6.5 Interpreting the Behavior of Accumulation Functions

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS local extrema 极值 • accumulation function 累积函数 • accumulation 累积

Objective

Build the skills to answer exam questions on the **behavior of accumulation functions** — reading features of $g(x) = \int_a^x f(t)\,dt$ from a graph of f.

You must be able to:

- decide where g is **increasing/decreasing** from the sign of f
- locate **local extrema 极值** of g where f crosses zero (sign change)
- decide **concavity** of g from whether f is increasing or decreasing

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ g rises where f is positive

Because $g'(x) = f(x)$, the accumulation function g **increases** on intervals where the graph of f is **above** the axis and **decreases** where f is **below** the axis.

■ Extrema of g at zeros of f

g has a **local maximum** where f changes from $+$ to $-$, and a **local minimum** where f changes from $-$ to $+$. A zero of f where the sign does **not** change is not an extremum of g.

■ Concavity of g from the slope of f

$g''(x) = f'(x)$, so g is **concave up** where f is **increasing** and **concave down** where f is **decreasing**. A point where f has a local max or min is a **point of inflection** of g.

■ Comparing values of g

To compare $g(3)$ and $g(0)$, look at the net signed area of f between them: $g(3) - g(0) = \int_0^3 f\,dt$. More area above the axis than below makes $g(3) > g(0)$.

2 Practice

Now apply the methods above. Let $g(x) = \displaystyle\int_0^x f(t)\,dt$, where f is continuous.

2.1 On an interval where $f(t) < 0$, state whether g is increasing or decreasing. [1]

2.2 At a point where f changes from positive to negative, what feature does g have? [1]

2.3 On an interval where f is increasing, state the concavity of g. [1]

3 Exam-style questions

3.1 $g(x) = \int_0^x f(t)\,dt$. The graph of f crosses from below the axis to above it at $x = 4$. At $x = 4$, g has a [1]

- **A** local maximum
- **B** local minimum
- **C** point of inflection
- **D** vertical asymptote

3.2 The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(a) On what interval is g increasing? [1]

(b) At what x does g attain a local maximum? Justify. [2]

3.3 For $g(x) = \int_0^x f(t)\,dt$, the graph of f is increasing on $(1, 5)$. State, with a reason, the concavity of g on $(1, 5)$. [2]

Solutions

2.1 Decreasing — $g'(x) = f(x) < 0$.

2.2 A local maximum.

2.3 Concave up — $g''(x) = f'(x) > 0$.

3.1 **B** — f changes $-$ to $+$, so g' changes sign the same way and g has a local minimum.

3.2 (a) g increases on $(0, 3)$ where $f > 0$. (b) At $x = 3$, because f (which is g') changes from positive to negative there.

3.3 Concave up — $g''(x) = f'(x) > 0$ since f is increasing.