AP CALCULUS BC • EXERCISE SHEET

7.2 Verifying Solutions for Differential Equations

Name: ______________ Class: __________ Date: ____________ **Total: 12 marks**

KEY WORDS initial condition 初始条件 • differential equation 微分方程

Objective

Build the skills to answer exam questions on **verifying solutions of differential equations** — checking that a function satisfies a given equation.

You must be able to:

- differentiate a proposed solution and **substitute** it into the equation
- confirm both sides are equal for all x (or t)
- check that an **initial condition** 初始条件 is also met

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Substitute and compare

To verify $y = e^{2x}$ solves $\dfrac{dy}{dx} = 2y$: differentiate, $\dfrac{dy}{dx} = 2e^{2x}$; the right side is $2y = 2e^{2x}$. They match, so $y = e^{2x}$ **is** a solution.

A solution with a constant

$y = Ce^{kt}$ solves $\dfrac{dy}{dt} = ky$ for **any** constant C: $\dfrac{dy}{dt} = kCe^{kt} = ky$. This is the general solution; each C gives one curve.

Checking an implicit solution

To check $y^2 = x^2 + C$ solves $\dfrac{dy}{dx} = \dfrac{x}{y}$: differentiate implicitly, $2y\dfrac{dy}{dx} = 2x$, so $\dfrac{dy}{dx} = \dfrac{x}{y}$. Verified.

Including an initial condition

If asked to verify a **particular** solution, also check the point. $y = 3e^{2x}$ with $y(0) = 3$: at $x = 0$, $y = 3e^0 = 3$. The condition holds.

2 Practice

Now apply the methods above.

2.1 Verify that $y = e^{5x}$ satisfies $\dfrac{dy}{dx} = 5y$. [2]

2.2 Show that $y = 4e^{-t}$ satisfies $\dfrac{dy}{dt} = -y$. [2]

2.3 For $y = Ce^{3x}$, state $\dfrac{dy}{dx}$ in terms of y. [1]

3 Exam-style questions

3.1 Which function satisfies $\dfrac{dy}{dx} = 2y$? [1]

- **A** $y = 2x$
- **B** $y = x^2$
- **C** $y = e^{2x}$
- **D** $y = 2e^x$

3.2 Consider $y = x^2 + 3x + C$.

(a) Show that $\dfrac{dy}{dx} = 2x + 3$. [1]

(b) Find the value of C so that the solution passes through $(1, 6)$. [2]

3.3 Verify that $y = \sin(2x)$ satisfies $\dfrac{d^2y}{dx^2} = -4y$. [3]

Solutions

2.1 $\dfrac{dy}{dx} = 5e^{5x}$; this equals $5y = 5e^{5x}$, so it is a solution.

2.2 $\dfrac{dy}{dt} = -4e^{-t}$; and $-y = -4e^{-t}$, so they match.

2.3 $\dfrac{dy}{dx} = 3Ce^{3x} = 3y$.

3.1 **C** — $\dfrac{d}{dx}e^{2x} = 2e^{2x} = 2y$.

3.2 (a) $\dfrac{dy}{dx} = 2x + 3$. (b) $6 = 1 + 3 + C \Rightarrow C = 2$.

3.3 $\dfrac{dy}{dx} = 2\cos(2x)$; $\dfrac{d^2y}{dx^2} = -4\sin(2x)$; this equals $-4y = -4\sin(2x)$.