

10 Infinite series — Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
sequence	数列	_____
series	级数	_____
geometric series	等比级数	_____

Recall

You met geometric sequences earlier – each term multiplies by a fixed ratio:

- 2, 4, 8, 16, ... doubles each time.
- What if you add up **all** the terms? Can an infinite sum have a finite answer?

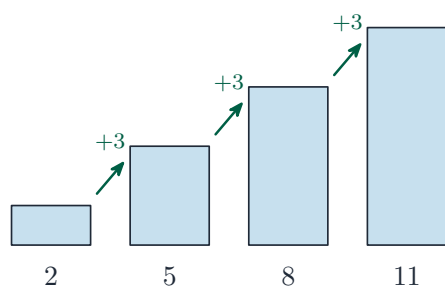
This sheet introduces infinite series. Each idea has a worked example.

New ideas

■ 1 Sequence versus series

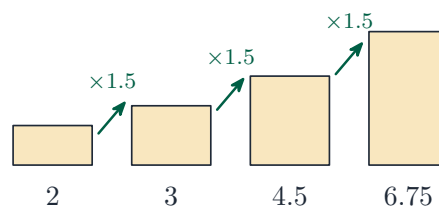
A **sequence** 数列 is a **list** of terms; a **series** 级数 is the **sum** of those terms.

arithmetic: add d each step



$$u_n = a + (n-1)d: \text{equal steps}$$

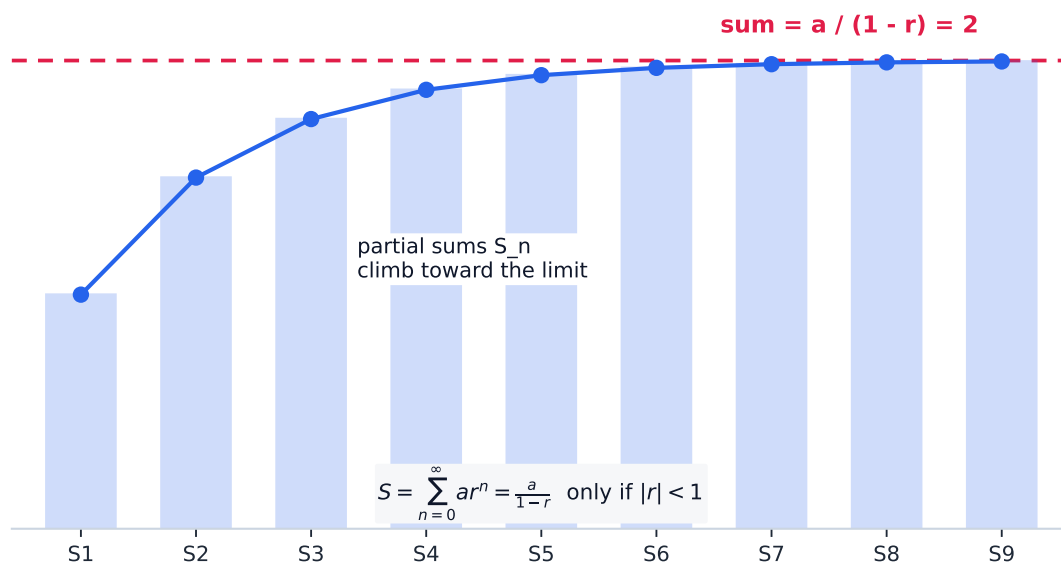
geometric: multiply by r each step



$$u_n = ar^{n-1}: \text{growing steps}$$

■ 2 Geometric series

A **geometric series** 等比级数 adds terms that each multiply by a ratio r : $a + ar + ar^2 + \dots$.



■ 3 Infinite sum

If $|r| < 1$, the terms shrink toward zero and the infinite sum **converges** to a finite value:

$$S = \frac{a}{1-r}.$$

Worked example. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ has $a = 1$, $r = \frac{1}{2}$, so $S = \frac{1}{1 - \frac{1}{2}} = 2$.

■ 4 Converge or diverge

If $|r| \geq 1$ the terms do not shrink, so the sum grows without limit – it **diverges** (no finite answer).

Watch out: an infinite sum only has a finite value when the terms **shrink fast enough** ($|r| < 1$ for a geometric series). If the terms stay the same size or grow, the sum diverges.

Practice

Try these in order. The methods are all above.

2.1 State the difference between a sequence and a series. [2]

2.2 Write the formula for the sum of an infinite geometric series with $|r| < 1$. [1]

2.3 Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ (with $a = 1$, $r = \frac{1}{3}$). [3]

2.4 Find the sum of $4 + 2 + 1 + \frac{1}{2} + \cdots$ (with $a = 4$, $r = \frac{1}{2}$). [2]

2.5 State whether $1 + 2 + 4 + 8 + \cdots$ converges, and why. [2]
