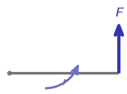
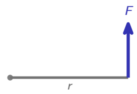
Torque & Rotational Dynamics

AP Physics 1 ▪ Unit 5

AP Physics 1



What we will cover



1 Rotational kinematics

Rotation mirrors straight-line motion

Spinning is described by angles:

- **angular velocity** 角速度 ω – rate the angle turns
- **angular acceleration** 角加速度 α – rate ω changes

For constant α, the equations have the same form as the linear ones. Rotation rate: $\omega = 2\pi f$.

Rotation links angle, angular speed, and linear speed

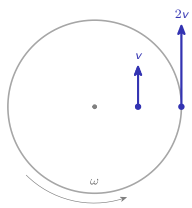
Linking linear and rotational motion

Every point on a spinning body moves along a curve, tied to the **radius** 半径 r.

Same ω, different v

$$v = r\omega, \quad a_t = r\alpha, \quad s = r\theta$$

All points share one ω, but points farther out move faster.

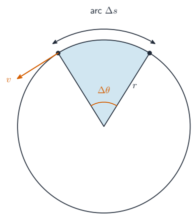


The angular quantities, and the moment arm

Angular displacement	**Angular displacement** $\Delta\theta$ in radians, with $s = r\Delta\theta$ linking it to the arc travelled.
Angular velocity	$\omega = d\theta/dt$, and $v = r\omega$ for a point on a rotating body – the further out, the faster.
Moment arm	The **moment arm** is the perpendicular distance from the axis to the line of the force: $\tau = r_\perp F = rF\sin\theta$.
Why perpendicular	A force pointing at the axis has zero moment arm, so it produces no torque however large it is.

Every rotational quantity mirrors a linear one. Write the linear equation first, then substitute the angular partner.

As the radius turns through an angle



As the radius turns through an angle, a point moves along an arc at speed v

2 Torque

Torque

Torque 力矩 is the rotational version of force – how strongly a force twists:

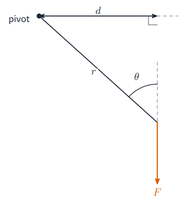
Twisting strength

$$\tau = rF\sin\theta$$

The **lever arm** 力臂 is the perpendicular distance to the force. A bigger lever arm gives more torque.

Torque is the rotational analogue of force

The moment of a force depends on



The moment of a force depends on the perpendicular distance from the pivot

3 Rotational inertia

Rotational inertia

Rotational inertia 转动惯量 I resists a change in spin – and depends on how the mass is spread:

Distance is squared

$$I = \sum mr^2$$

Parallel-axis theorem 平行轴定理: $I = I_{cm} + md^2$.

Move mass out to $2\times$ the radius $\Rightarrow I$ grows $4\times$.

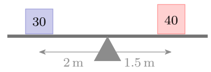
arms in $\Rightarrow$ smaller I

4 Equilibrium

Rotational equilibrium

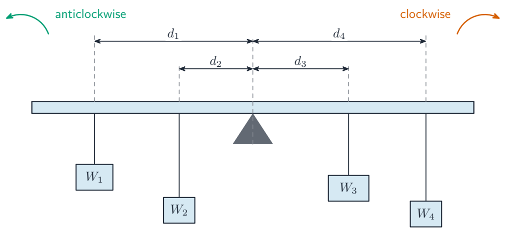
In **rotational equilibrium** 转动平衡 the **net torque** 净力矩 is zero. Full **static equilibrium** 静平衡: both net force and net torque zero.

- pick the axis at an unknown force to drop it out
- set clockwise torque = counterclockwise torque



Heavier child sits closer to balance.

At balance the clockwise and anticlockwise moments



> At balance the clockwise and anticlockwise moments about the pivot are equal

5 Rotational Newton's law

Newton's second law for rotation

A nonzero net torque gives an angular acceleration – just like $F = ma$:

Rotational form

$$\tau_{net} = I\alpha$$

torque ↔ force, I ↔ mass, α ↔ acceleration.

Worked example

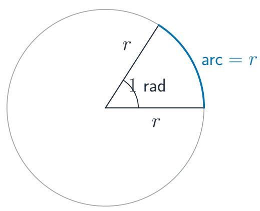
$\tau = 12\,\mathrm{N\,m}$ on $I = 3\,\mathrm{kg\,m^2}$:

$$\alpha = \frac{\tau}{I} = \frac{12}{3} = 4\,\mathrm{rad/s^2}$$

Double $I \Rightarrow$ half the α – harder to spin up.

Torque & Rotational Dynamics
└Rotational Newton's law

Rotational Kinematics



one **radian**: the angle whose arc is exactly one radius long

π rad $= 180°$, so 1 rad $\approx 57.3°$

degrees → radians: $\times \frac{\pi}{180}$ radians → degrees: $\times \frac{180}{\pi}$

> One radian is the angle whose arc length equals the radius

Exam tips

- Torque $\tau = Fr_{\perp}$ uses the **perpendicular** distance from the pivot; a force through the pivot gives zero torque.
- For balance, set clockwise torque = anticlockwise torque; choosing the pivot at an unknown force removes it from the equation.
- Use the rotational analogues: $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$, so $\sum \tau = I\alpha$ mirrors $\sum F = ma$.
- **Rotational inertia** depends on **how far** the mass sits from the axis, not just its amount – a hoop resists spinning more than a disc of equal mass.
- Convert angles to **radians** and link linear to angular with $v = r\omega$, $a_t = r\alpha$.

6 Recap

Unit 5 – key takeaways

1 Analogy

$x, v, a, m, F \leftrightarrow \theta, \omega, \alpha, I, \tau$; $v = r\omega$

2 Torque

$\tau = rF\sin\theta$; the lever arm sets the twist

3 Inertia

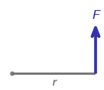
$I = \sum mr^2$; mass farther out counts far more

4 Dynamics

equilibrium: net $\tau = 0$; otherwise $\tau = I\alpha$

Check yourself

1. Push a door near the hinge vs at the handle – which gives more torque?
2. Move a mass to twice the radius – what happens to I?
3. On a balanced seesaw, who sits closer to the pivot?
4. $\tau = 20\,\mathrm{N\,m}$ on $I = 5\,\mathrm{kg\,m^2}$ – find α.



Answers **1.** the handle (bigger lever arm) **2.** it quadruples **3.** the heavier one **4.** $4\,\mathrm{rad/s^2}$