10.14 Finding Taylor or Maclaurin Series for a Function

Name: ____________ Class: ________ Date: __________ **Total: 12 marks**

KEY WORDS geometric series 几何级数

Objective

Build the skills to answer exam questions on **finding Taylor and Maclaurin series**.

You must be able to:

- recall the key **Maclaurin series** for e^x, $\sin x$, $\cos x$, $\dfrac{1}{1-x}$
- build new series by **substitution** into a known one
- write a series in summation form

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The must-know Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

■ Building by substitution

For e^{x^2}, substitute x^2 into the e^x series:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}.$$

■ Another substitution

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^n x^n.$$

■ **Writing out terms**

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$. Being able to write the first few terms **and** the general summation form is expected.

2 Practice

Now apply the methods above.

2.1 Write the Maclaurin series for e^x (summation form). [1]

2.2 Write the first three nonzero terms of $\cos x$. [2]

2.3 Use substitution to write the Maclaurin series for e^{-x}. [2]

3 Exam-style questions

3.1 The Maclaurin series for $\dfrac{1}{1-x}$ is [1]

- **A** $\sum(-1)^n x^n$
- **B** $\sum x^n$
- **C** $\sum \dfrac{x^n}{n!}$
- **D** $\sum nx^n$

3.2 Find the Maclaurin series for $\dfrac{1}{1+x^2}$ (substitute into the geometric series). [3]

3.3 Find the first three nonzero terms of the Maclaurin series for $x \sin x$. [3]

Solutions

2.1 $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

2.2 $1 - \dfrac{x^2}{2} + \dfrac{x^4}{24}$.

2.3 $\displaystyle\sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$.

3.1 B — $\dfrac{1}{1-x} = \sum x^n$.

3.2 $\displaystyle\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$.

3.3 $\sin x = x - \dfrac{x^3}{6} + \dfrac{x^5}{120} - \cdots$; multiply by x: $x^2 - \dfrac{x^4}{6} + \dfrac{x^6}{120} - \cdots$.