AP CALCULUS BC • EXERCISE SHEET

10.2 Working with Geometric Series

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS ratio 比值 • converges 收敛 • geometric series 几何级数 • diverges 发散

Objective

Build the skills to answer exam questions on **geometric series** — the one series you can sum exactly.

You must be able to:

- identify the first term a and common **ratio 比值** r
- apply the convergence test $|r| < 1$
- compute the sum $\dfrac{a}{1-r}$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The geometric sum

A geometric series $\sum_{n=0}^{\infty} ar^n$ converges **exactly when** $|r| < 1$, and then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

Reading a and r

For $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$: $a = 3$, $r = \frac{1}{2}$. Since $|r| < 1$, sum $= \dfrac{3}{1-\frac{1}{2}} = 6$.

Watch the starting index

If the sum starts at $n = 1$, the first term is ar, not a. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \dfrac{1/3}{1-1/3} = \dfrac{1/3}{2/3} = \frac{1}{2}$.

■ **Divergence**

If $|r| \geq 1$ the terms do not shrink, so the series diverges. $\sum 2^n$ has $r = 2$, diverges.

2 Practice

Now apply the methods above.

2.1 State a and r for $\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$. [2]

2.2 Find the sum of $\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$. [2]

2.3 Does $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ converge? Give a reason. [1]

3 Exam-style questions

3.1 A geometric series $\sum ar^n$ converges when [1]

- **A** $r > 0$
- **B** $|r| < 1$
- **C** $|r| > 1$
- **D** $a < 1$

3.2 Consider $\sum_{n=0}^{\infty} 6\left(-\frac{1}{2}\right)^n$.

(a) State a and r. [2]

(b) Find the sum. [2]

3.3 Find the sum of $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$ (note the index starts at 1). [3]

Solutions

2.1 $a = 5$, $r = \frac{1}{3}$.

2.2 $\dfrac{5}{1 - 1/3} = \dfrac{5}{2/3} = \frac{15}{2}$.

2.3 No — $|r| = \frac{5}{4} > 1$, so it diverges.

3.1 **B** — converges iff $|r| < 1$.

3.2 (a) $a = 6$, $r = -\frac{1}{2}$. (b) $\dfrac{6}{1 - (-\frac{1}{2})} = \dfrac{6}{3/2} = 4$.

3.3 First term ($n = 1$) is $4 \cdot \frac{2}{5} = \frac{8}{5}$, ratio $\frac{2}{5}$; sum $= \dfrac{8/5}{1 - 2/5} = \dfrac{8/5}{3/5} = \frac{8}{3}$.