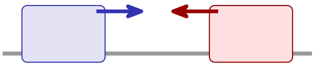
Linear Momentum

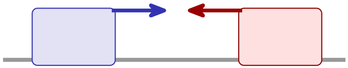
AP Physics 1 ▪ Unit 4

AP Physics 1



What we will cover

1. Linear momentum
2. Impulse and its theorem
3. Conservation of momentum
4. Elastic and inelastic collisions



1 Linear momentum

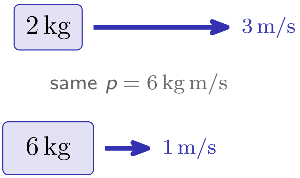
Linear momentum

Mass in motion

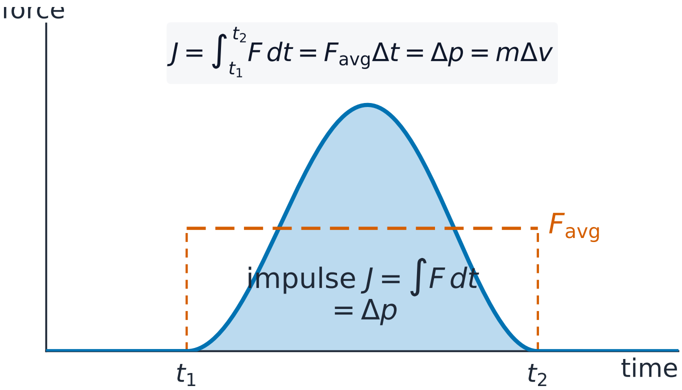
$$\vec{p} = m\vec{v}$$

A **vector** 矢量 – points the way the velocity does. Total momentum = the vector sum.

> Momentum grows with both mass and speed – a slow truck and a fast bullet can match.



Impulse is the area under the force-time curve



Impulse is the area under the force-time curve, equal to the average force times the contact time

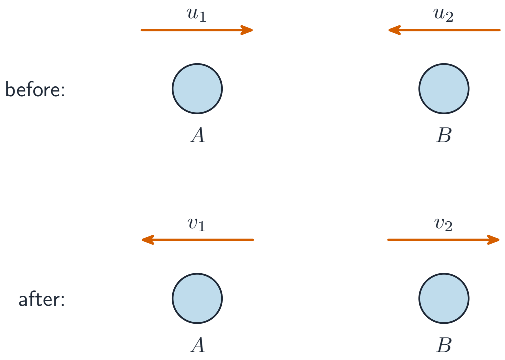
Worked example – a ball bouncing off a wall

A 0.15 kg ball hits a wall at 20 m/s and bounces straight back at 15 m/s. The contact lasts 0.020 s. Find the average force.

- take toward the wall as positive: $\Delta p = m(v_f - v_i) = 0.15(-15 - 20)$
- $= -5.25\ \mathrm{kg\,m/s}$
- $F = \dfrac{\Delta p}{\Delta t} = \dfrac{-5.25}{0.020} = -260\ \mathrm{N}$, i.e. 260 N away from the wall

The rebound makes v_f negative, so Δp is larger than either speed alone would suggest. Forgetting that sign halves the answer.

A head-on collision



A head-on collision: total momentum before equals total momentum after

2 Impulse

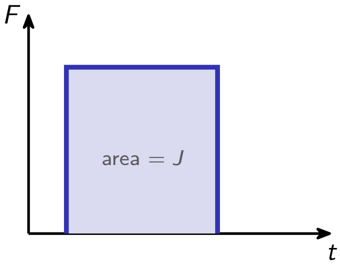
Impulse

Force over time

$$\vec{J} = \vec{F}_{net}\,\Delta t = \Delta\vec{p}$$

The **impulse-momentum theorem** 冲量动量定理. Impulse = area under an F–t graph.

For a fixed Δp, a longer **contact time** 接触时间 means a smaller force.



Airbag 安全气囊: $10\times$ the time $\Rightarrow \frac{1}{10}$ the force.

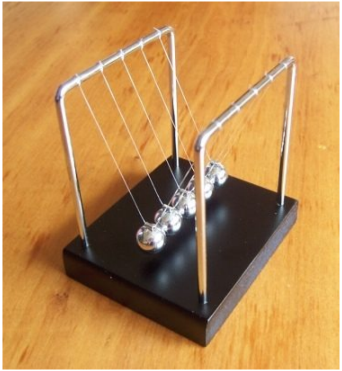
3 Conservation

Conservation of momentum

With zero net external force, a system is **isolated** 孤立的 and its total momentum stays constant.

- internal forces cancel (Newton's third law)
- only external forces change total p

> Holds for **collisions** 碰撞 and **explosions** 爆炸 alike.



Momentum is conserved in collisions

Worked example – recoil on frictionless ice

A 60 kg skater, at rest, throws a 2.0 kg ball at 8.0 m/s. Find the skater's recoil speed.

- total momentum before $= 0$, and no external horizontal force acts
- $0 = m_1 v_1 + m_2 v_2 = (60) v_1 + (2.0)(8.0)$
- $v_1 = -\frac{16}{60} = -0.27$ m/s – backwards

Recoil 反冲 is conservation of momentum from a zero total. The heavier body moves more slowly by exactly the mass ratio.

4 Collisions

Elastic and inelastic collisions

Momentum is conserved in every collision; kinetic energy is not.

- **elastic** 弹性碰撞 – *KE* conserved
- **inelastic** 非弹性碰撞 – some $KE \rightarrow$ heat, sound
- **perfectly inelastic** 完全非弹性碰撞 – they stick

Worked example (stick together)

$2\,\mathrm{kg}$ at $3\,\mathrm{m/s}$ hits a $4\,\mathrm{kg}$ at rest:

$$2(3) = (2{+}4)v_f \;\Rightarrow\; v_f = 1\,\mathrm{m/s}$$

KE: $9\,\mathrm{J} \rightarrow 3\,\mathrm{J}$; the missing $6\,\mathrm{J}$ became heat.

Which conservation law to write, and when

always	write momentum conservation – it holds in every collision
only if stated elastic	add kinetic energy conservation as a second equation
perfectly inelastic	the bodies stick together, so they share one final velocity – one unknown instead of two
the elastic fact worth memorising	for a head-on elastic collision, the relative velocity of approach equals the relative velocity of separation
centre-of-mass velocity 质心速度	$\vec{v}_{\text{cm}} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$ – unchanged by any internal collision
why that helps	the centre of mass moves at constant velocity throughout, whatever the bodies do to each other

> Assuming kinetic energy is conserved when the question has not said “elastic” is the standard error. Momentum first, energy only on invitation.

The centre-of-mass velocity

the definition	$\vec{v}_{\text{cm}} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$ – the mass-weighted average velocity
what it does	a whole collection of objects can be described by this single velocity
the key property	internal forces cannot change it, so it is unchanged by any collision or explosion
the consequence	in a collision with no external force, the centre of mass keeps moving at constant velocity throughout
the shortcut	in the centre-of-mass frame the total momentum is zero, which makes many collisions symmetric
an explosion	fragments fly apart, and their centre of mass carries on along the original path

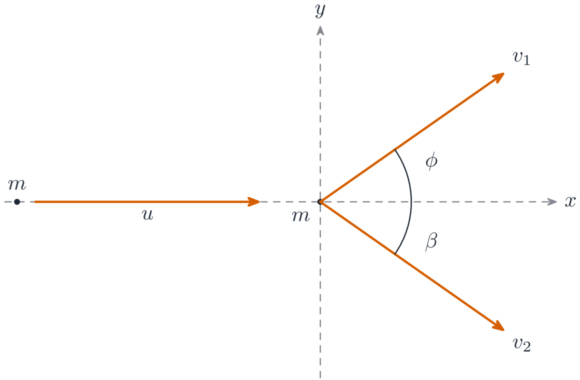
Conservation of momentum and constant $\vec{v}_{\text{cm}}$ are the same statement. Using the second is often the faster route through a two-body problem.

The centre of mass keeps moving

Center-of-mass velocity	The **center-of-mass velocity** is $\vec{v}_{cm} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$ – the total momentum divided by the total mass.
Why it matters	With no external force it does not change, whatever the parts do to each other in a collision or an explosion.
Using it	In a perfectly inelastic collision the objects end up moving *at* $\vec{v}_{cm}$ – which gives the final velocity with no algebra.
A check	Work in the centre-of-mass frame and the total momentum is zero, before and after.

> "Momentum is conserved" and "the centre of mass keeps its velocity" are the same statement.

A glancing collision



A glancing collision, resolved along two perpendicular axes

Exam tips

- Momentum is a **vector** – assign $+/-$ signs before adding; a ball that **rebounds** reverses its velocity, giving a large Δp.
- Momentum is conserved in **every** collision (no external force); kinetic energy is conserved **only** if the collision is stated to be **elastic**.
- In a **perfectly inelastic** collision the objects stick and move with one common velocity.
- Impulse $= F\Delta t = \Delta p =$ the **area** under a force–time graph; spreading a collision over a longer time reduces the force (airbags, bending knees).
- For recoil/explosions, set the total momentum equal before and after (often zero before).

5 Recap

Unit 4 – key takeaways

1 Momentum

$\vec{p} = m\vec{v}$, a vector; add momenta as vectors

2 Impulse

$\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$; longer time, smaller force

3 Conservation

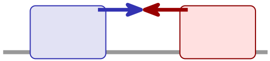
zero net external force keeps total momentum constant

4 Collisions

momentum always conserved; KE only if elastic

Check yourself

1. Why does an airbag reduce the force on you?
2. Two skaters push off from rest – total momentum?
3. In a collision, is momentum always conserved?
4. Which collision conserves kinetic energy?



Answers **1.** longer contact time **2.** zero **3.** yes **4.** elastic