State whether two resistors in parallel give a total larger or smaller than the smallest one. [1]



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

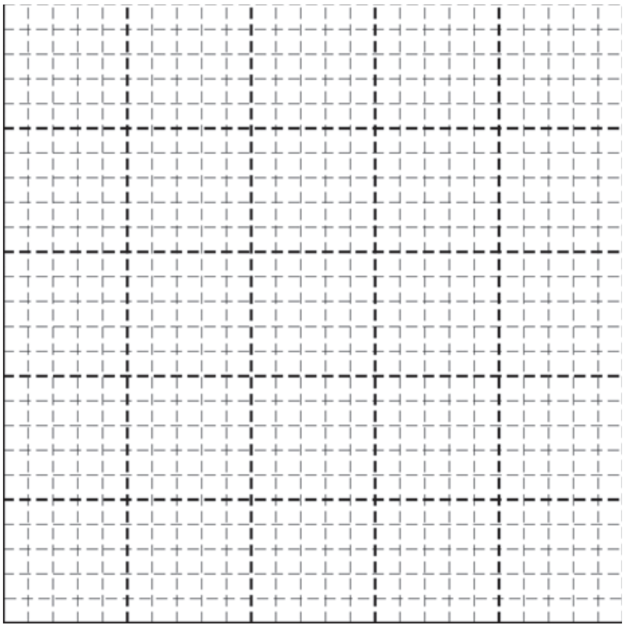
Trial #	Resistance (Ω)	Voltage (V)	$1/R$ ($1/\Omega$)	$1/V$ (1/V)
1	0.50	5.6	2.00	0.179
2	1.0	7.4	1.00	0.135
3	2.0	9.4	0.50	0.106
4	3.0	10.6	0.33	0.094
5	5.0	10.9	0.20	0.092
6	10	11.4	0.10	0.088

(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

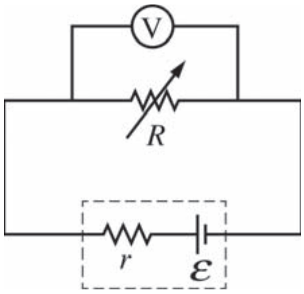
(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.



E&M.2.

A student performs an experiment to determine the emf $\mathcal{E}$ and internal resistance r of a given battery. The student connects the battery in series to a variable resistance R, with a voltmeter across the variable resistor, as shown in the figure above, and measures the voltmeter reading V as a function of the resistance R. The data are shown in the table below.

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4	3.0	10.6	0.33	0.094
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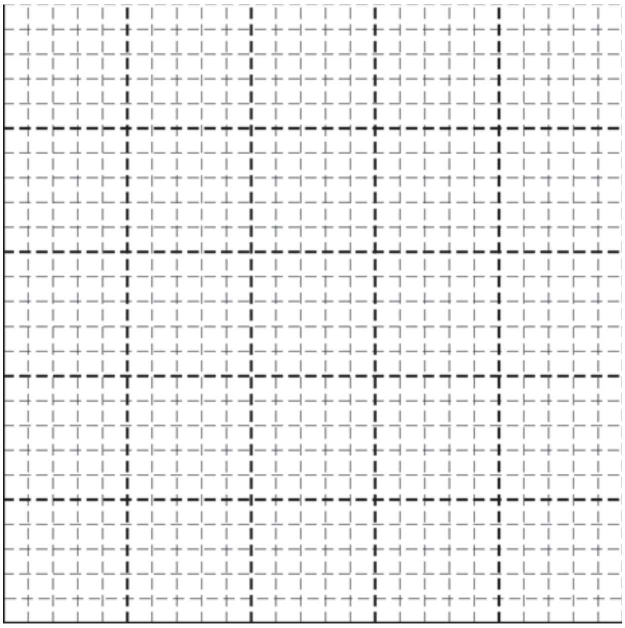
(a)

i. Derive an expression for the measured voltage V. Express your answer in terms of R, $\mathcal{E}$, r, and physical constants, as appropriate.

ii. Rewrite your expression from part (a)-i to express $1/V$ as a function of $1/R$.

(b) On the grid below, plot data points for the graph of $1/V$ as a function of $1/R$. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



(c) Use the straight line from part (b) to obtain values for the following.

i. $\mathcal{E}$

ii. r

(d) Using the results of the experiment, calculate the maximum current that the battery can provide.

(e) A voltmeter is to be used to determine the emf of the battery after removing the battery from the circuit. Two voltmeters are available to take this measurement—one with low internal resistance and one with high internal resistance. Indicate which voltmeter will provide the most accurate measurement.

____ The voltmeter with low resistance will provide the most accurate measurement.

____ The voltmeter with high resistance will provide the most accurate measurement.

____ The two voltmeters will provide equal accuracy.

Justify your answer.

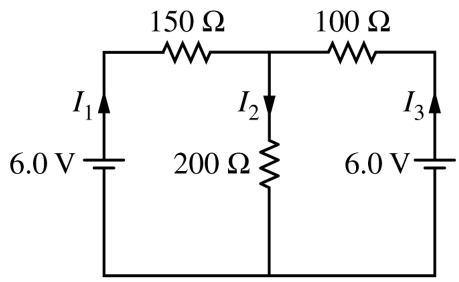


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

   (a)

      i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

      ii. Calculate the current in the $200\ \Omega$ resistor.

      iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

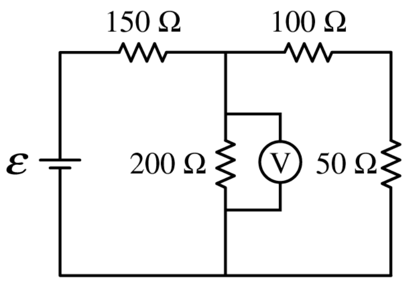


Figure 2

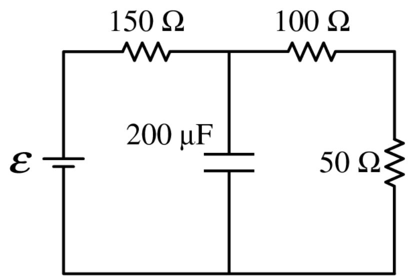
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

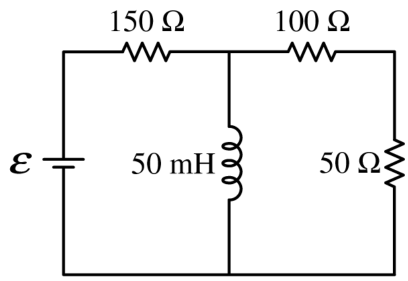
(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.

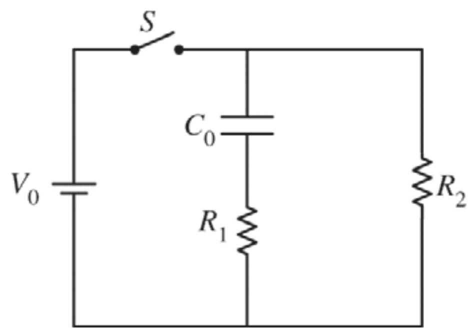


ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ____ Less than ____ Equal to

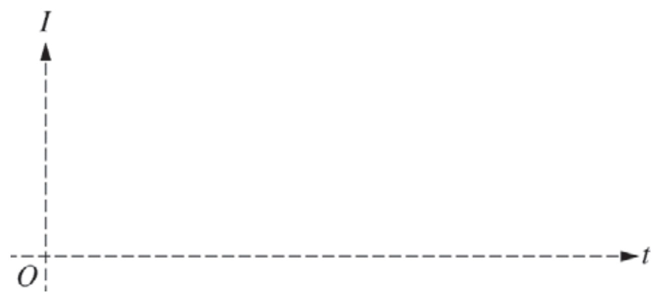
Justify your answer.



2. In the circuit above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. On the axes below, sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

(b) On the axes below, sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2, the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The circuit is constructed using an ideal 1.5 V battery, an 80 μF capacitor, and resistors $R_1 = 150\ \Omega$ and $R_2 = 100\ \Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) The time it takes to charge the capacitor to 50% of its maximum charge is Δt_C. The time it takes for the capacitor to discharge to 50% of its maximum charge is Δt_D. Which of the following correctly relates the two time intervals?

____ $\Delta t_C > \Delta t_D$ ____ $\Delta t_C = \Delta t_D$ ____ $\Delta t_C < \Delta t_D$

Justify your answer.

(d)

i. Calculate the current through resistor R_2 immediately after the switch is opened.

ii. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

____ Increasing ____ Decreasing ____ Constant

Justify your answer.

(e)

i. Calculate the energy stored in the capacitor immediately after the switch is opened.

ii. Calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

12 Magnetic fields and forces – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
magnetic field	磁场	
magnetic force	磁力	

Recall

You have played with magnets:

- A magnet has a north and a south pole; like poles repel, opposite attract.
- Magnets pull on iron and steel.
- A coil of wire carrying a current becomes a magnet.

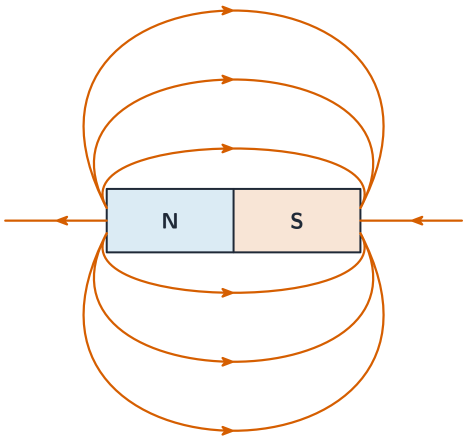
This sheet adds the forces magnetic fields make. Each idea has a worked example.

New ideas

1 Magnetic fields

A **magnetic field** 磁场 B surrounds every magnet. Field lines run from the **north** pole to the **south** pole outside the magnet.

field lines run from **N** to **S** outside the magnet

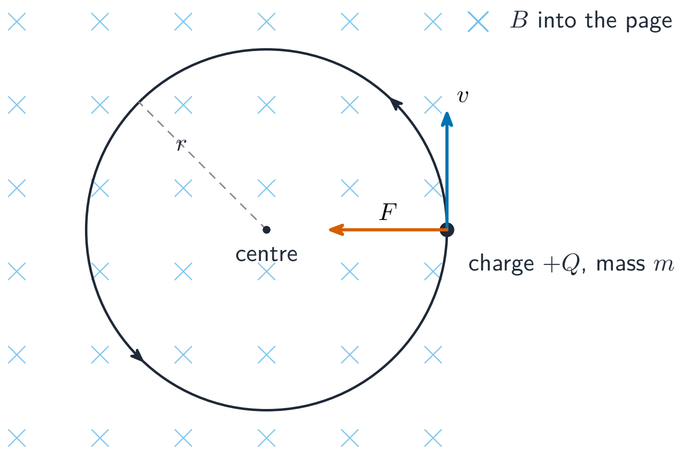


2 Force on a moving charge

A charge moving **across** a magnetic field feels a **magnetic force** 磁力:

$$F = qvB\sin\theta.$$

The force is at right angles to the motion, so it bends the charge into a **circle**.



$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

Worked example. A charge of 1.6×10^{-19} C moving at 2.0×10^5 m/s across a 0.50 T field feels $F = qvB = 1.6 \times 10^{-19} \times 2.0 \times 10^5 \times 0.50 = 1.6 \times 10^{-14}$ N.

■ **3 Force on a wire**

A current is moving charge, so a magnetic field pushes on a **current-carrying wire**: $F = BIL$. This push turns an electric motor.

■ **4 No force when parallel**

Watch out: a charge that is **not moving**, or one moving **along** the field lines, feels **no** magnetic force. The charge must move **across** the field to feel a push.

Practice

Try these in order. The methods are all above.

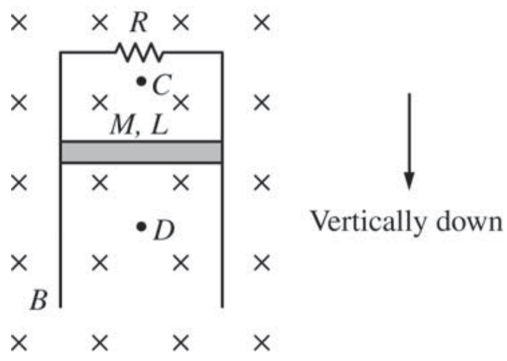
2.1 State the direction field lines run outside a bar magnet. [1]

2.2 A charge of 2.0×10^{-19} C moves at 1.0×10^{5} m/s across a 0.40 T field. Find the force. [3]

2.3 State the shape of the path of a charge moving across a uniform magnetic field. [1]

2.4 A 0.50 m wire carries 3.0 A across a 0.10 T field. Find the force on it ($F = BIL$). [2]

2.5 State when a moving charge feels no magnetic force. [2]



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

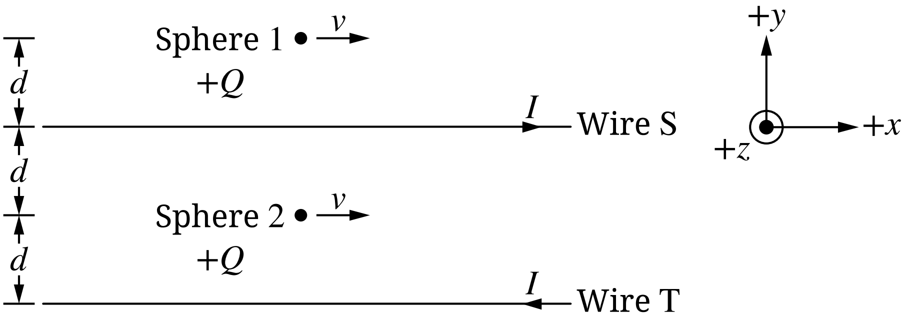
STOP

END OF EXAM

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

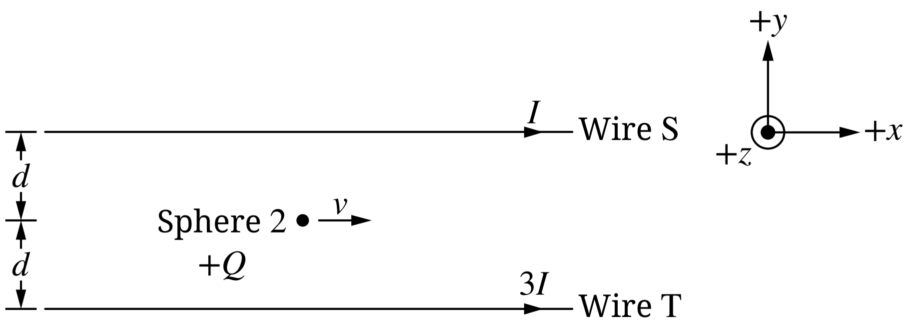
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance *d* below Wire S and is moving with speed *v* in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP

END OF EXAM

12 Magnetic fields from currents – Part 2

Name: ______________ Class: __________ Date: ____________

Recall

In Part A a magnetic field pushed on a moving charge. Now the reverse link:

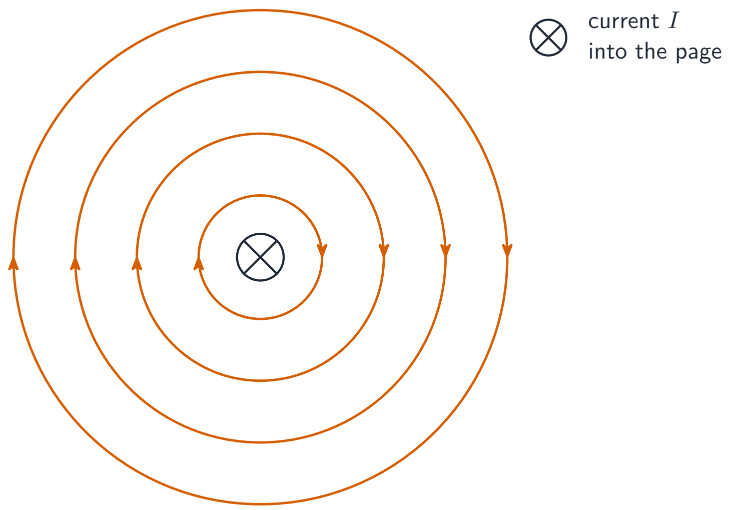
- A current does not just feel magnetic forces – it **creates** a magnetic field.
- This is how electromagnets work.

Each idea has a worked example before the Practice.

New ideas

1 A current makes a field

A current in a straight wire creates a magnetic field that **circles around** the wire.



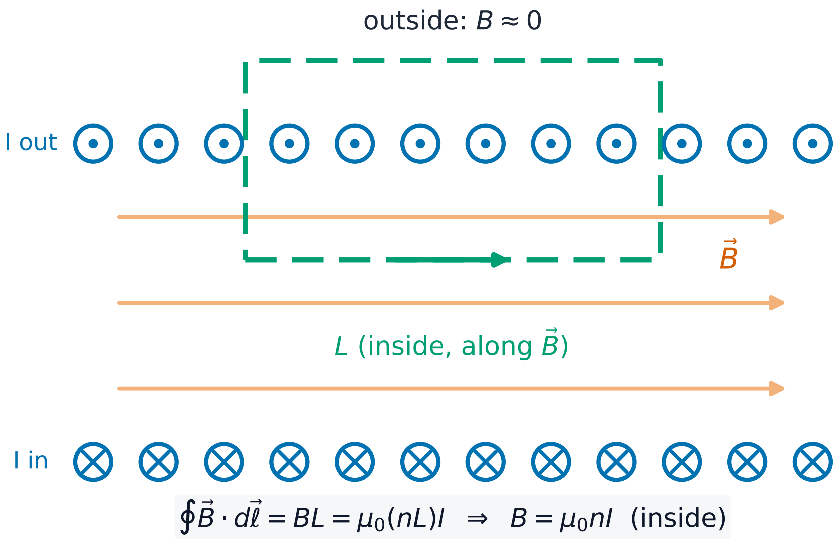
circular field lines (right-hand grip rule)

2 Stronger with more current

The field is stronger with a bigger current and weaker further from the wire.

3 Coils and solenoids

Winding the wire into a coil (a **solenoid**) makes the fields add up, giving a strong, uniform field inside – just like a bar magnet.



4 Electromagnets

Because the field comes from the current, you can **switch it off** by cutting the current – the basis of electromagnets used in scrapyard cranes and doorbells.

Watch out: the magnetic field circles **around** the wire, not along it. The field lines form closed loops around the current, so there is no "north end" of a straight wire.

Practice

Try these in order. The methods are all above.

2.1 State the shape of the magnetic field around a straight current-carrying wire. [2]

2.2 State two things that make the field around a wire stronger. [2]

2.3 State what a solenoid produces inside it. [2]

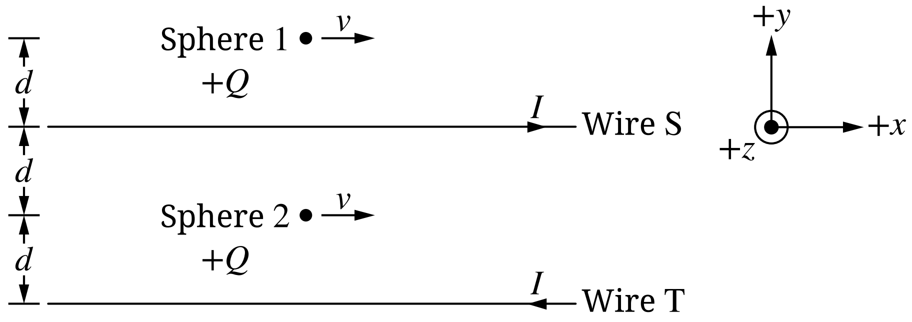
2.4 State one advantage of an electromagnet over a permanent magnet. [2]

2.5 Give one everyday device that uses an electromagnet. [1]

Question 4

4. Long, parallel wires S and T are a distance $2d$ apart. Both wires carry equal currents I, but the currents are in opposite directions. Both wires are parallel to the x-axis. At the instant shown in Figure 1, Sphere 1 is a distance d above Wire S, Sphere 2 is a distance d below Wire S, and both spheres are moving with speed v in the $+x$-direction. Each sphere has positive charge $+Q$. Gravitational effects are negligible.

Figure 1



A. F_1 is the magnitude of the magnetic force exerted on Sphere 1 due to the currents in wires S and T. F_2 is the magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Indicate whether F_2 is greater than, less than, or equal to F_1 by writing one of the following.

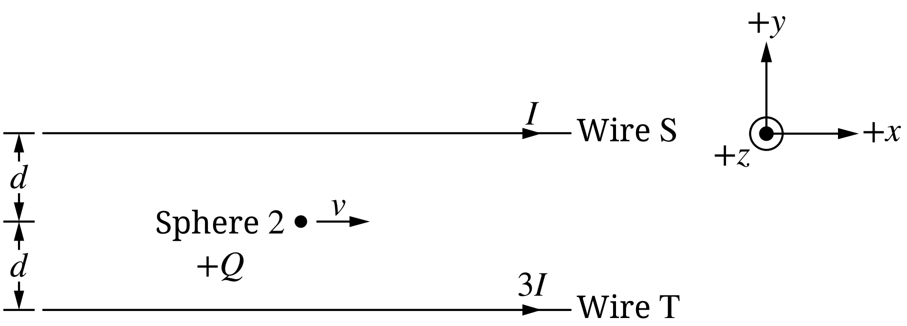
- $F_2 > F_1$
- $F_2 < F_1$
- $F_2 = F_1$

Justify your answer.

B. **Derive** an expression for the magnitude B_{tot} of the magnetic field at the location of Sphere 2 due to the currents in wires S and T in terms of d, I, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. Later, Wire T carries current $3I$ in the $+x$-direction. At the instant shown in Figure 2, Sphere 2 is a distance d below Wire S and is moving with speed v in the $+x$-direction. F_{new} is the new magnitude of the magnetic force exerted on Sphere 2 due to the currents in wires S and T.

Figure 2



Indicate whether F_{new} is greater than, less than, or equal to F_2 by writing one of the following.

- $F_{\text{new}} > F_2$
- $F_{\text{new}} < F_2$
- $F_{\text{new}} = F_2$

Briefly **justify** your answer by referencing your derivation in part B.

STOP

END OF EXAM

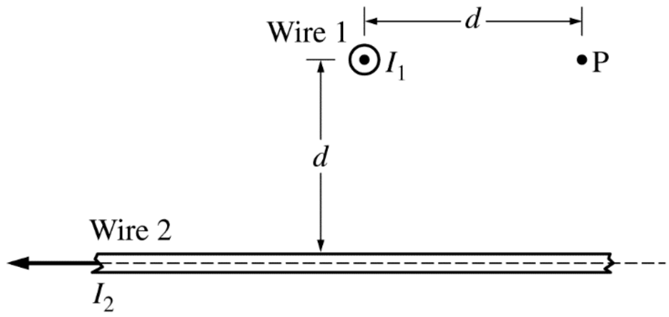


Figure 1. Side view

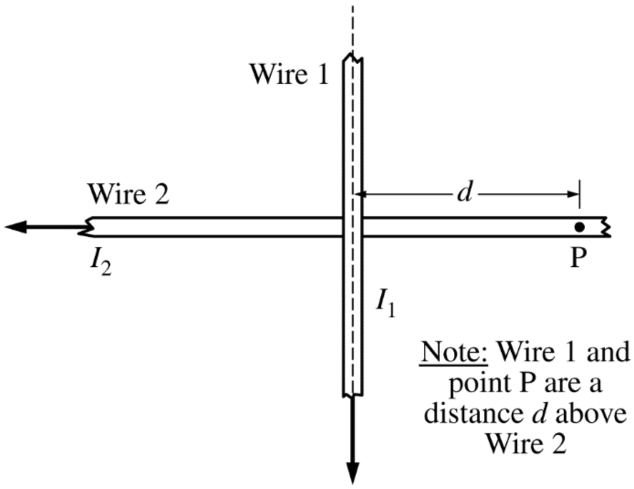


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

   (a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

   (b) Derive an expression for the magnitude of the net magnetic field at point P.

   (c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

   (d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

   ____ Wire 1 will not move.

   ____ Wire 1 will move upward as viewed in Figure 1.

   ____ Wire 1 will move downward as viewed in Figure 1.

   ____ Wire 1 will rotate clockwise as viewed in Figure 2.

   ____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

   Justify your answer.

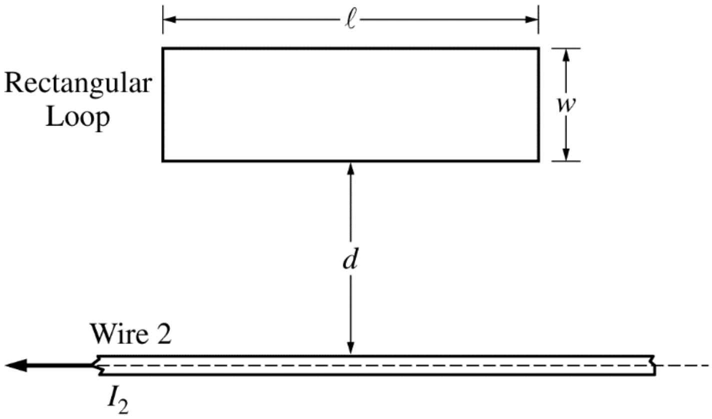


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM

13 Electromagnetic induction and flux – Part 1

Name: ______________ Class: __________ Date: ____________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
electromagnetic induction	电磁感应			
magnetic flux	磁通量			

Recall

You saw that a current makes a magnetic field. The reverse is just as important:

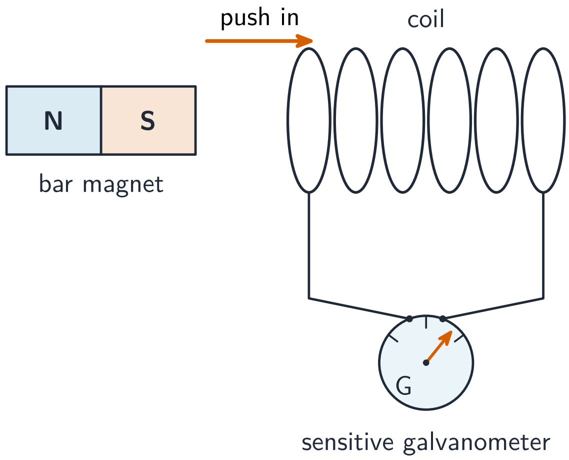
- Move a magnet near a coil of wire and a current appears – with no battery.
- This is how every generator and power station makes electricity.

This sheet lines up electromagnetic induction. Each idea has a worked example.

New ideas

■ 1 Electromagnetic induction

A **changing** magnetic field through a coil drives a current – **electromagnetic induction** 电磁感应. Move a magnet into or out of a coil and a current flows while the magnet is moving.

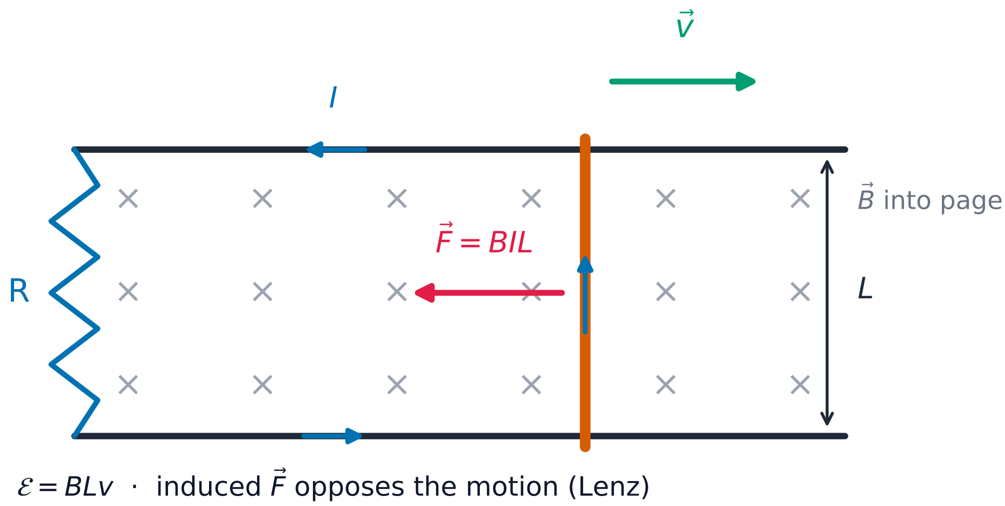


■ 2 Magnetic flux

The **magnetic flux** 磁通量 $\Phi = BA$ measures how much field passes through a loop of area A. The flux changes if the field, the area, or the loop's angle changes.

■ 3 A moving rod

A rod sliding along rails through a field also has a changing flux, so it generates a voltage.



motional emf: $\mathcal{E} = B\ell v$ · $F_{mag} = I\ell B$ opposes

■ 4 The size of the effect

Worked example. If the flux through a single loop changes from 0.020 Wb to 0.008 Wb in 0.030 s, the average voltage is $\dfrac{0.020 - 0.008}{0.030} = 0.40$ V.

Watch out: it is the **change** in flux that induces a current, not the flux itself. A magnet sitting still inside a coil induces **nothing** – the magnet has to be moving.

Practice

Try these in order. The methods are all above.

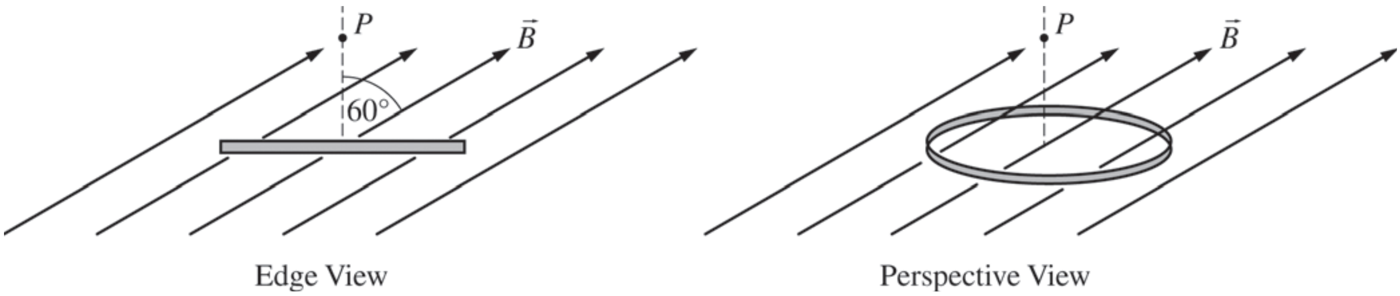
2.1 State what must happen for a coil to have a current induced in it. [1]

2.2 State what magnetic flux measures. [2]

2.3 The flux through a single loop rises from 0.010 Wb to 0.030 Wb in 0.040 s. Find the induced voltage. [3]

2.4 Name two things that could change the flux through a loop. [2]

2.5 Explain why a magnet held still inside a coil induces no current. [2]



E&M. 3.

A circular wire loop with radius 0.10 m and resistance 50 Ω is held in place horizontally in a magnetic field $\vec{B}$ directed upward at an angle of 60° with the vertical, as shown in the figure above. The magnetic field in the direction shown is given as a function of time t by $B(t) = a(1 - bt)$, where $a = 4.0$ T and $b = 0.20\ s^{-1}$.

(a) Derive an expression for the magnetic flux through the loop as a function of time t.

(b) Calculate the numerical value of the induced emf in the loop.

(c)

i. Calculate the numerical value of the induced current in the loop.

ii. What is the direction of the induced current in the loop as viewed from point P ?

____ Clockwise ____ Counterclockwise

Justify your answer.

(d) Assuming the loop stays in its current position, calculate the energy dissipated in the loop in 4.0 seconds.

(e) Indicate whether the net magnetic force and net magnetic torque on the loop are zero or nonzero while the loop is in the magnetic field.

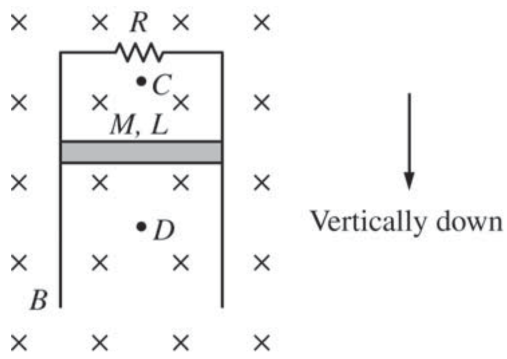
Net magnetic force: ____ Zero ____ Nonzero

Net magnetic torque: ____ Zero ____ Nonzero

Justify both of your answers.

STOP

END OF EXAM



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM

13 Faraday's and Lenz's laws – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Faraday's law	法拉第定律			
Lenz's law	楞次定律			

Recall

In Part A you saw that a changing magnetic flux drives a current. Now the two laws that describe it:

- One law gives the **size** of the induced voltage.
- The other gives its **direction**.

Each idea has a worked example before the Practice.

New ideas

1 Faraday's law

Faraday's law 法拉第定律 says the induced voltage equals how fast the flux changes:

$$\varepsilon = \frac{\Delta\Phi}{\Delta t}.$$

A coil of N turns gives N times as much.

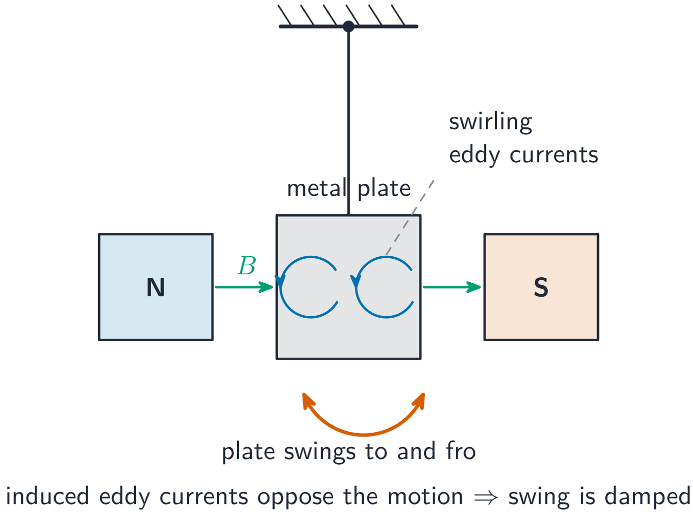
Worked example. If one turn would give 0.40 V, a 50-turn coil gives $50 \times 0.40 = 20$ V.

2 Lenz's law

Lenz's law 楞次定律 gives the direction: the induced current flows so as to **oppose** the change that caused it.

■ **3 It is energy conservation**

Lenz's law is really energy conservation – you must do work to push the magnet against this opposing effect, and that work becomes the electrical energy.



■ **4 Generators**

Spinning a coil in a magnetic field constantly changes the flux, inducing a current – this is how a **generator** turns motion into electricity.

Watch out: the induced current always **opposes** the change (Lenz's law). If it helped the change instead, you would get energy for free – which is impossible.

Practice

Try these in order. The methods are all above.

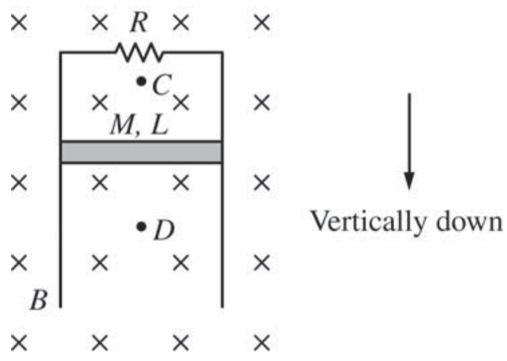
2.1 State what Faraday's law says the induced voltage depends on. [2]

2.2 A coil has 20 turns; one turn would give 0.30 V. Find the coil's voltage. [2]

2.3 State what Lenz's law tells you about the direction of an induced current. [2]

2.4 Explain why Lenz's law is really a statement of energy conservation. [2]

2.5 State how a generator produces electricity. [2]



E&M.3.

A conducting bar of mass M, length L, and negligible resistance is connected to two long vertical conducting rails of negligible resistance. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is located in a magnetic field of magnitude B directed into the page, as shown in the figure above. The bar is released from rest and slides without friction down the rails.

(a) What is the direction of the current in the resistor?

____ Left ____ Right

(b)

i. Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. While the bar is above point D, is the magnitude of the net magnetic field at point D greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Express your answers to parts (c) and (d) in terms of M, L, R, B, and physical constants, as appropriate.

(c) Write, but do NOT solve, a differential equation that could be used to determine the velocity of the falling bar as a function of time t.

(d) Determine an expression for the terminal velocity v_T of the bar.

Express your answers to parts (e) and (f) in terms of v_T, M, L, R, B, and physical constants, as appropriate.

(e) Derive an expression for the power dissipated in the resistor when the bar is falling at terminal velocity.

(f) Using your differential equation from part (c), derive an expression for the speed of the falling bar $v(t)$ as a function of time t.

STOP

END OF EXAM

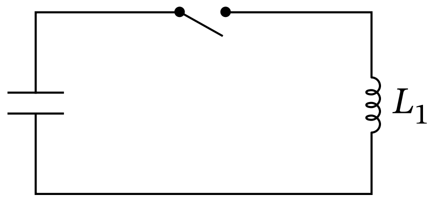
Question 3

The following information applies to parts A and B.

---

3. In Experiment 1, students are asked to collect data that can be graphed to determine the inductance L_1 of an inductor. The students have access to multiple charged capacitors, each of a different known capacitance, and a voltmeter that measures potential difference as a function of time. In each trial, the inductor is connected to a charged capacitor and an initially open switch, as shown in Figure 1.

Figure 1



Part A

i. **Indicate** quantities that could be measured by the students that would allow them to determine L_1 using a linear graph.

ii. Briefly **describe** a method to reduce experimental uncertainty.

Part B

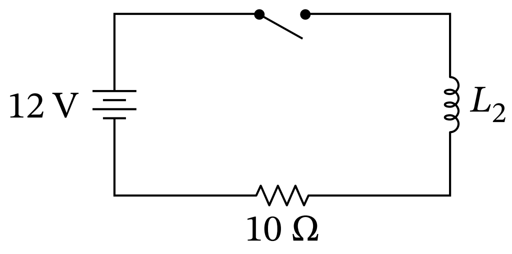
i. **Indicate** what quantities the students could graph on the horizontal and vertical axes to create a linear graph that can be used to determine L_1.

ii. Briefly **describe** the relationship between L_1 and a feature of the graph from part B (i). Your answer may include an equation that relates L_1 and the chosen feature of the graph.

The following information applies to parts C and D.

In Experiment 2, the students are asked to use a graph to determine the inductance L_2 of a new inductor. The inductor is connected to a 12 V battery, a 10 Ω resistor, and an initially open switch, as shown in Figure 2.

Figure 2



After the switch is closed, the current I in the circuit is measured at different times and the corresponding rate of change of the current, $\frac{dI}{dt}$, is determined. The collected data are shown in Table 1.

Table 1

I (A)	$\frac{dI}{dt}$ (A/s)
0.40	280
0.50	230
0.75	150
0.90	100
1.00	68

Part C

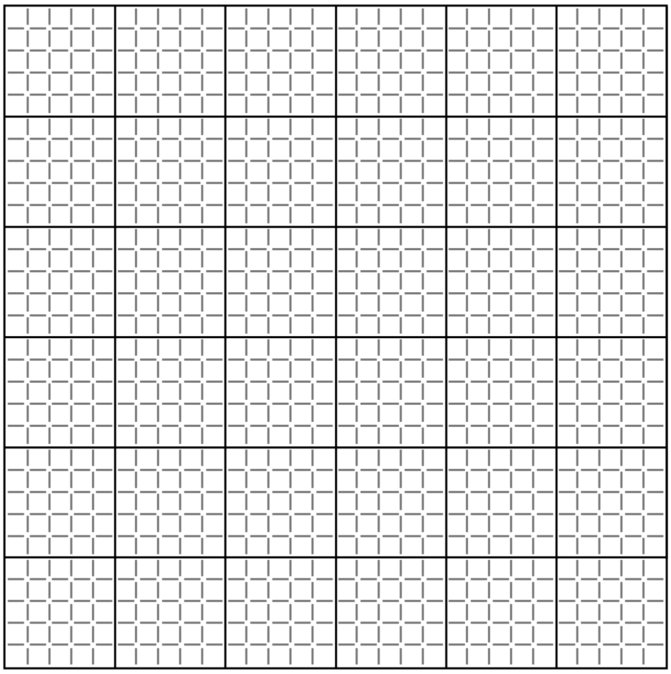
i. **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine L_2.

ii. On the grid provided, create a graph of the quantities indicated in part C (i).

- Clearly **label** each axis with a numerical scale.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

Quantity (units, if appropriate)

____________ ____________



Quantity (units, if appropriate)

____________ ____________

iii. **Draw** a best-fit line for the data plotted in part C (ii).

Part D

Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for L_2.

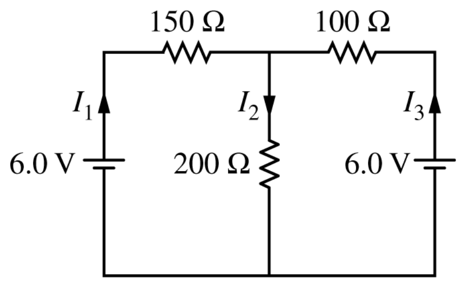


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

   (a)

   i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

   ii. Calculate the current in the $200\ \Omega$ resistor.

   iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

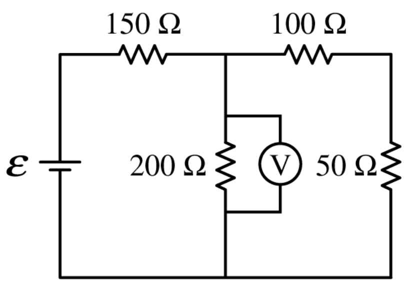


Figure 2

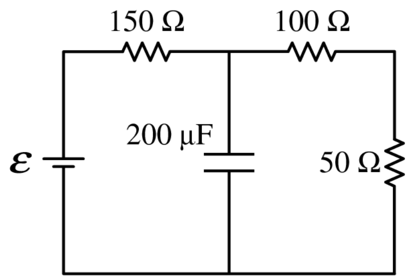
The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

(d)

i. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with a $200\ \mu F$ capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the $50\ \Omega$ resistor.



ii. The $200\ \Omega$ resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the $50\ \Omega$ resistor greater than, less than, or equal to the current calculated in part (b) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.