AP CALCULUS BC • EXERCISE SHEET

8.2 Connecting Position, Velocity, and Acceleration Using Integrals

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS total distance 总路程 • displacement 位移

Objective

Build the skills to answer exam questions on **motion with integrals** — recovering velocity and position, and finding distance.

You must be able to:

- integrate acceleration to get velocity and velocity to get position
- find **displacement** 位移 $\int_a^b v\,dt$ and **total distance** 总路程 $\int_a^b |v|\,dt$
- use an initial condition to fix the constant

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Integrating the motion chain

Acceleration integrates to velocity; velocity integrates to position:

$$v(t) = \int a(t)\,dt, \qquad s(t) = \int v(t)\,dt.$$

Each integration adds a constant, fixed by an initial condition.

■ Displacement

Displacement over $[a, b]$ is the net change in position:

$$\int_a^b v(t)\,dt = s(b) - s(a).$$

It can be negative (ended left of the start).

■ **Total distance**

Total distance counts all motion regardless of direction:

$$\int_a^b |v(t)|\,dt.$$

Split the integral where v changes sign, integrate each piece, and add the **absolute** amounts.

■ **A worked distance**

$v(t) = t - 2$ on $[0, 3]$: $v < 0$ on $[0, 2)$, $v > 0$ on $(2, 3]$. Distance $= \left|\int_0^2 (t-2)\,dt\right| + \int_2^3 (t-2)\,dt = |-2| + \frac{1}{2} = 2.5$.

2 Practice

Now apply the methods above.

2.1 A particle has velocity $v(t) = 6t$. Find its displacement over $0 \le t \le 3$. [2]

2.2 With $a(t) = 4$ and $v(0) = 1$, find $v(t)$. [2]

2.3 A particle has $v(t) = 2t - 6$. State the time it changes direction. [1]

3 Exam-style questions

3.1 The total distance travelled over $[a, b]$ is [1]

- **A** $\displaystyle\int_a^b v\,dt$
- **B** $\displaystyle\int_a^b |v|\,dt$
- **C** $v(b) - v(a)$
- **D** $\displaystyle\int_a^b a\,dt$

3.2 A particle moves with velocity $v(t) = 3t^2 - 12$ m/s.

(a) Find the displacement over $0 \le t \le 3$. [2]

(b) At what time does the particle change direction? [2]

3.3 A particle has velocity $v(t) = t - 3$ on $0 \le t \le 5$.

(a) Find the displacement. [2]

(b) Find the total distance travelled. [3]

Solutions

2.1 $\displaystyle\int_0^3 6t\,dt = \big[3t^2\big]_0^3 = 27.$

2.2 $v(t) = 4t + C$; $v(0) = C = 1$, so $v(t) = 4t + 1$.

2.3 $v = 0$ at $2t - 6 = 0$, i.e. $t = 3$.

3.1 **B** — total distance is $\int |v|\,dt$.

3.2 (a) $\displaystyle\int_0^3 (3t^2 - 12)\,dt = \big[t^3 - 12t\big]_0^3 = 27 - 36 = -9$ m. (b) $v = 0$: $3t^2 = 12 \Rightarrow t = 2$.

3.3 (a) $\displaystyle\int_0^5 (t - 3)\,dt = \big[\tfrac{t^2}{2} - 3t\big]_0^5 = 12.5 - 15 = -2.5$. (b) $v < 0$ on $[0, 3)$, $v > 0$ on $(3, 5]$; distance $= \left|\int_0^3\right| + \int_3^5 = |-4.5| + 2 = 6.5$.