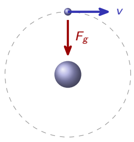
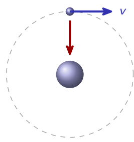
Energy & Momentum of Rotating Systems

AP Physics 1 ▪ Unit 6

AP Physics 1



What we will cover



1 Rotational kinetic energy

Rotational kinetic energy

A spinning object stores energy even if its centre stays put:

Energy of spin

$$K_{rot} = \frac{1}{2} I\omega^2$$

Mass $\to$ I, speed $\to$ ω. Double the spin, quadruple the energy.

A rolling object has both: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

Down a ramp

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

A shape with mass near the rim (a hoop) puts more energy into spin, so it moves *slower* at the bottom.

2 Work and power

Work and power of a torque

A torque does work when it turns an object through an angle:

Rotational work and power

$$W = \tau\,\Delta\theta, \qquad P = \tau\omega$$

$W_{net} = \Delta K_{rot}$. A changing torque: work = area under a τ–θ graph.

Worked example

Motor: $\tau = 15\,\mathrm{N\,m}$ over $\Delta\theta = 8\,\mathrm{rad}$:

$$W = 15 \times 8 = 120\,\mathrm{J}$$

At $\omega = 20\,\mathrm{rad/s}$:

$$P = 15 \times 20 = 300\,\mathrm{W}$$

3 Angular momentum

Angular momentum and impulse

Angular momentum 角动量 is the rotational twin of momentum:

Spin that resists change

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

Angular impulse 角冲量 = net torque × time.

Worked example

$L_i = 30$; a torque $\tau = 5\,\mathrm{N\,m}$ acts for $4\,\mathrm{s}$:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50\,\mathrm{kg\,m^2/s}$$

4 Conservation

Conservation of angular momentum

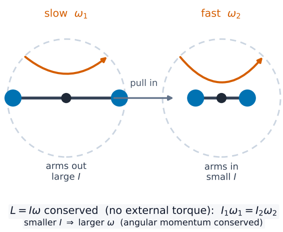
With zero net external torque, angular momentum is conserved – even as the body changes shape:

Before = after

$$I_i\omega_i = I_f\omega_f$$

A skater pulls their arms in: I drops, so ω rises – they spin faster.

Pulling mass inward lowers I



Pulling mass inward lowers I, so ω rises to conserve L = I

5 Rolling

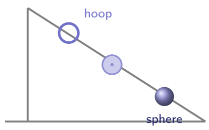
Rolling without slipping

Rolling without slipping 无滑滚动 locks forward speed to spin:

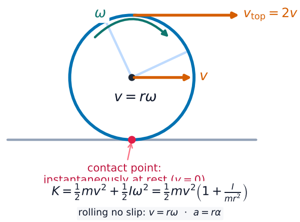
The rolling condition

$$v = \omega r, \qquad K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Race down a ramp: the solid sphere wins, the hoop comes last – more I means more energy lost to spin.



In rolling without slipping the contact point



In rolling without slipping the contact point is at rest, so v = r

6 Satellites

Orbiting satellites

Gravity 引力 supplies the centripetal force. Set them equal:

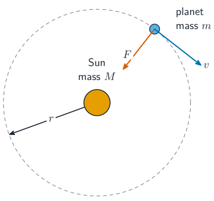
Orbital speed

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \;\Rightarrow\; v = \sqrt{\frac{GM}{r}}$$

- speed is independent of the satellite's mass
- a bigger orbit means a slower speed

Satellites are continuous free-fall around Earth

Motion of Orbiting Satellites



Gravity provides the centripetal force that keeps a satellite in orbit

Escape velocity

the definition	the launch speed that just lets an object leave for good
the condition	its total mechanical energy is exactly zero – it arrives at infinity with no speed left
so	$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$, giving $v_{esc} = \sqrt{\frac{2GM}{r}}$
the relation	$v_{esc} = \sqrt{2}\times$ the circular orbital speed at the same radius
no mass in it	the escaping object's own mass cancels, so it is the same for a pebble and a rocket

Escape speed is set by where you start, not by what you are. Launching from a higher orbit needs less of it, which is why staged launches work.

Exam tips

- Conserve **angular momentum** $L = I\omega$ when no external torque acts: a smaller I (arms pulled in) gives a larger ω.
- A rolling object splits its energy between $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$, linked by $v = r\omega$ – so it accelerates down a ramp **more slowly** than a sliding one.
- For a circular orbit set gravity equal to the centripetal requirement: $v = \sqrt{GM/r}$, so a larger orbit is **slower** and the satellite's mass cancels.
- Pulling in raises the spin **and** the kinetic energy – the extra energy comes from the work done pulling inward; L is unchanged.
- Watch which rotational quantity is conserved: L (no torque) versus energy (no friction) are different conditions.

7 Recap

Unit 6 – key takeaways

1 Rotational energy

$K_{rot} = \frac{1}{2}I\omega^2$; rolling splits energy in two

2 Angular momentum

$L = I\omega$; angular impulse $\tau\,\Delta t = \Delta L$

3 Conservation

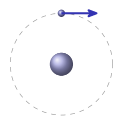
no external torque $\Rightarrow I_i\omega_i = I_f\omega_f$ (skater)

4 Orbits

gravity is the centripetal force; $v = \sqrt{GM/r}$

Check yourself

1. A skater pulls their arms in – what happens to their spin rate?
2. Down a ramp: hoop, disk, or sphere – which wins?
3. Does orbital speed depend on the satellite's mass?
4. A bigger orbit radius means a faster or slower speed?



Answers **1.** it speeds up **2.** the sphere **3.** no **4.** slower