

4.4 Introduction to Related Rates

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS time 时间 • related rates 相关变化率

Objective

Build the skills to answer exam questions on **introduction to related rates**.

You must be able to:

- differentiate a relation with respect to **time** 时间 to link two rates

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Related rates

When two quantities are related, their rates are related: differentiate the relation with respect to t (using the chain rule).

■ Example

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

2 Practice

2.1 State the first step in a related-rates problem. [1]

2.2 Differentiate $A = \pi r^2$ with respect to t . [2]

2.3 State what links the two rates. [1]

3 Exam-style questions

3.1 In related rates, you differentiate the relation with respect to [1]

- **A** x
 - **B** time t
 - **C** r
 - **D** nothing
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3.2 Differentiating $A = \pi r^2$ with respect to t gives [1]

- **A** $2\pi r$
 - **B** $2\pi r \frac{dr}{dt}$
 - **C** πr^2
 - **D** $2r$
-

3.3 $V = s^3$ (a cube).

(a) Differentiate with respect to t . [1]

(b) State the chain-rule factor. [1]

(c) Write $\frac{dV}{dt}$. [1]

Solutions

2.1 write the relation connecting the quantities.

2.2 $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.

2.3 the derivative of the relation with respect to t .

3.1 B.

3.2 B.

3.3 (a) $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$. (b) $\frac{ds}{dt}$. (c) $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$.