

3.1 The Chain Rule

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS chain rule 链式法则 • composite function 复合函数

Objective

Build the skills to answer exam questions on **the chain rule**.**You must be able to:**

- differentiate a **composite function** 复合函数 with the **chain rule** 链式法则

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

— derivative of the outer (evaluated at the inner) times the derivative of the inner.

■ Example

$$\frac{d}{dx}[\sin(x^2)] = \cos(x^2) \cdot 2x = 2x \cos(x^2).$$

2 Practice

2.1 State the chain rule. [1]

2.2 Differentiate $(x^2 + 1)^3$. [2]

2.3 State what the chain rule is for. [1]

3 Exam-style questions

3.1 $\frac{d}{dx}[f(g(x))]$ equals [1]

- **A** $f'(x)g'(x)$
 - **B** $f'(g(x)) \cdot g'(x)$
 - **C** $f'(g(x))$
 - **D** $g'(f(x))$
-

3.2 $\frac{d}{dx}[\sin(x^2)]$ equals [1]

- **A** $\cos(x^2)$
 - **B** $2x \cos(x^2)$
 - **C** $2x \sin(x^2)$
 - **D** $\cos(2x)$
-

3.3 $y = (3x + 1)^4$.

- (a) Identify the outer and inner functions. [1]
- (b) Apply the chain rule. [1]
- (c) State y' . [1]

Solutions

2.1 $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$

2.2 $3(x^2 + 1)^2 \cdot 2x = 6x(x^2 + 1)^2.$

2.3 differentiating a composite (a function of a function).

3.1 B.

3.2 B.

3.3 (a) outer u^4 , inner $3x + 1$. (b) $4(3x + 1)^3 \cdot 3$. (c) $y' = 12(3x + 1)^3.$