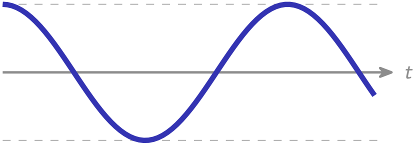
Oscillations

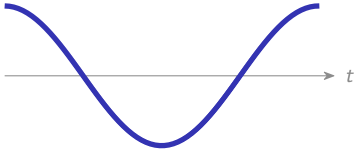
AP Physics 1 ▪ Unit 7

AP Physics 1



What we will cover

1. Defining simple harmonic motion
2. Frequency and period
3. Representing and analyzing SHM
4. Energy of an oscillator



1 Defining SHM

Simple harmonic motion

Simple harmonic motion 简谐运动 happens when the **restoring force** 回复力 is proportional to the displacement:

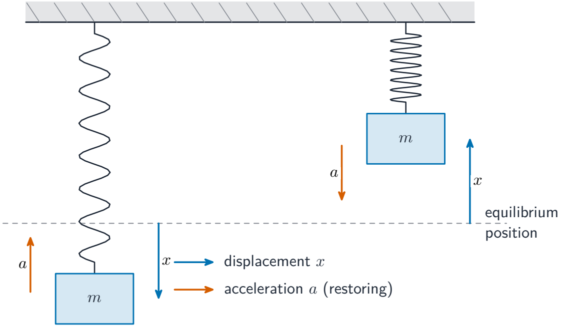
The defining law

$$F = -kx$$

> Pull it twice as far, and it pulls back twice as hard – always toward the middle.

SHM restores toward equilibrium

Defining Simple Harmonic Motion



In SHM the acceleration always points back towards equilibrium, opposite the displacement

2 Period

Frequency and period

An oscillation repeats every **period** 周期 T; the **frequency** 频率 $f = 1/T$.

Period depends on the system, not the swing

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}$$

> The period does not depend on the **amplitude** 振幅 – a big swing and a small swing take the same time.

$0.5\,\mathrm{kg}$ on $k = 50\,\mathrm{N/m}$: $T \approx 0.63\,\mathrm{s}$, whether you pull it 2 or $10\,\mathrm{cm}$.

A pendulum's period ignores mass entirely.

Three ways to say how fast it oscillates

Period — T: seconds for one complete cycle.

Frequency — $f = 1/T$: cycles per second, in hertz.

Angular frequency — **Angular frequency** $\omega = 2\pi f = \dfrac{2\pi}{T}$, in radians per second – the one that appears inside $x = A\cos(\omega t + \phi)$.

Why three — ω makes the algebra clean, f and T are what an experiment measures.

> For a spring $\omega = \sqrt{k/m}$; for a simple pendulum $\omega = \sqrt{g/L}$. Neither depends on the amplitude.

Worked example – two periods

(a) A 0.25 kg mass on a spring with $k = 100$ N/m. (b) A pendulum clock ticking with period exactly 2.0 s: how long is it?

- (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31$ s
- (b) rearrange $T = 2\pi\sqrt{L/g}$:
  $L = g\left(\frac{T}{2\pi}\right)^2$
- $= 9.8\left(\frac{2.0}{2\pi}\right)^2 = 0.99$ m

A one-metre pendulum has a two-second period – one second per swing. That is not a coincidence: the metre was very nearly defined that way.

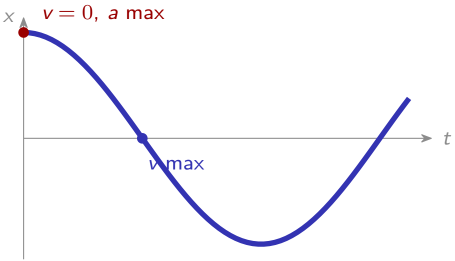
The period of SHM does not depend on amplitude

the claim	for simple harmonic motion, period and frequency depend only on the system, not on how far it swings
a spring	$T_{\text{spring}} = 2\pi\sqrt{\dfrac{m}{k}}$ – mass and stiffness only
a pendulum	$T_{\text{pendulum}} = 2\pi\sqrt{\dfrac{L}{g}}$ – length and gravity only, with no mass in it
why it matters	a pendulum clock keeps time as its swing decays, which is the whole reason it works
the small-angle limit	the pendulum formula holds only for small angles, where $\sin\theta \approx \theta$

> "Independent of amplitude" is the defining property of SHM. If a question changes the amplitude and asks about the period, the answer is no change.

3 Analyzing SHM

Reading an oscillation



Displacement, velocity and acceleration trace smooth sine curves:

- velocity greatest at the **equilibrium** 平衡位置 (middle)
- acceleration greatest at the extremes

The **phase** 相位 sets where in the cycle it is: v peaks a quarter-cycle before x.

Reading the oscillation graph as a story

at the turning point	x is largest; v has just fallen to zero; a is at its most negative, hauling the mass back
passing through the centre	$x = 0$; $a = 0$; v is at its maximum
the phases	v leads x by a quarter cycle; a is a half cycle out of phase with x
a against x	a straight line through the origin with a negative gradient – and that graph *is* the definition of SHM
the energy	U is greatest at the ends, K greatest at the centre, and the total is constant

> Never have maximum speed and maximum acceleration at the same point. If your answer does, a quarter cycle has gone missing.

4 Energy

Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

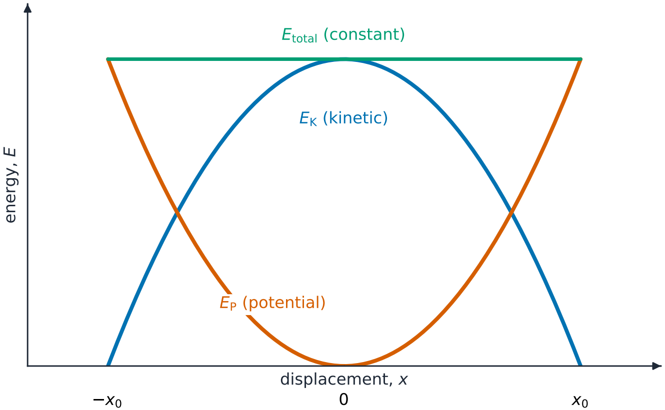
Set by the amplitude

$$E = \frac{1}{2}kA^2$$

- **kinetic energy** 动能 greatest in the middle
- **potential energy** 势能 greatest at the extremes

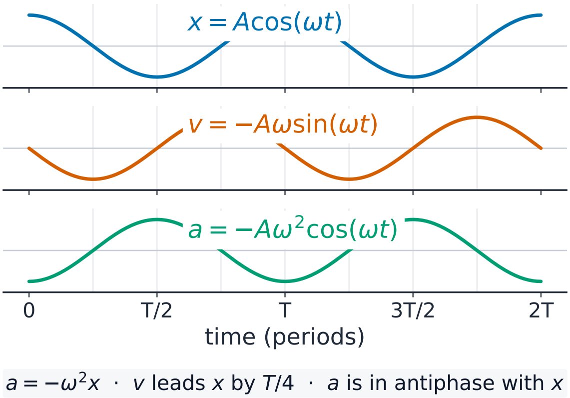
Oscillators store and exchange energy

Energy of an oscillator, continued



Kinetic and potential energy swap over a cycle while the total energy stays constant

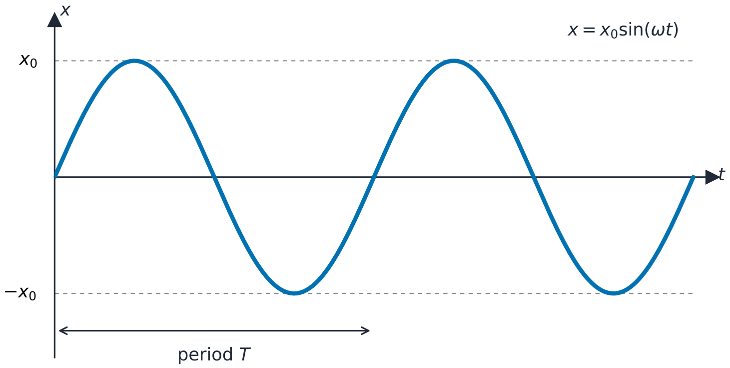
Displacement – one



Displacement, velocity, and acceleration in SHM, each a quarter-cycle apart

> Displacement, velocity, and acceleration in SHM, each a quarter-cycle apart

Displacement – two



Displacement varies sinusoidally with time in simple harmonic motion

> Displacement varies sinusoidally with time in simple harmonic motion

Exam tips

- Test for **SHM**: the acceleration must be proportional to the displacement and directed back toward the middle ($a = -\frac{k}{m}x$).
- The **period does not depend on amplitude** – use $T = 2\pi\sqrt{m/k}$ (spring) or $T = 2\pi\sqrt{L/g}$ (pendulum, small angles).
- Speed is **maximum at the middle** (all kinetic) and **zero at the extremes** (all potential); acceleration is largest at the extremes.
- Use energy ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$) to find the maximum speed quickly: $v_{\max} = A\sqrt{k/m}$.
- Total energy $\propto A^2$, so doubling the amplitude quadruples the energy.

5 Recap

Unit 7 – key takeaways

1 **SHM**

$F = -kx$: restoring force proportional to displacement

2 **Period**

$2\pi\sqrt{m/k}$ or $2\pi\sqrt{L/g}$; independent of amplitude

3 **Motion**

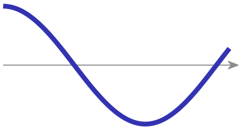
v max in the middle; a max at the extremes

4 **Energy**

$E = \frac{1}{2}kA^2$; trades between KE and PE

Check yourself

1. Double the amplitude – does the period change?
2. Where in the swing is the velocity greatest?
3. Where is the acceleration greatest?
4. Does a pendulum's period depend on the mass?



Answers **1.** no **2.** in the middle (equilibrium) **3.** at the extremes **4.** no