

3.2 Implicit Differentiation

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS implicit differentiation 隐函数求导 • implicit relation 隐式关系 • chain rule 链式法则

Objective

Build the skills to answer exam questions on **implicit differentiation**.

You must be able to:

- differentiate an **implicit relation** 隐式关系 by attaching $\frac{dy}{dx}$ to every y
- solve for $\frac{dy}{dx}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Implicit differentiation

Differentiate both sides with respect to x , writing y' each time you differentiate y (the chain rule), then solve for y' .

■ Example

$$x^2 + y^2 = 25 \Rightarrow 2x + 2y y' = 0 \Rightarrow y' = -\frac{x}{y}.$$

2 Practice

2.1 State what to attach when differentiating y . [1]

2.2 Differentiate $x^2 + y^2 = 25$ implicitly. [2]

2.3 State the final step. [1]

3 Exam-style questions

3.1 In implicit differentiation, differentiating y^2 gives [1]

- **A** $2y$
 - **B** $2y y'$
 - **C** y'
 - **D** $2x$
-

3.2 After differentiating, you [1]

- **A** stop
 - **B** solve for $\frac{dy}{dx}$
 - **C** set $y = 0$
 - **D** integrate
-

3.3 $x^2 + y^2 = 25$.

(a) Differentiate both sides. [1]

(b) Isolate the y' term. [1]

(c) Solve for y' . [1]

Solutions

2.1 the factor $\frac{dy}{dx}$ (i.e. y').

2.2 $2x + 2y y' = 0$, so $y' = -\frac{x}{y}$.

2.3 solve the equation for $\frac{dy}{dx}$.

3.1 B.

3.2 B.

3.3 (a) $2x + 2y y' = 0$. (b) $2y y' = -2x$. (c) $y' = -\frac{x}{y}$.