

8.10 Volume with Disc Method: Revolving Around Other Axes

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS disc method 圆盘法

Objective

Build the skills to answer exam questions on the **disc method around other axes** — when the axis is a line like $y = k$.

You must be able to:

- adjust the **radius** to the distance from the curve to a non-coordinate axis
- write $R = |f(x) - k|$ for a horizontal axis $y = k$ (or $|x(y) - k|$ for $x = k$)
- integrate πR^2 as before

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Radius to a shifted axis

When the axis of revolution is $y = k$ (not the x -axis), the radius is the **distance from the curve to that line**:

$$R = |f(x) - k|, \quad V = \pi \int_a^b (f(x) - k)^2 dx.$$

■ A worked example

Revolve $y = \sqrt{x}$, $0 \leq x \leq 4$, about the line $y = -1$. The radius is $\sqrt{x} - (-1) = \sqrt{x} + 1$:

$$V = \pi \int_0^4 (\sqrt{x} + 1)^2 dx = \pi \int_0^4 (x + 2\sqrt{x} + 1) dx.$$

■ Revolving about a vertical line

For an axis $x = k$, use $R = |x(y) - k|$ and integrate in y . Always draw a picture and measure the radius from the curve **to** the axis.

■ **Watch the direction**

If the region lies **below** the axis line, the distance is $k - f(x)$; squaring makes the sign irrelevant, but set the radius as a genuine (positive) distance.

2 Practice

Now apply the methods above.

2.1 A curve $y = f(x)$ is revolved about $y = 3$. Write the radius. [1]

2.2 Revolving $y = 2$ (constant) region about $y = -1$: state the radius. [1]

2.3 Expand $(\sqrt{x} + 1)^2$. [1]

3 Exam-style questions

3.1 Revolving $y = f(x)$ about the line $y = k$ uses radius [1]

- **A** $f(x)$
 - **B** $f(x) - k$
 - **C** k
 - **D** $f(x) + k$ always
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3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.

(a) Write the radius $R(x)$. [1]

(b) Set up the volume integral. [2]

3.3 The region under $y = \sqrt{x}$ on $[0, 4]$ is revolved about the line $y = 2$. Set up the volume

integral for the solid (radius = $2 - \sqrt{x}$).

[3]

Solutions

2.1 $R = |f(x) - 3|$.

2.2 $R = 2 - (-1) = 3$.

2.3 $x + 2\sqrt{x} + 1$.

3.1 B — the radius is the distance $f(x) - k$ to the line.

3.2 (a) $R = x^2 - (-1) = x^2 + 1$. (b) $\pi \int_0^2 (x^2 + 1)^2 dx$.

3.3 $R = 2 - \sqrt{x}$; $V = \pi \int_0^4 (2 - \sqrt{x})^2 dx$.