9.7 Defining Polar Coordinates and Differentiating in Polar Form

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS polar 极坐标

Objective

Build the skills to answer exam questions on **polar coordinates** and differentiating in polar form.

You must be able to:

- convert between **polar** 极坐标 (r, θ) and Cartesian (x, y)
- treat $r = f(\theta)$ as a parametric curve $x = r\cos\theta$, $y = r\sin\theta$
- find $\dfrac{dy}{dx}$ for a polar curve

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Polar to Cartesian

A point (r, θ) has $x = r\cos\theta$, $y = r\sin\theta$. So $(2, \frac{\pi}{3})$ is $x = 2\cos\frac{\pi}{3} = 1$, $y = 2\sin\frac{\pi}{3} = \sqrt{3}$.

■ A polar curve is parametric in θ

For $r = f(\theta)$, use θ as the parameter:

$$x = f(\theta)\cos\theta, \qquad y = f(\theta)\sin\theta.$$

■ Slope of a polar curve

The slope is the parametric slope with θ as parameter:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}.$$

Compute $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ from the product rule.

■ **A worked conversion**

The circle $r = 4$ is $x = 4\cos\theta$, $y = 4\sin\theta$ — the circle of radius 4. The line $\theta = \frac{\pi}{4}$ is $y = x$ (for $r > 0$).

2 Practice

Now apply the methods above.

2.1 Convert the polar point $(3, \frac{\pi}{2})$ to Cartesian coordinates. [2]

2.2 Write x and y for the polar curve $r = 2\cos\theta$. [2]

2.3 Convert the Cartesian point $(0, 5)$ to polar coordinates. [2]

3 Exam-style questions

3.1 For a polar curve $r = f(\theta)$, $x =$ [1]

- **A** $r\sin\theta$
- **B** $r\cos\theta$
- **C** $\dfrac{r}{\cos\theta}$
- **D** $r + \cos\theta$

3.2 For the polar curve $r = 1 + \cos\theta$:

(a) Write x and y as functions of θ. [2]

(b) Find the point (in Cartesian coordinates) at $\theta = 0$. [2]

3.3 For $r = 2\sin\theta$, find $\dfrac{dx}{d\theta}$. [3]

Solutions

2.1 $x = 3\cos\frac{\pi}{2} = 0,\ y = 3\sin\frac{\pi}{2} = 3$: $(0, 3)$.

2.2 $x = 2\cos\theta\cos\theta = 2\cos^2\theta,\ y = 2\cos\theta\sin\theta$.

2.3 $r = 5,\ \theta = \frac{\pi}{2}$: $(5, \frac{\pi}{2})$.

3.1 **B** — $x = r\cos\theta$.

3.2 (a) $x = (1+\cos\theta)\cos\theta,\ y = (1+\cos\theta)\sin\theta$. (b) at $\theta = 0$: $r = 2,\ x = 2,\ y = 0$: $(2, 0)$.

3.3 $x = 2\sin\theta\cos\theta = \sin(2\theta)$; $\dfrac{dx}{d\theta} = 2\cos(2\theta)$. (Or via product rule on $2\sin\theta\cos\theta$.)