Past-paper walk-through

Kinetic Theory of Temperature and Pressure ▪ 9.1

eXercise

Questions in this set

- 2026 Q1
- 2025 Q2
- 2023 Q3
- 2021 Q1
- 2021 Q2
- 2019 Q4
- 2018 Q4
- 2017 Q1
- 2016 Q1
- 2015 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2026 ▪ Q1

Part A

A sample of n moles of a monatomic, ideal gas is in a large, sealed, thermally conducting container with fixed volume. A small sphere of mass m_S is in the container. The volume of the sphere is much less than the volume of the container.

The gas is initially in State X with pressure P and volume V, as shown in Figure 1.

[Figure 1: A sealed square container labelled "Gas n, P, V" with a small sphere labelled "Sphere m_S" inside near the bottom, connected by an arrow from the "Sphere" label to a dot inside the container. Caption below the figure: "State X". Note: Figure not drawn to scale.]

The sphere is initially in thermal equilibrium with the gas. The gas undergoes a heating process until the gas is in State Y with pressure $3P$ and the sphere is again in

Question 1

thermal equilibrium with the gas. The total energy transferred to the sphere during the heating process is Q_S.

(i) The graph in Figure 2 represents the number of atoms per unit speed as a function of atom speed for the gas when the gas is in State X. On Figure 3, **sketch** a curve that could represent the number of atoms per unit speed as a function of atom speed for the gas when the gas is in State Y.

[Figure 2: Axes labelled "Number of Atoms per Unit Speed" (vertical) vs. "Atom Speed" (horizontal), captioned "State X". A Maxwell-Boltzmann-type curve starts at the origin, rises to a single rounded peak (marked with a dashed horizontal line to the peak height and a dashed vertical line to the corresponding atom speed), then decreases back toward the horizontal axis at higher speeds.]

[Figure 3: Blank axes labelled "Number of Atoms per Unit Speed" (vertical) vs. "Atom Speed" (horizontal), captioned "State Y", with the same dashed reference lines

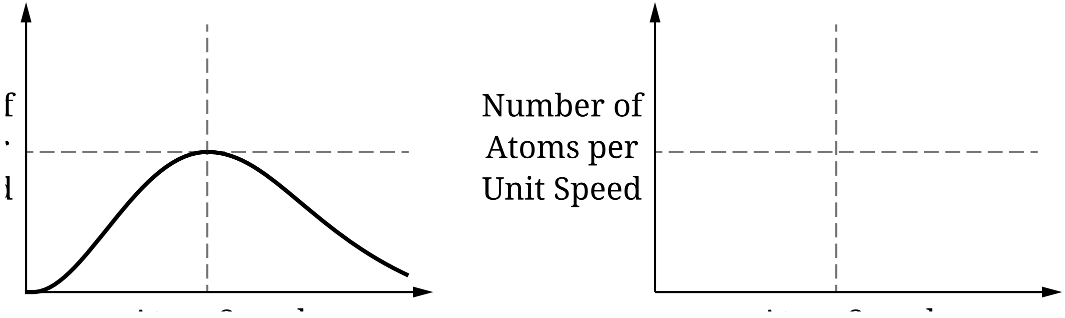
Question 1

(horizontal at the State X peak height, vertical at the State X peak speed) shown for reference, but no curve drawn — to be sketched by the student.]

(ii) **Derive** an expression for the change ΔT in temperature of the gas for the heating process that the gas undergoes as the gas changes from State X to State Y. Express your answer in terms of n, P, V, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

(iii) **Derive** an expression for the specific heat c_S of the sphere. Express your answer in terms of n, m_S, P, V, Q_S, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 1



Question 1

2026 ▪ Q1

Part A

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[Figure 1: A sealed square container labelled "Gas n, P, V" with a small sphere labelled "Sphere m_S" inside near the bottom, connected by an arrow from the "Sphere" label to a dot inside the container. Caption below the figure: "State X". Note: Figure not drawn to scale.]

The sphere is initially in thermal equilibrium with the gas. The gas undergoes a heating process until the gas is in State Y with pressure $3P$ and the sphere is again in thermal equilibrium with the gas. The total energy transferred to the sphere during the heating process is Q_S.

(b) Part B

Question 1

An insulated container is filled with a liquid of mass m_L and specific heat c_L. The original sphere is submerged in the liquid. The sphere has mass m_S, where $m_S < m_L$, and specific heat c_S, where $c_S < c_L$. The initial temperature of the sphere is greater than the initial temperature of the liquid. Later, the liquid and the sphere reach thermal equilibrium. The absolute values of the changes in the temperatures of the liquid and the sphere are $|\Delta T_L|$ and $|\Delta T_S|$, respectively.
Indicate whether $|\Delta T_S|$ is greater than, less than, or equal to $|\Delta T_L|$. Include one of the following relationships in your response.
- $|\Delta T_S| > |\Delta T_L|$ - $|\Delta T_S| < |\Delta T_L|$ - $|\Delta T_S| = |\Delta T_L|$
Justify your answer. Your justification may include expressions/equations but must include conceptual reasoning beyond algebraic solutions.

Question 2

2025 ▪ Q2

A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.

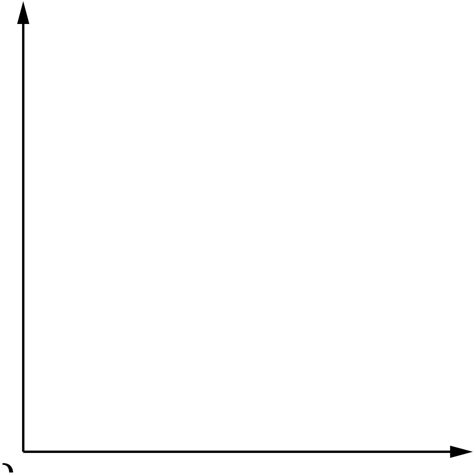
[Figure: Side View. A container (open at the top) sits inside a larger tank labeled "Water Bath". Inside the container, a horizontal bar labeled "Piston" sits partway up, with the region below it labeled "Gas" and the region above it open to the air. The tank surrounding the container is filled with the water bath.]

Note: Figure not drawn to scale.

Question 2

(a) On the dot shown, representing the piston, **draw** and **label** the forces that are exerted on the piston. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.
[Figure: a single dot representing the piston, for the student to draw force vectors from.]

Question 2



Question 2

2025 ▪ Q2

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Note: Figure not drawn to scale.

(b) Derive an expression for the internal energy of the gas in terms of M, A, V_0, P_{atm}, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2025 ▪ Q2

A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.
[Figure: Side View. A container (open at the top) sits inside a larger tank labeled "Water Bath". Inside the container, a horizontal bar labeled "Piston" sits partway up, with the region below it labeled "Gas" and the region above it open to the air. The tank surrounding the container is filled with the water bath.]

Note: Figure not drawn to scale.

(c) A block, also of mass M, is placed on the piston at time $t = t_0$ and is slowly lowered. The piston comes to rest at $t = t_f$ when the block is completely released.

Question 2

On the axes provided, **sketch** the expected relationship between the pressure P and volume V of the gas for the thermodynamic process that the gas undergoes during time interval $t_0 \leq t \leq t_f$. **Draw** an arrow on your sketch to represent the direction of the thermodynamic process.
[Graph: axes with vertical axis P (origin labeled O) and horizontal axis V; both axes unlabeled with values — a blank grid for the student to sketch the P-V process and direction arrow.]

Question 2

2025 ▪ Q2

A sample of a monatomic ideal gas is sealed in a thermally conducting container by a movable piston of mass M and area A. The container is in a large water bath that is held at a constant temperature T_0. The piston is free to move with negligible friction. At the instant shown, the gas is in thermal equilibrium with the water bath, the piston is at rest, and the gas occupies volume V_0. The pressure of the air above the piston is P_{atm}.
[Figure: Side View. A container (open at the top) sits inside a larger tank labeled "Water Bath". Inside the container, a horizontal bar labeled "Piston" sits partway up, with the region below it labeled "Gas" and the region above it open to the air. The tank surrounding the container is filled with the water bath.]

Note: Figure not drawn to scale.

(d) With the block still on the piston, the temperature of the water bath is changed to a new constant temperature T_{new}. The gas occupies the original volume V_0 when the sample of gas and the water bath come to thermal equilibrium.

Question 2

Indicate whether T_{new} is greater than, less than, or equal to T_0.

________ $T_{new} > T_0$

________ $T_{new} < T_0$

________ $T_{new} = T_0$

Briefly **justify** your answer by referencing at least one feature of your answers to parts A, B, or C.

Solution 2

(a)

Mark scheme

- Three forces act on the piston, each drawn as a distinct arrow starting on the dot: F_g (gravity/weight of the piston) points downward; F_atm (force of the atmosphere pushing down on the top of the piston) points downward; F_gas (force of the gas pushing up on the bottom of the piston) points upward. All three arrows must be correctly labeled with the correct direction (F_g down, F_atm down, F_gas up) to earn full credit – one point per correctly drawn/labeled force.

Solution 2

(b) Mark scheme

- Derive U starting from Newton's second law for the piston in equilibrium and the ideal gas law. a_sys = sum(F)/m_sys = F_net/m_sys, and since the piston is in equilibrium sum(F) = 0. Using P = F_perp/A on the piston: P*A - P_atm*A - F_g = 0, i.e. P*A - P_atm*A - Mg = 0, so P - P_atm - Mg/A = 0, giving P = P_atm + Mg/A. Then using U = (3/2)nRT together with PV = nRT gives U = (3/2)PV. Substituting the pressure found above and the original volume V0: U = (3/2)*(P_atm + Mg/A)*V0. Full credit requires: (1) a multistep derivation that includes U=(3/2)nRT (or U=(3/2)N*kB*T), PV=nRT (or PV=N*kB*T), and Newton's second law / P=F_perp/A, or equivalent relevant equations; (2) correctly relating U to PV, e.g. U=(3/2)PV; (3) a correct expression for the

Solution 2

absolute pressure P = P_atm + Mg/A (or consistent with an incorrect force diagram from part A); (4) the final expression for U consistent with that pressure, e.g. U = (3/2)(P_atm + Mg/A)*V0. A correct isolated final expression alone earns the last three points.

Solution 2

(c) Mark scheme

- On the P-V axes, sketch a curve/line connecting a point in the lower-right region of the diagram (larger V, smaller P – the initial state before the block's weight is fully added) to a point in the upper-left region (smaller V, larger P – the final state once the block is completely resting on the piston), i.e. the gas is compressed as pressure increases. The curve should be concave up. Draw an arrow on the curve pointing from the larger-volume/lower-pressure point toward the smaller-volume/higher-pressure point (i.e., from lower-right to upper-left), showing volume decreasing (or pressure increasing) as the process proceeds. Full credit requires: (1) a line/curve connecting lower-right to upper-left; (2) the curve

Solution 2

is concave up; (3) an arrow indicating volume decreasing / pressure increasing along the curve.

Solution 2

(d) Mark scheme

- T_new > T0 (or an indication consistent with incorrect features of parts A, B, or C). Justification: with the block's weight now supported by the gas but the gas back at the original volume V0, the pressure of the gas must be greater than the original pressure (to support the added weight of the block in equilibrium) – e.g. using the part B derivation, since U = (3/2)*P*V and V = V0 is unchanged while P has increased (due to the block's added weight, P = P_atm + Mg/A becomes larger with the extra weight), the internal energy – and hence temperature – increases. Therefore T_new > T0. (Any justification consistent with the student's own diagram in part A, derivation in part B, or graph in part C that correctly relates a pressure increase at equal volume, or a volume increase at equal

Solution 2

pressure, to a temperature increase earns full credit; a justification consistent with an incorrect earlier part is also accepted for the reasoning point.)

Question 3

2023 ▪ Q3

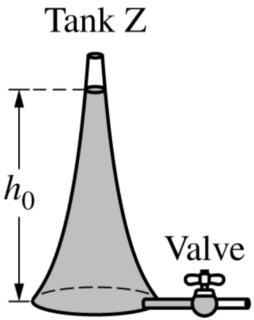
(12 points, suggested time 25 minutes)
[Figure 1: Tank X is a large cylindrical tank, partially filled with water (shaded), with two blocks labeled A and B floating at the bottom-left in front of the tank, and a "Valve" labeled at the bottom-right of the tank. Note: Figure not drawn to scale.]
Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.
(a)(i) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

Question 3

The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

(a)(ii) The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

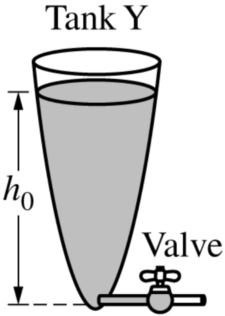
Question 3



Note: Figure not drawn to scale.

Figure 3

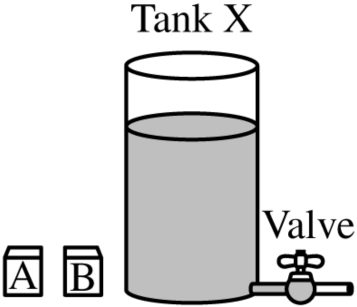
Question 3



Note: Figure not drawn to scale.

Figure 2

Question 3



Note: Figure not drawn to scale.

Figure 1

Question 3

2023 ▪ Q3

(12 points, suggested time 25 minutes)
[Figure 1: Tank X is a large cylindrical tank, partially filled with water (shaded), with two blocks labeled A and B floating at the bottom-left in front of the tank, and a “Valve” labeled at the bottom-right of the tank. Note: Figure not drawn to scale.]

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(b)(i) [Figure 2: Tank Y is a large tank shaped like an inverted cone (wide at the top, narrowing toward the bottom), open to the air at the top, filled with water to height h_0 above a horizontal pipe at the bottom, which has a closed "Valve." Note: Figure not drawn to scale.]

Question 3

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

At time $t = 0$, the valve is opened.

Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that

$$v_p = \sqrt{v_s^2 + 2gh}.$$

(b)(ii) Derive the relationship between v_p and the changing radius R of the top surface of the water to show that

Question 3

$$v_p = \frac{R^2}{r^2} v_s.$$

(b)(iii) When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as

$$v_p = \sqrt{2gh}.$$

Justify this claim.

Question 3

2023 ▪ Q3

(12 points, suggested time 25 minutes)
[Figure 1: Tank X is a large cylindrical tank, partially filled with water (shaded), with two blocks labeled A and B floating at the bottom-left in front of the tank, and a “Valve” labeled at the bottom-right of the tank. Note: Figure not drawn to scale.]

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(c) [Figure 3: Tank Z is a large tank whose top is open to the air, shaped like a cone that is wider at the top and narrows sharply partway down before widening again near a horizontal pipe at the bottom fitted with a closed "Valve." Water fills the tank to height h_0 above the horizontal pipe. Note: Figure not drawn to scale.]

Question 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe. At time $t = 0$, the valve of Tank Z is opened. Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

Solution 3

(a)(i) Mark scheme

- The water and air are modeled as particles in continuous random motion that collide with the block. The air particles collide with the top of the block and exert downward forces on the block (Point 1, part a). The water particles collide with the bottom of the block and exert upward forces on the block (Point 1, part b — both directional statements needed for this point). The force exerted by the water particles is greater than the force exerted by the air particles (Point 2), since water is denser and its particles strike the bottom surface more frequently/with greater momentum transfer. Therefore, the net result of these particle collisions is an upward buoyant force on the block.

Solution 3

(a)(ii)

Mark scheme

- Block A has the greater density. Because Block A displaces a greater volume of fluid than Block B does at the moment Block A touches bottom while B is still floating higher, the buoyant force on Block A is greater than the buoyant force on Block B. Because the buoyant force and gravitational force are balanced for both (floating) blocks, Block A must weigh more than Block B. Because the blocks have identical dimensions (same volume), Block A must be more dense than Block B.

Solution 3

(b)(i)

Mark scheme

- Using Bernoulli's equation between the top surface (speed v_s, height h above the pipe) and the pipe (speed v_p, height 0), with $P_1 = P_2$ (both open to atmosphere) (Point: $P_2 = P_1$): $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ (Point: using Bernoulli's equation to derive the v_p–h relationship). Substituting $y_1 = h,\ v_1 = v_s,\ y_2 = 0,\ v_2 = v_p$ (Point: correct substitutions of heights and speeds): $\frac{1}{2}\rho v_s^2 + \rho g h = \rho g(0) + \frac{1}{2}\rho v_p^2 \Rightarrow v_p^2 = v_s^2 + 2gh \Rightarrow v_p = \sqrt{v_s^2 + 2gh}$.

Solution 3

(b)(ii)

Mark scheme

- Using the continuity equation (Point 1) with the tank's top surface (radius R, speed v_s) and the pipe (radius r, speed v_p), substituting the circular cross-sectional areas and speeds (Point 2):

$$A_1 v_1 = A_2 v_2 \Rightarrow \pi R^2 v_s = \pi r^2 v_p \Rightarrow v_p = \frac{R^2}{r^2} v_s.$$

Solution 3

(b)(iii) Mark scheme

- When the cross-sectional area of the tank is very large compared to the cross-sectional area of the pipe ($R \gg r$), conservation of volume flow rate ($\pi R^2 v_s = \pi r^2 v_p$) requires $v_s \ll v_p$ (Point 1: using conservation principles to justify $v_s \ll v_p$ when $R \gg r$). As a result, the speed of the surface of the water can be approximated as zero — it has a negligible effect on v_p (Point 2) — so from $v_p = \sqrt{v_s^2 + 2gh}$, the speed of water exiting the pipe can be approximated as $v_p = \sqrt{2gh}$.

Solution 3

(c) Mark scheme

- From part (b)(i), $v_p = \sqrt{v_s^2 + 2gh}$: as h decreases, v_p decreases (Point 1: correctly relating the decrease in v_p to the decrease in surface height h). From part (b)(ii) solved for v_s, $v_s = \frac{r^2}{R^2} v_p$: an increase in R results in a decrease in v_s, and since v_p also decreases as h decreases, v_s decreases on both counts (Point 2: correctly relating the decrease in v_s to the increase in radius R). Equivalently, solving the two equations simultaneously gives $v_s = \sqrt{\frac{2gh}{R^4/r^4 - 1}}$, so as h decreases and R increases, v_s decreases.

Question 1

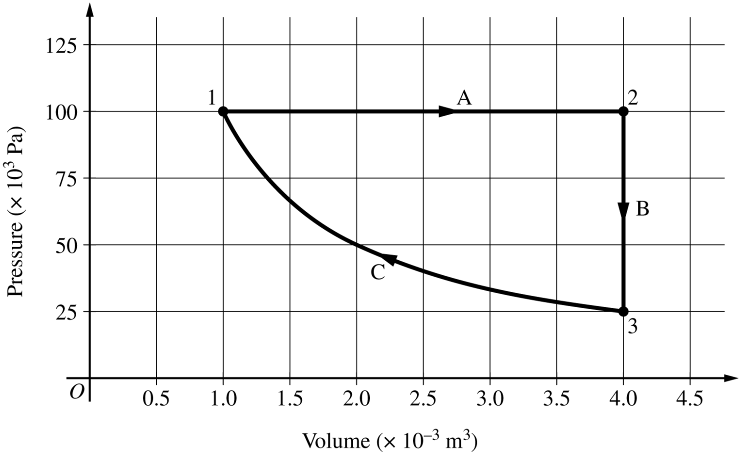
2021 ▪ Q1

A sample of ideal gas is taken through the thermodynamic cycle shown above. Process C is isothermal.
[Graph: Pressure ($\times 10^3$ Pa) on the y-axis (gridlines at 25, 50, 75, 100, 125) vs. Volume ($\times 10^{-3}$ m^3) on the x-axis (gridlines at 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5). State 1 is at approximately $(1.0, 100)$. Process A is a horizontal arrow from state 1 at $(1.0, 100)$ to state 2 at $(4.0, 100)$, pointing right. Process B is a vertical arrow from state 2 at $(4.0, 100)$ down to state 3 at $(4.0, 25)$, pointing down. Process C is a curve from state 3 at $(4.0, 25)$ back to state 1 at $(1.0, 100)$, passing through a labeled point near $(2.1, 42)$, with an arrow pointing up and to the left toward state 1.]
(a) Consider the portion of the cycle that takes the gas from state 1 to state 3 by processes A and B. Calculate the magnitude of the following and indicate the sign of any nonzero quantities.

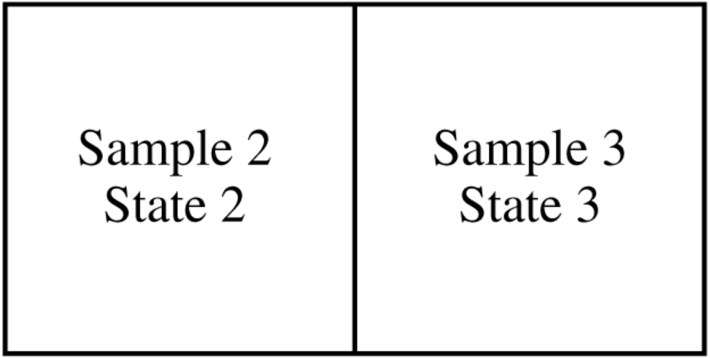
Question 1

- The net change in internal energy ΔU of the gas - The net work W done on the gas - The net energy Q transferred to the gas by heating

Question 1


Pressure (× 10³ Pa)
125
100
75
50
25
O
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
Volume (× 10⁻³ m³)
1
2
3
A
B
C

Question 1



Question 1

2021 ▪ Q1

A sample of ideal gas is taken through the thermodynamic cycle shown above. Process C is isothermal.

[Graph: Pressure ($\times 10^3$ Pa) on the y-axis (gridlines at 25, 50, 75, 100, 125) vs. Volume ($\times 10^{-3}$ m^3) on the x-axis (gridlines at 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5). State 1 is at approximately $(1.0, 100)$. Process A is a horizontal arrow from state 1 at $(1.0, 100)$ to state 2 at $(4.0, 100)$, pointing right. Process B is a vertical arrow from state 2 at $(4.0, 100)$ down to state 3 at $(4.0, 25)$, pointing down. Process C is a curve from state 3 at $(4.0, 25)$ back to state 1 at $(1.0, 100)$, passing through a labeled point near $(2.1, 42)$, with an arrow pointing up and to the left toward state 1.]

(b)(i) Consider isothermal process C.

Question 1

Compare the magnitude and sign of the work W done on the gas in process C to the magnitude and sign of the work in the portion of the cycle in part (a). Support your answer using features of the graph.
(b)(ii) Explain how the microscopic behavior of the gas particles and changes in the size of the container affect interactions on the microscopic level and produce the observed pressure difference between the beginning and end of process C.

Question 1

2021 ▪ Q1

A sample of ideal gas is taken through the thermodynamic cycle shown above. Process C is isothermal.

[Graph: Pressure ($\times 10^3$ Pa) on the y-axis (gridlines at 25, 50, 75, 100, 125) vs. Volume ($\times 10^{-3}$ m^3) on the x-axis (gridlines at 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5). State 1 is at approximately $(1.0, 100)$. Process A is a horizontal arrow from state 1 at $(1.0, 100)$ to state 2 at $(4.0, 100)$, pointing right. Process B is a vertical arrow from state 2 at $(4.0, 100)$ down to state 3 at $(4.0, 25)$, pointing down. Process C is a curve from state 3 at $(4.0, 25)$ back to state 1 at $(1.0, 100)$, passing through a labeled point near $(2.1, 42)$, with an arrow pointing up and to the left toward state 1.]

(c) Consider two samples of the gas, each with the same number of gas particles. Sample 2 is in state 2 shown in the graph, and sample 3 is in state 3 shown in the graph. The samples are put into thermal contact, as shown above.

Question 1

[Diagram: two adjacent boxes labeled “Sample 2 State 2” and “Sample 3 State 3”, placed in contact with each other.]
Indicate the direction, if any, of energy transfer between the samples. Support your answer using macroscopic thermodynamic principles.

Solution 1

(a) Mark scheme

- $\Delta U = 0$ J (temperatures at states 1 and 3 are equal, so internal energy is unchanged). Net work done ON the gas: $W = -P\Delta V = -(100\times10$^3 Pa)(3×10^-3 m^3) $= -300$ J (volume increases from 1×10^-3 m^3 to 4×10^-3 m^3 at constant pressure 100×10^3 Pa; answer must show the negative sign or state the work is done BY the gas). Applying the first law $\Delta U = Q + W$ with ΔU=0 and W=$-$300 J gives $Q = +300$ J transferred TO the gas by heating (or equivalently, showing |Q|=|W| with opposite sign). Award 1 point each for: ΔU=0; W=$-$300 J with correct sign/units; correctly using the first law to obtain Q.

Solution 1

(b)(i)

Mark scheme

- The magnitude of the work in isothermal process C is LESS than in part (a) ($|W\_C| < 300$ J), because there is less area under the P–V curve for process C than for the combined processes A+B (1 point, must reference area under the curve). The sign is OPPOSITE that of part (a): in part (a) the volume increased so W (on gas) was negative; in process C the volume decreases (shown by the arrow direction on the graph), so the work done ON the gas is POSITIVE (1 point, must reference the sign of ΔV or the direction of the process on the graph).

Solution 1

(b)(ii) Mark scheme

- Process C is isothermal, so temperature is constant and the average kinetic energy/average speed of the gas molecules does NOT change (1 point). The volume of the container decreases, which changes something relevant to the collision rate — acceptable references include volume, surface area, or time to traverse the container (1 point). With unchanged molecular speed but decreased volume, molecules collide with the container walls more frequently, producing more net force per unit area on the walls — the response must explicitly refer to collisions with the walls (1 point). Example: temperature (and thus molecular speed) is unchanged; smaller volume increases the density of gas molecules, so they collide with the walls more often, giving more net force due to wall collisions

Solution 1

(and the smaller volume also means less wall surface area), producing higher pressure.

Solution 1

(c)

Mark scheme

- The temperature of the gas in state 2 (sample 2) is HIGHER than the temperature in state 3 (sample 3), as read from their positions on the P–V diagram/isotherms (1 point). When placed in thermal contact, energy spontaneously flows from the hotter sample to the cooler sample, so energy transfers from sample 2 to sample 3 (1 point, must reference the macroscopic principle that heat flows hot$\rightarrow$cold).

Question 2

2021 ▪ Q2

[Diagram: a cylindrical container with a movable piston. A downward-pointing arrow labeled "Cylinder" points to the outer container wall, and a downward-pointing arrow labeled "Piston" points to a shaded disk (the piston) inside the cylinder near the top.]
A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

Question 2

(a)(i) Describe the measurements the students should take and a procedure they could use to collect the data needed to create the graph. Specifically indicate how the students could keep the temperature constant. Include enough detail that another student could follow the procedure and obtain similar data.

(a)(ii) Determine an expression for the absolute pressure of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

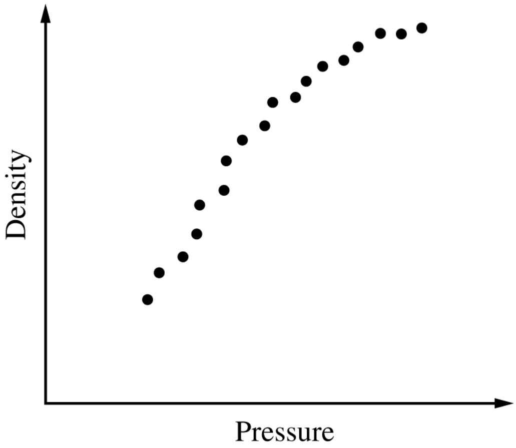
(a)(iii) Determine an expression for the density of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

(a)(iv) [Graph: Density on the y-axis vs. Pressure on the x-axis, no numerical scale shown. Data points form a curve starting near the lower left, rising steeply at first,

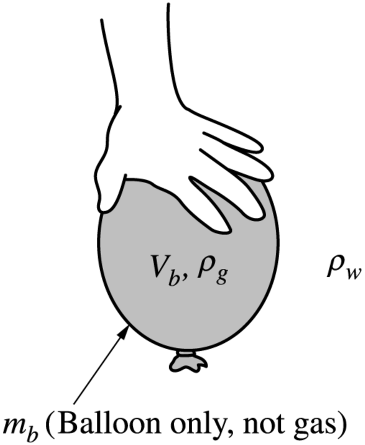
Question 2

then curving to rise more gradually (concave down) as pressure increases, ending near the upper right.]
The graph above represents the students' data. Does the data indicate that the gas is ideal? Describe the application of physics principles in an analysis of the graph that can be used to arrive at your answer.

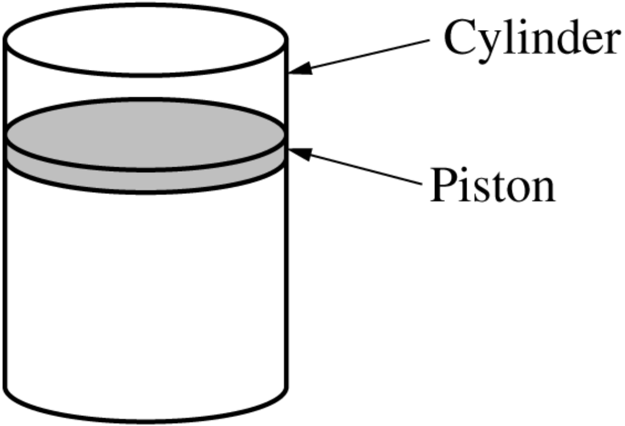
Question 2



Question 2



Question 2



Question 2

2021 ▪ Q2

[Diagram: a cylindrical container with a movable piston. A downward-pointing arrow labeled "Cylinder" points to the outer container wall, and a downward-pointing arrow labeled "Piston" points to a shaded disk (the piston) inside the cylinder near the top.]
A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

Question 2

(b) Another group of students propose that the relationship between density and pressure could also be obtained by filling a balloon with the gas and submerging it to increasing depths in a deep pool of water.
Why could submerging the balloon to increasing depths be useful for determining the relationship between the density and pressure of the gas?

Question 2

2021 ▪ Q2

[Diagram: a cylindrical container with a movable piston. A downward-pointing arrow labeled "Cylinder" points to the outer container wall, and a downward-pointing arrow labeled "Piston" points to a shaded disk (the piston) inside the cylinder near the top.]
A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

Question 2

(c) [Diagram: a hand pushing down on a shaded spherical balloon submerged in water. The balloon is labeled with V_b, ρ_g inside it, ρ_w labeled outside to the right (the surrounding water), and m_b (Balloon only, not gas) labeled below with an arrow pointing to the balloon.]
The balloon is kept underwater in the deep pool by a student pushing down on the balloon, as shown above. Let V_b represent the volume of the inflated balloon, m_b represent the mass of just the balloon (not including the mass of the gas), ρ_g represent the density of the gas in the balloon, and ρ_w represent the density of the water. Derive an expression for the force the student must exert to hold the balloon at rest under the water, in terms of the quantities given in this part and physical constants, as appropriate.

Solution 2

(a)(i) Mark scheme

- A valid procedure should include: (1) a method to keep temperature constant, e.g. submerge the piston/container in an ice-water bath (1 point); (2) a valid use of objects of known mass to change the pressure, e.g. place objects of known mass on top of the piston one at a time to add weight (1 point); (3) explicitly measuring the piston height h AND the piston radius r (or diameter) for each trial, so volume/area can be computed (1 point); (4) explicitly obtaining more than two data points, i.e. repeating with several (e.g. eight) different masses/trials (1 point). Example: 'Place the container in an ice bath so the part below the piston is submerged. For eight different objects of known mass, add each object to the piston and measure the height (and radius) of the piston for each object.'

Solution 2

(a)(ii)

Mark scheme

- An equation for the absolute pressure of the gas, consistent with the procedure in (a)(i), e.g.: $P\_tot = P\_atm + (m\_p + N \cdot m\_o)g$ / $A =$ $P\_atm + (m\_p + N \cdot m\_o)g$ / (πr^2), where m_p is the piston's mass, N is the number of objects placed on the piston, m_o is the mass of one object, g is the gravitational acceleration, $A = \pi r^2$ is the piston's cross-sectional area, and r is the piston radius. Any newly introduced symbol must be defined.

Solution 2

(a)(iii)

Mark scheme

- An equation for the density of the gas in terms of measured quantities, e.g.: $\rho = M\_g / V = M\_g / (\pi r^2 h)$, where M_g is the mass of the enclosed gas, $V = \pi r^2 h$ is the gas volume, r is the piston radius, and h is the piston/gas-column height. Any newly introduced symbol must be defined.

Solution 2

(a)(iv) Mark scheme

- By the ideal gas law (or Boyle's Law) at constant temperature and fixed amount of gas, PV $=$ constant, so P is inversely proportional to V (1 point, must reference the ideal gas law/Boyle's law and use it to derive the inverse proportionality). Since the mass of gas is constant, density $\rho =$ M/V, so for an ideal gas ρ should be directly proportional to P — a straight line through the origin on a density-vs-pressure graph. Because the actual graph is a curve (concave down, not a straight line), pressure and density are NOT directly proportional, so the data indicate the gas is NOT ideal (1 point, conclusion must be grounded in the shape/slope of the graph and a correct P–ρ relationship).

Solution 2

(b)

Mark scheme

- Submerging the balloon to increasing depths increases the water pressure on the balloon (and hence the gas pressure inside it), since pressure in a fluid increases with depth: $P = P\_atm + \rho\_w$ g d. This gives the students another controllable way to vary the gas pressure (and hence density/volume) continuously with depth, providing additional data points for the density-vs-pressure relationship without needing more discrete masses.

Solution 2

(c)

Mark scheme

- Applying Newton's second law (equilibrium) to the balloon+gas system held underwater by the student's downward push: $\Sigma F = 0 = F\_B - W\_balloon - W\_gas - F\_student$ (1 point; must be a correct application including at least one of the two weight terms). The buoyant force is $F\_B = \rho\_w\ V\_b\ g$, where ρ_w is water density and V_b is the balloon's volume (1 point). The total weight is $W = W\_balloon + W\_gas = m\_b\ g + \rho\_g\ V\_b\ g$, where m_b is the mass of the balloon alone and ρ_g is the gas density (1 point). Solving gives $F\_student = \rho\_w\ V\_b\ g - (\rho\_g\ V\_b\ g + m\_b\ g)$.

Question 4

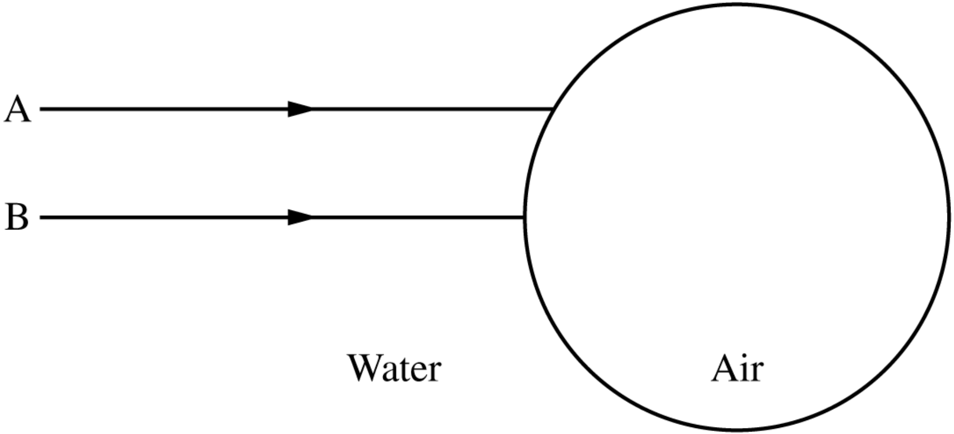
2019 ▪ Q4

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(a) In the figure below, the circle represents one such air bubble, and two incoming rays of light, A and B, are shown. Ray B points toward the center of the circle. On the diagram, draw the paths of rays A and B as they go through the bubble and back into the water. Your diagram should clearly show what happens to the rays at each interface.

[Figure: A large circle labeled "Air" on the right side, with "Water" labeled to its left outside the circle. Two horizontal rays labeled A (upper) and B (lower) enter from the left and strike the circle's boundary; ray B is aimed at the center of the circle. Arrowheads show the rays traveling rightward toward the circle.]

Question 4



Question 4

2019 ▪ Q4

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(b) The bubble has a volume V_1, the air inside it has density ρ_A, and the water around it has density ρ_W. The bubble starts at rest and has a speed v_f when it has risen a height h. Assume that the change in the bubble's volume is negligible. Derive an expression for the mechanical energy dissipated by drag forces as the bubble rises this distance. Express your answer in terms of the given quantities and fundamental constants, as appropriate.

Question 4

2019 ▪ Q4

A student notices many air bubbles rising through the water in a large fish tank at an aquarium.

(c)(i) At a particular instant, one bubble is 4.5 m below the water's surface. The surface of the water is at sea level, and the density of the water is $1000\ \text{kg/m}^3$. Determine the absolute pressure in the bubble at this location.

(c)(ii) The bubble has a volume V_1 when it is 4.5 m below the water's surface. Assume that the temperature of the air in the bubble remains constant as it rises. In terms of V_1, calculate the volume of the bubble when it is just below the surface of the water.

(c)(iii) If the air in the bubble cooled as it rose, the volume of the bubble would be less than the value calculated in part (c)(ii). Use physics principles to briefly explain why.

Solution 4

(a) Mark scheme

- Ray B is aimed at the center of the circular bubble, so it strikes the water–air interface along the normal (radial direction) and passes straight through undeviated, continuing in a straight line through the bubble and back into the water on the far side (1 pt for ray B going straight through). Ray A strikes the first interface at an angle to the normal and bends away from the normal as it refracts from water into air (light speeds up going into the lower-index medium, air) (1 pt for ray A bending away from the normal entering the air from the water). At the second interface, as ray A exits the air bubble back into the water, it bends the opposite way relative to the normal there (toward the normal, since it is now going from a lower-index medium, air, into a higher-index medium, water)

Solution 4

compared to how it refracted at the first interface (1 pt for ray A bending the opposite direction relative to the normal on exiting compared to entering).

Solution 4

(b) Mark scheme

- Apply the work-energy theorem to the rising bubble:
  $\Delta K = W_{net} = |W_b| - |W_g| - |W_{diss}|$, where W_b is the work done by the buoyant force, $|W_g|$ is the magnitude of the work done against gravity, and $|W_{diss}|$ is the mechanical energy dissipated by drag (1 pt for a valid application of the work-energy theorem). The buoyant force equals the weight of displaced water, $\rho_W V_1 g$, so the work it does over height h is $W_b = \rho_W V_1 gh$ (1 pt for finding the work done by the buoyant force). Substituting $\Delta K = \frac{1}{2}\rho_A V_1 v_f^2$ (starting from rest) and $|W_g| = \rho_A V_1 gh$ (weight of the bubble's air) with consistent relative signs: $\frac{1}{2}\rho_A V_1 v_f^2 = \rho_W V_1 gh - \rho_A V_1 gh - |W_{diss}|$, so

Solution 4

$|W_{diss}| = \rho_W V_1 gh - \rho_A V_1 gh - \frac{1}{2}\rho_A V_1 v_f^2$ (1 pt for correct substitution into an equation with consistent relative signs for the terms).

Solution 4

(c)(i)

Mark scheme

- $P_{4.5\text{m}} = P_{atm} + \rho_W g d = (1.0 \times 10^5 \text{ Pa}) + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.5 \text{ m})$. $P_{4.5\text{m}} = 1.44 \times 10^5$ Pa (or 1.45×10^5 Pa using $g = 10 \text{ m/s}^2$). (1 pt for a correct answer with units.)

Solution 4

(c)(ii)

Mark scheme

- Applying the ideal gas law at constant temperature between the two locations (so PV is constant): $P_{4.5\text{m}} V_1 = P_{atm} V_{surface}$, giving $V_{surface} = P_{4.5\text{m}} V_1 / P_{atm}$ (1 pt for applying the ideal gas law at two locations to determine the new bubble volume). Substituting the pressure from part (c)(i): $V_{surface} = (1.44 \times 10^5 \text{ Pa})\, V_1 / (1 \times 10^5 \text{ Pa}) = 1.44\, V_1$ (or $1.45\, V_1$ using $g = 10 \text{ m/s}^2$) (1 pt for substituting pressures consistent with part (i)).

Solution 4

(c)(iii) Mark scheme

- By the ideal gas law, $P_{4.5\mathrm{m}} V_1 / T_1 = P_{atm} V_{surface} / T_{surface}$, so $V_{surface} = P_{4.5\mathrm{m}} V_1 T_{surface} / (P_{atm} T_1)$; the pressures are the same as before, but now $T_{surface} < T_1$ (the air cooled), so the volume at the surface comes out smaller than the constant-temperature value found in part (c)(ii). (Equivalent microscopic explanation: as the bubble cools, its air molecules move slower and exert less outward force on the bubble's inner surface; the resulting unbalanced inward force from the water causes the bubble to expand less than — i.e., contract relative to — the constant-temperature case.) Full credit for a correct explanation, whether qualitative or quantitative, macroscopic or microscopic. (1 pt.)

Question 4

2018 ▪ Q4

[Diagram: a large boat viewed from above/side, labeled with an arrow pointing to steel bars stacked on its deck, labeled "Steel Bars". A small cabin structure sits at the rear of the boat.]

A large boat like the one shown above has a mass M_b and can displace a maximum volume V_b. The boat is floating in a river with water of density ρ_{water} and is being loaded with steel beams each of density ρ_{steel} and volume V_{steel}. The boat owners want to be able to carry as many beams as possible.

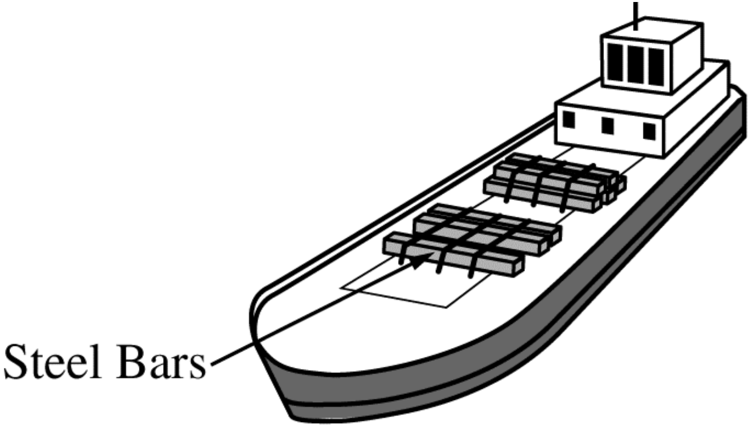
(a) Derive an expression for the maximum number N of steel beams that can be loaded on the boat without exceeding the maximum displaced volume, in terms of the given quantities and physical constants, as appropriate.

(b) The captain realizes that oil is leaking from the boat, creating a thin film of oil on the water surface. In one area of the oil film the surface looks mostly green. Explain in

Question 4

detail how constructive interference contributes to the green appearance. Assume the index of refraction of the oil is greater than the index of refraction of the water.
(c) Later the boat is floating down the river with the water current, heading for a town. The river has a width of 60 m and a constant depth and flows at a speed of 5 km/hr. Partway to the town, the river narrows to a width of 30 m while its depth remains the same. Calculate the speed of the water in the narrow section.

Question 4



Solution 4

(a)

Mark scheme

- At maximum load, the boat floats with the maximum displaced volume V_b, so the buoyant force equals the total weight of the boat-plus-beams system: $F_b = W_{system}$. Weight of the system: $W_{system} = (M_b + N\rho_{steel}V_{steel})g$, where N is the number of steel beams. Buoyant force (Archimedes' principle): $F_b = \rho_{water}gV_b$. Setting these equal and solving for N: $(M_b + N\rho_{steel}V_{steel})g = \rho_{water}gV_b \Rightarrow N = \dfrac{\rho_{water}V_b - M_b}{\rho_{steel}V_{steel}}$.

Solution 4

(b) Mark scheme

- The green appearance results from constructive interference between the light wave reflected at the air-oil interface and the light wave reflected at the oil-water interface — interference between these two reflected waves. Because the oil has a higher refractive index than air, the wave reflecting off the air-oil interface undergoes a 180° phase shift on reflection, while the reflection at the oil-water interface (oil index greater than water index, as given) does not gain that same extra shift — so there is a net relative phase shift arising from one of the two reflections. The wavelength of the light inside the oil film is shorter than in air/vacuum ($\lambda_{oil} = \lambda_{air}/n_{oil}$), which sets the phase accumulated as the wave travels through the film. There is also a path-length difference between the two

Solution 4

reflected rays, equal to twice the oil film's thickness (the extra distance traveled by the ray that penetrates to the oil-water interface and reflects back). When the combined phase difference — from the path-length difference through the oil (using the wavelength in oil) together with the reflection phase shift — corresponds to constructive interference specifically for green wavelengths, the reflected light appears green.

Solution 4

(c)

Mark scheme

- By conservation of mass for an incompressible fluid (the continuity equation), $A_{wide}v_{wide} = A_{narrow}v_{narrow}$, where cross-sectional area $=$ width $\times$ depth; since the depth is the same in both sections it cancels, leaving width in place of area: $(60\ \text{m})(\text{depth})(5\ \text{km/hr}) = (30\ \text{m})(\text{depth})(v_{narrow})$. Solving: $v_{narrow} = \dfrac{(60\ \text{m})(5\ \text{km/hr})}{30\ \text{m}} = 10\ \text{km/hr}$.

Question 1

2017 ▪ Q1

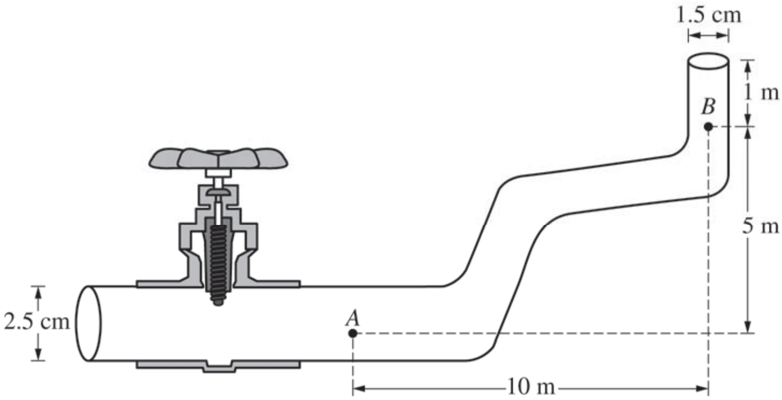
Two students observe water flowing from left to right through the section of pipe shown above, which decreases in diameter and increases in elevation. The pipe ends on the right, where the water exits vertically. At point A the water is known to have a speed of 0.50 m/s and a pressure of 2.0×10^5 Pa. The density of water is 1000 kg/m^3. [Diagram: A section of horizontal pipe with a valve (wheel handle) near the left end. The pipe has diameter 2.5 cm at the left section (where point A is located, 10 m to the right of the valve), then bends and rises, decreasing to diameter 1.5 cm at point B near the top right, where the water exits vertically upward out of the open pipe end. Point B is located 1 m below the top opening of the vertical pipe, and the vertical pipe extends 5 m above the level of point A. Note: Figure not drawn to scale.]

(a) The students disagree about the water pressure and speed at point B. They make the following claims.

Question 1

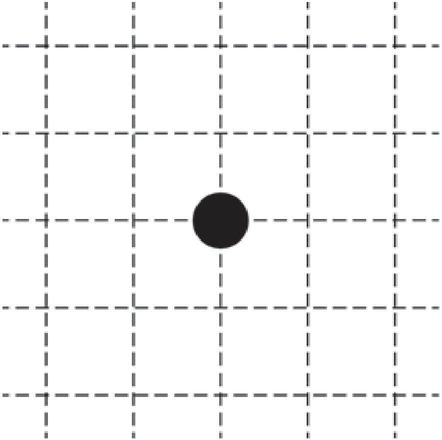
Student Y claims that the pressure at point B is greater than that at point A because the water is moving faster at point B.
Student Z claims the speed of the water is less at point B than that at point A because by conservation of energy, some of the water's kinetic energy has been converted to potential energy of the Earth-water system.
i. Indicate any aspects of student Y's claim that are correct.
ii. Indicate any aspects of student Y's claim that are incorrect. Support your answer using appropriate physics principles.
iii. Indicate any aspects of student Z's claim that are correct.
iv. Indicate any aspects of student Z's claim that are incorrect. Support your answer using appropriate physics principles.

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2017 ▪ Q1

Two students observe water flowing from left to right through the section of pipe shown above, which decreases in diameter and increases in elevation. The pipe ends on the right, where the water exits vertically. At point A the water is known to have a speed of 0.50 m/s and a pressure of 2.0×10^5 Pa. The density of water is 1000 kg/m^3.

[Diagram: A section of horizontal pipe with a valve (wheel handle) near the left end. The pipe has diameter 2.5 cm at the left section (where point A is located, 10 m to the right of the valve), then bends and rises, decreasing to diameter 1.5 cm at point B near the top right, where the water exits vertically upward out of the open pipe end. Point B is located 1 m below the top opening of the vertical pipe, and the vertical pipe extends 5 m above the level of point A. Note: Figure not drawn to scale.]

(b)(i) Calculate the following at point B.

The speed of the water

Question 1

(b)(ii) Calculate the following at point B.
The pressure in the pipe

Question 1

2017 ▪ Q1

Two students observe water flowing from left to right through the section of pipe shown above, which decreases in diameter and increases in elevation. The pipe ends on the right, where the water exits vertically. At point A the water is known to have a speed of 0.50 m/s and a pressure of 2.0×10^5 Pa. The density of water is 1000 kg/m^3.

[Diagram: A section of horizontal pipe with a valve (wheel handle) near the left end. The pipe has diameter 2.5 cm at the left section (where point A is located, 10 m to the right of the valve), then bends and rises, decreasing to diameter 1.5 cm at point B near the top right, where the water exits vertically upward out of the open pipe end. Point B is located 1 m below the top opening of the vertical pipe, and the vertical pipe extends 5 m above the level of point A. Note: Figure not drawn to scale.]

(c)(i) A valve to the left of point A now closes off that end of the pipe. The section of pipe shown is still full of water, but the water is no longer flowing.

Question 1

Calculate the absolute pressure at point A (the pressure that includes the effect of the atmosphere).

(c)(ii) An air bubble forms at point A. On the figure below, where the dot represents the air bubble, draw a free-body diagram showing and labeling the forces (not components) exerted on the bubble. Draw the relative lengths of all vectors to reflect the relative magnitudes of the forces.

[Figure: A dashed grid with a single filled black dot at the center, representing the air bubble at point A, on which the free-body diagram is to be drawn.]

Solution 1

(a) Mark scheme

- Part (a) has four sub-parts (i)-(iv), $1+2+1+1 = 5$ points total.
- (i) 1 pt: Student Y is correct that the water moves faster at point B (no other aspect needs to be indicated).
- (ii) 2 pts: Student Y's claim that $P_B > P_A$ is NOT correct. 1 pt for a correct indication of how height affects pressure using Bernoulli's equation (conservation of energy); 1 pt for a correct indication of how speed affects pressure using Bernoulli's equation. Example: The pressure at B is not greater. Because the water at B is moving faster and is higher, the kinetic energy and gravitational

Solution 1

potential energy terms in Bernoulli's equation are both greater. Because the sum of pressure and these energy terms is a constant, the pressure must be less.

- (iii) 1 pt: For indicating one of: Student Z is correct that the potential energy of the water-Earth system has increased; OR Student Z is correct that conservation of energy applies; OR stating that nothing is correct (with justification in (iv)).
- (iv) 1 pt: For indicating that Student Z is incorrect that the speed is less at B (not indicating any other aspect), using continuity or Bernoulli's equation to show the speed is actually greater at B, OR (if the third bullet of (iii) applies) indicating that work is done on the water due to the pressure difference, so the energy is not constant.

Solution 1

(b)(i)

Mark scheme

- Use the continuity equation: $A_A v_A = A_B v_B$.
- $v_B = A_A v_A / A_B = r_A^2 v_A / r_B^2 = (2.5\ \text{cm})^2 (0.5\ \text{m/s}) / (1.5\ \text{cm})^2$
- $v_B = 1.4\ \text{m/s}$.
- 1 pt for a correct application of the continuity equation including substitutions; 1 pt for a correct answer with units.

Solution 1

(b)(ii) Mark scheme

- Apply Bernoulli's equation between A and B, consistent with the speed found in (b)(i):
- $P_A + \rho g y_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g y_B + \frac{1}{2}\rho v_B^2$
- $P_B = P_A + \rho g(y_A - y_B) + \frac{1}{2}\rho(v_A^2 - v_B^2)$
- $P_B = 2 \times 10^5 + (1000)(10)(-5) + \frac{1}{2}(1000)(0.5^2 - 1.4^2) = 2 \times 10^5 - 50000 - 855$
- $P_B = 1.5 \times 10^5$ Pa.
- 1 pt for an application of Bernoulli's equation with substitutions consistent with (b)(i).

Solution 1

(c)(i)

Mark scheme

- $P = P_0 + \rho g h_A = 1 \times 10^5\ \mathrm{Pa} + (1000\ \mathrm{kg/m^3})(10\ \mathrm{m/s^2})(6\ \mathrm{m})$
- $P = 1.6 \times 10^5\ \mathrm{Pa}$.
- 1 pt for correctly substituting into the appropriate hydrostatic-pressure equation.

Solution 1

(c)(ii)

Mark scheme

- Free-body diagram of the air bubble at point A: draw the buoyant force pointing toward the top of the page and gravity pointing toward the bottom of the page, with the buoyant-force arrow drawn longer than the gravity arrow (net upward force, since the bubble is less dense than the surrounding water). The student may instead draw several pressure forces around the dot rather than a single buoyant force, as long as no arrow is explicitly labeled 'buoyant force' and the forces sum to a net upward force longer than the gravitational force. 1 pt total.

Question 1

2016 ▪ Q1

[Graph: P
$(10^5 Pa) ontheverticalaxis, V(m^3) onthehorizontalaxis. Gridlinesontheverticalaxisat 0.5, 1.0, 1.5; gr$
pointAisat(0.04, 1.0); *ahorizontallinewithanarrowpointsrightfromAtopointBat*(0.10,
1.0); *alinewithanarrowpointsdownfromBtopointCat*(0.10,
0.5); *adiagonallinewithanarrowpointsfromCbackuptoA*.]

Two moles of a monatomic ideal gas are enclosed in a cylinder by a movable piston. The gas is taken through the thermodynamic cycle shown in the figure above. The piston has a cross-sectional area of 5×10^{-3} m^2.

(a)(i) Calculate the force that the gas exerts on the piston in state A, and explain how the collisions of the gas atoms with the piston allow the gas to exert a force on the piston.

Question 1

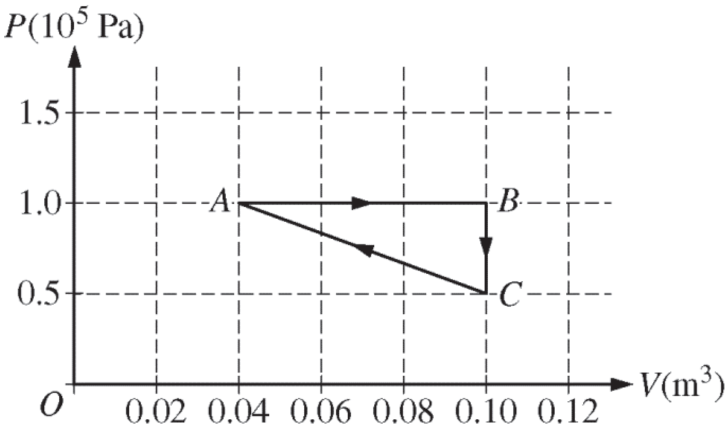
(a)(ii) Calculate the temperature of the gas in state B, and indicate the microscopic property of the gas that is characterized by the temperature.

(b)(i) Predict qualitatively how the internal energy of the gas changes as it is taken from state A to state B. Justify your prediction.

(b)(ii) Calculate the energy added to the gas by heating as it is taken from state A to state C along the path ABC.

(c) Determine the change in the total kinetic energy of the gas atoms as the gas is taken directly from state C to state A.

Question 1



Solution 1

(a)(i)

Mark scheme

- F = PA = (1.0×10^5 Pa)(5×10^-3 m^2) = 500 N. Explanation: the gas atoms colliding with the piston undergo a change in momentum/velocity on each collision; by Newton's third law this momentum exchange means each colliding atom exerts a force on the piston, and the sum of these collisions over time produces the net force on the piston. Full credit needs (1) the calculation with correct units and (2) an explanation that identifies some change in the atoms' momentum or velocity to justify the force between atoms and piston.

Solution 1

(a)(ii)

Mark scheme

- Using $PV = nRT$: $T = PV/(nR) = (1.0\times10\hat{}5 \text{ Pa})(0.10 \text{ m}\hat{}3)/[(2 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})] = 602$ K. Temperature characterizes the AVERAGE speed (equivalently the average kinetic energy or the RMS velocity) of the gas molecules — not the speed of any single molecule. Full credit needs (1) the calculation with correct units and (2) identifying that temperature reflects an average/RMS microscopic quantity.

Solution 1

(b)(i)

Mark scheme

- The internal energy increases from state A to state B. Because the process A$\rightarrow$B occurs at constant pressure while the volume increases, $PV = nRT$ shows the temperature must increase. An increase in temperature means an increase in the average kinetic energy of the gas molecules, i.e., an increase in the total internal energy of the gas. Full credit needs (1) identifying that temperature increases because volume increases at constant pressure and (2) relating that temperature increase to an internal-energy increase.

Solution 1

(b)(ii) Mark scheme

- Work done in process ABC (area under the P-V path): W_AB = -(1.0×10^5 Pa)(0.10 m^3 - 0.04 m^3) = -6000 J, and W_BC = 0 (constant volume). Temperatures: T_A = P_A V_A/(nR) = (1.0×10^5 Pa)(0.04 m^3)/[(2 mol)(8.31 J/mol $\cdot$ K)] = 241 K; T_C = P_C V_C/(nR) = (0.5×10^5 Pa)(0.10 m^3)/[(2 mol)(8.31 J/mol $\cdot$ K)] = 301 K. Internal energy change: ΔU = (3/2)nRΔT = (3/2)(P_C V_C - P_A V_A) = (3/2)[(0.5×10^5 Pa)(0.10 m^3) - (1.0×10^5 Pa)(0.04 m^3)] = 1500 J (equivalently ΔU = (3/2)k_B ΔT n N_0 = (3/2)(1.38×10^-23 J/K)(301 K - 241 K)(2 mol)(6.02×10^23 mol^-1) = 1500 J). First law: Q = ΔU - W = 1500 J - (-6000 J) = 7500 J. The energy added to the gas by heating along ABC is Q = 7500 J. Full credit needs (1) W_AB = -6000

Solution 1

J and W_BC $= 0$, (2) T_A, T_C (or ΔT) used to get $\Delta U = 1500$ J, and (3) correctly substituting ΔU and W into the first law (with units) to get $Q = 7500$ J.

Solution 1

(c)

Mark scheme

- The change in total kinetic energy of the gas atoms for process C→A has the same magnitude as ΔU found in part (b)(ii) but the opposite sign (since C→A is the reverse temperature change of A→C): $\Delta K\_total = (3/2)k\_B \Delta T\ n\ N\_0 = (3/2)(1.38\times10\hat{}-23\ J/K)(241\ K - 301\ K)(2\ mol)(6.02\times10\hat{}23\ mol\hat{}-1) = -1500\ J$. So $\Delta K\_total = -1500$ J (the kinetic energy of the gas atoms decreases by 1500 J).

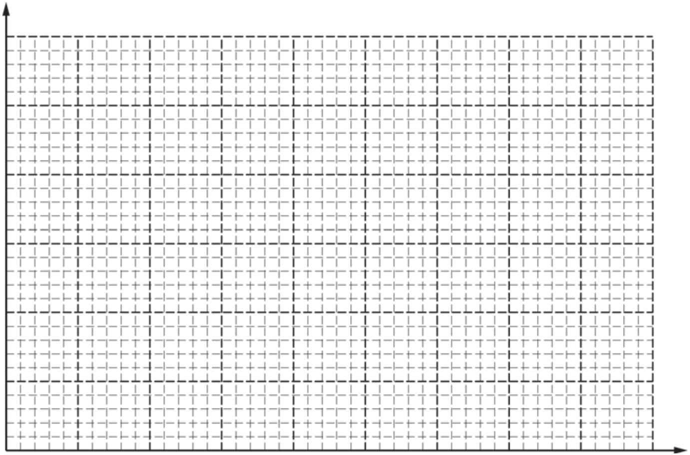
Question 3

2015 ▪ Q3

Students are watching a science program about the North Pole. The narrator says that cold air sinking near the North Pole causes high air pressure. Based on the narrator's statement, a student makes the following claim: "Since cold air near the North Pole is at high pressure, temperature and pressure must be inversely related."

(a) Do you agree or disagree with the student's claim about the relationship between pressure and temperature? Justify your answer.

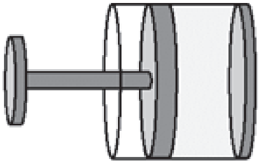
Question 3



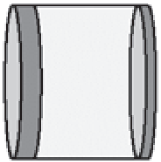
Question 3

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Volume (cm^3)	10.0	5.0	4.0	3.0	5.0	4.0	10.0	5.0	3.0	4.0	5.0	10.0	3.0	5.0
Pressure (kPa)	100	200	250	330	220	270	110	230	380	290	240	120	420	250
Temperature (°C)	0	0	0	0	20	20	20	40	40	40	60	60	70	70

Question 3



Cylinder with Movable Piston



Cylinder with Fixed Lid

Question 3

2015 ▪ Q3

Students are watching a science program about the North Pole. The narrator says that cold air sinking near the North Pole causes high air pressure. Based on the narrator's statement, a student makes the following claim: "Since cold air near the North Pole is at high pressure, temperature and pressure must be inversely related."

(b) After hearing the student's hypothesis, you want to design an experiment to investigate the relationship between temperature and pressure for a fixed amount of gas. The following equipment is available.

[Figure: Two labeled apparatus drawings side by side. Left: a cylindrical container with a movable piston (a rod inserted into one end of the cylinder), labeled "Cylinder with Movable Piston". Right: a cylindrical container with both ends capped/fixed, labeled "Cylinder with Fixed Lid".]

$A^{cylinder with a movable piston, shown above on the left} A_{cylinder with a fixed lid, shown above on the right} Note: The two cylinders have gaskets through which measurement insti$

Question 3

Put a check in the blank next to each of the items above that you would need for your investigation. Outline the experimental procedure you would use to gather the necessary data. Make sure the outline contains sufficient detail so that another student could follow your procedure.

The table below shows data from a different experiment in which the volume, temperature, and pressure of a sample of gas are varied.

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Volume (cm^3)	10.0	5.0	4.0	3.0	5.0	4.0	10.0	5.0	3.0	4.0	5.0	10.0	3.0	5.0
Pressure (kPa)	100	200	250	330	220	270	110	230	380	290	240	120	420	250
Temperature (°C)	0	0	0	0	20	20	20	40	40	40	60	60	70	70

Question 3

2015 ▪ Q3

Students are watching a science program about the North Pole. The narrator says that cold air sinking near the North Pole causes high air pressure. Based on the narrator's statement, a student makes the following claim: "Since cold air near the North Pole is at high pressure, temperature and pressure must be inversely related."

(c) What subset of the experimental trials would be most useful in creating a graph to determine the relationship between temperature and pressure for a fixed amount of gas? Explain why the trials you selected are most useful.

Question 3

2015 ▪ Q3

Students are watching a science program about the North Pole. The narrator says that cold air sinking near the North Pole causes high air pressure. Based on the narrator's statement, a student makes the following claim: "Since cold air near the North Pole is at high pressure, temperature and pressure must be inversely related."

(d) [Blank grid: a set of labeled coordinate axes (x-axis horizontal, y-axis vertical) with fine gridlines but no axis labels or numbers filled in, provided for the student to plot data.]

Plot the subset of data chosen in part (c) on the axes below. Be sure to label the axes appropriately. Draw a curve or line that best represents the relationship between the variables.

Question 3

2015 ▪ Q3

Students are watching a science program about the North Pole. The narrator says that cold air sinking near the North Pole causes high air pressure. Based on the narrator's statement, a student makes the following claim: "Since cold air near the North Pole is at high pressure, temperature and pressure must be inversely related."

(e) What can be concluded from your curve or line about the relationship between temperature and pressure?

Solution 3

(a) Mark scheme

- Disagree with the student's claim (credit is for the physical reasoning, not simply picking agree/disagree). Justification: apply the ideal gas law $PV = nRT$ (or $PV = NkT$) — for a fixed amount of gas at constant volume, pressure and absolute temperature are directly proportional, so as temperature decreases, pressure decreases too (1 point for invoking the ideal gas law and attempting to use it). State that the volume and the number of moles/particles of gas are held constant for this direct proportionality to apply (1 point); the student need not specify that P and T are proportional only when T is in Kelvin. [Equally-creditable alternate route: as temperature decreases, the density of a fixed gas sample increases (holding P and N constant), and denser gas sinks below less-dense gas (1

Solution 3

point); the gas near the North Pole is not a closed system, so its pressure increases as additional sinking, denser gas molecules are continually added to it (1 point).]

Solution 3

(b) Mark scheme

- Design a controlled experiment using the cylinder with the movable piston (not the fixed-lid cylinder), and indicate or imply that the piston is held at a fixed position so volume is held constant (1 point). Select all the equipment described in the procedure (e.g. thermometer/temperature sensor, pressure sensor, water bath/heat source) and no extraneous equipment (1 point). Describe a method of measuring the temperature of the enclosed gas (1 point). Describe a method for measuring the pressure at more than just two temperatures (1 point). Example procedure: insert a thermometer and pressure sensor into the gasket/cylinder to measure the enclosed gas's temperature and pressure; place the cylinder (piston fixed, so volume is constant) into a water bath, starting hot (or cold); record

Solution 3

paired temperature-pressure readings periodically as the bath water cools (or heats) over time.

Solution 3

(c) Mark scheme

- Select the subset of trials in which volume was held constant, and explain that volume must be held fixed across the chosen trials to isolate and test the pressure-temperature relationship alone (1 point). Among the constant-volume trials, select those at 5.0 cm^3 specifically, explaining that this volume has the most recorded trials, and using the most trials gives the most reliable test (1 point). [Equally-creditable alternate route: select the FULL set of trials across all volumes, explaining that the effect of varying volume can be accounted for by multiplying pressure by volume (or plotting P/T as a function of 1/V, etc.) (1 point); and explain that using the most trials gives the most reliable test, OR that

Solution 3

using the widest range of pressure values gives the most precise determination of the proportionality constant relating pressure and temperature (1 point).]

Solution 3

(d) Mark scheme

- Plot the trials selected in part (c) as pressure P vs. temperature T (or an equivalent linearizing pair, e.g. T vs. $1/P$, or PV vs. T, or P vs. V/T if using the full data set) for each selected trial (1 point). Use appropriate axis labels with units and appropriate scales that spread the data across the grid (1 point). Draw a single best-fit straight line (or smooth curve, for a non-linear pair) through the plotted points, not merely connecting the dots (1 point). Example (constant-volume subset): Pressure (kPa) on the y-axis from about 190 to 250, Temperature on the x-axis from 0 to about 70, with points rising roughly linearly from about (0, 200) through (20, 220), (40, 230), (60, 240) to about (70, 250), and a single straight best-fit line through them.

Solution 3

(e)

Mark scheme

- Correctly describe, in words, the relationship shown by the part (d) graph, consistent with the curve drawn. Examples: 'the relationship between P and T is linear' (for the constant-volume P vs. T plot); or 'the relationship between P and V/T is hyperbolic' (for the alternate full-data-set plot). The description must match whichever axes/curve were actually used in part (d).