

10 Power and Taylor series – Part 2

Name: _____ Class: _____ Date: _____

Recall

In Part A you added up infinitely many terms. Now use that to **approximate functions**:

- A complicated function can be replaced by a simple polynomial.
- Adding more terms makes the approximation better.

Each idea has a worked example before the Practice.

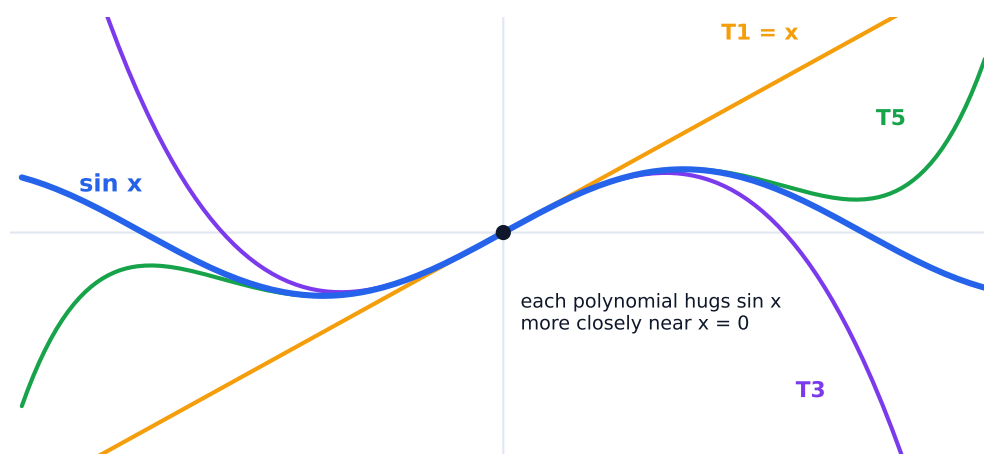
New ideas

■ 1 Power series

A **power series** adds up powers of x : $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$. It behaves like an "infinitely long polynomial".

■ 2 Taylor and Maclaurin series

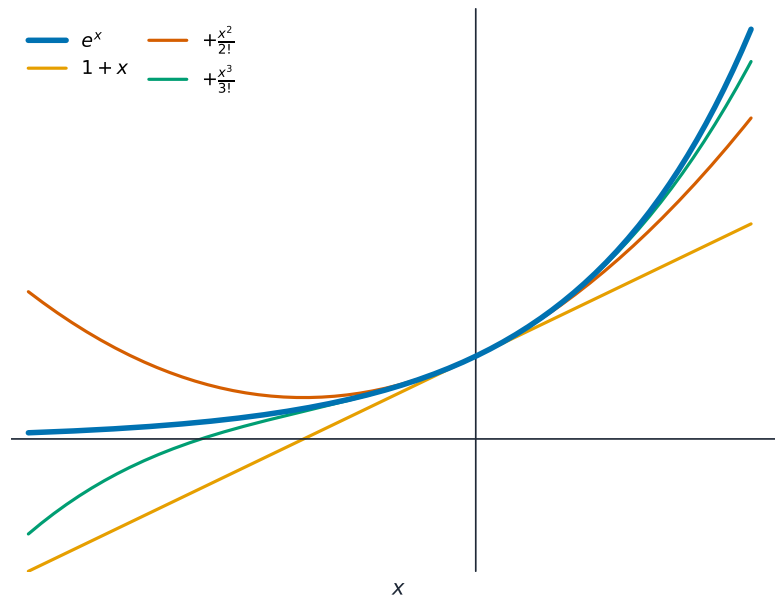
A **Taylor series** (a **Maclaurin series** when centred at 0) builds a power series that matches a function near a point – approximating it by a polynomial.



$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k \quad \cdot \quad \text{for } \sin x: x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

■ 3 More terms, better fit

The more terms you keep, the more closely the polynomial hugs the real function.



■ 4 Why it matters

Polynomials are easy to add, differentiate, and integrate, so replacing a hard function with its series makes calculations far simpler – calculators use this idea.

Watch out: a Taylor approximation is most accurate **near the point** it is built around. Far from that point, you need more terms to keep it accurate.

Practice

Try these in order. The methods are all above.

2.1 State what a power series is. [2]

2.2 State what a Taylor (or Maclaurin) series does. [2]

2.3 State what happens to the approximation as you keep more terms. [1]

2.4 Explain why replacing a function with its power series is useful. [2]

2.5 State where a Taylor approximation is most accurate. [1]
