AP CALCULUS BC • EXERCISE SHEET

8.9 Volume with Disc Method: Revolving Around the x- or y-Axis

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS disc method 圆盘法

Objective

Build the skills to answer exam questions on the **disc method** — volumes of revolution with no hole.

You must be able to:

- revolve a region around an axis and integrate $\pi(\text{radius})^2$ (the **disc method** 圆盘法)
- set the radius as the distance from the curve to the axis
- integrate in x (around the x-axis) or y (around the y-axis)

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The disc method

Revolving a region under $y = f(x)$ around the x-axis makes discs of radius $f(x)$:

$$V = \pi \int_a^b \big(f(x)\big)^2 \, dx.$$

■ A worked disc volume

Revolve $y = \sqrt{x}$, $0 \le x \le 4$, around the x-axis: radius $\sqrt{x}$, so

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx = \pi \cdot 8 = 8\pi.$$

■ Around the y-axis

Revolving around the y-axis uses $x = f(y)$ and integrates in y: $V = \pi \displaystyle\int_c^d \big(x(y)\big)^2 \, dy$.

For $y = x^2$ (so $x = \sqrt{y}$) from $y = 0$ to 4: $V = \pi \int_0^4 (\sqrt{y})^2 \, dy = \pi \int_0^4 y \, dy = 8\pi$.

> ■ **Radius is a distance**
>
> The radius is always the **distance from the curve to the axis** of revolution — here just $f(x)$ or $x(y)$ because the axis is a coordinate axis.

2 Practice

Now apply the methods above.

2.1 Write the disc-method integral for revolving $y = x$ on $[0, 2]$ about the x-axis. [1]

2.2 Evaluate $\pi \displaystyle\int_0^2 x^2\,dx$. [2]

2.3 Find the volume when $y = \sqrt{x}$ on $[0, 9]$ is revolved about the x-axis. [3]

3 Exam-style questions

3.1 Revolving $y = f(x)$ about the x-axis gives the volume [1]

- **A** $\displaystyle\int_a^b f(x)\,dx$
- **B** $\pi \displaystyle\int_a^b f(x)\,dx$
- **C** $\pi \displaystyle\int_a^b \big(f(x)\big)^2\,dx$
- **D** $2\pi \displaystyle\int_a^b f(x)\,dx$

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x-axis.

(a) Write the volume integral. [2]

(b) Evaluate it. [3]

3.3 The region bounded by $x = y^2$, $y = 0$ to $y = 2$, and the y-axis is revolved about the y-axis. Find the volume. [4]

Solutions

2.1 $\pi \displaystyle\int_0^2 x^2\,dx$.

2.2 $\pi\left[\frac{x^3}{3}\right]_0^2 = \frac{8\pi}{3}$.

2.3 $\pi \displaystyle\int_0^9 x\,dx = \pi\left[\frac{x^2}{2}\right]_0^9 = \frac{81\pi}{2}$.

3.1 **C** — the disc method integrates $\pi(\text{radius})^2$.

3.2 (a) $\pi \displaystyle\int_0^2 (x^2)^2\,dx = \pi \int_0^2 x^4\,dx$. (b) $\pi\left[\frac{x^5}{5}\right]_0^2 = \frac{32\pi}{5}$.

3.3 radius $= x = y^2$; $V = \pi \displaystyle\int_0^2 (y^2)^2\,dy = \pi \int_0^2 y^4\,dy = \pi\left[\frac{y^5}{5}\right]_0^2 = \frac{32\pi}{5}$.